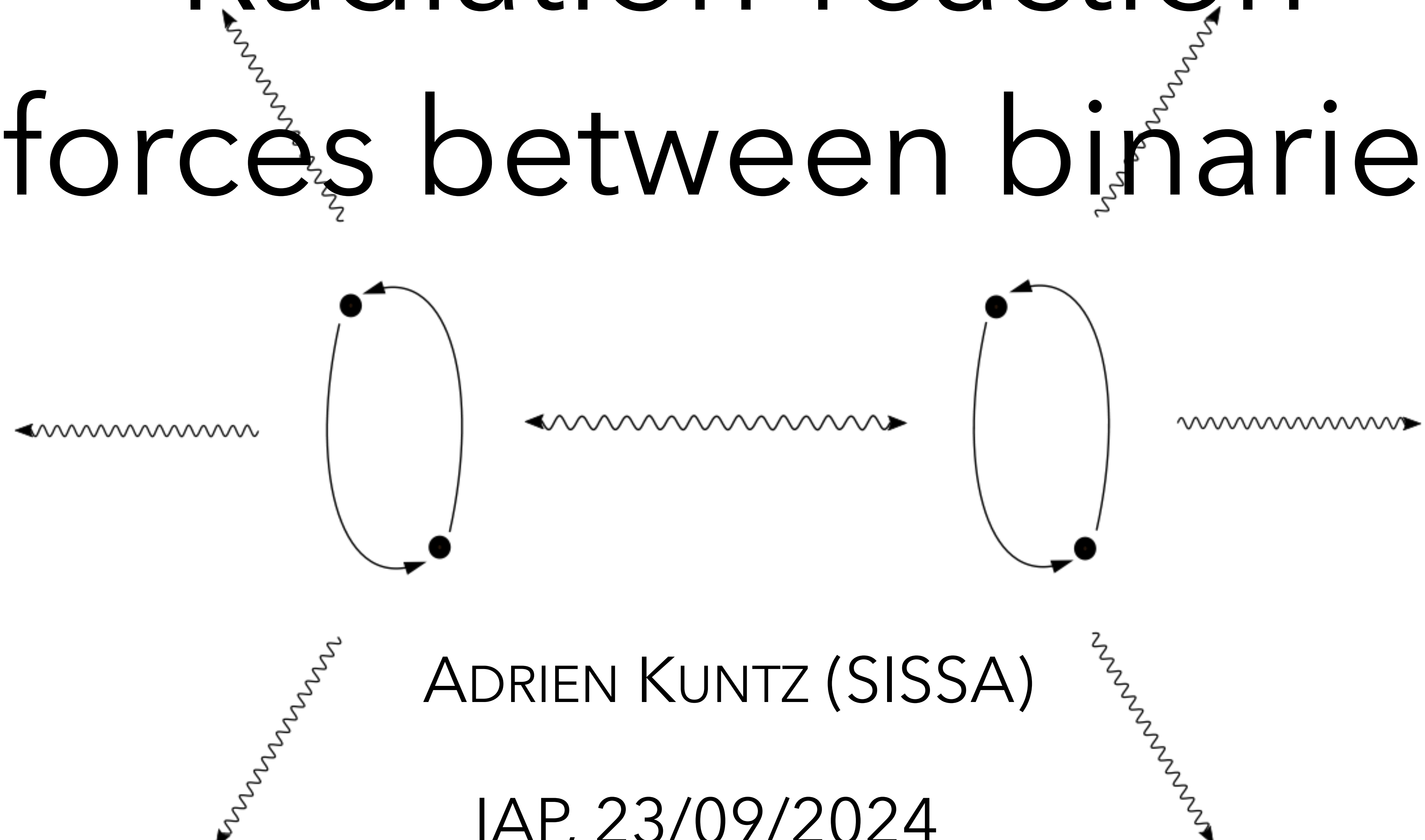
Radiation-reaction forces between binaries



Adrien Kuntz (SISSA)

IAP, 23/09/2024

Based on ArXiv:2302.08518

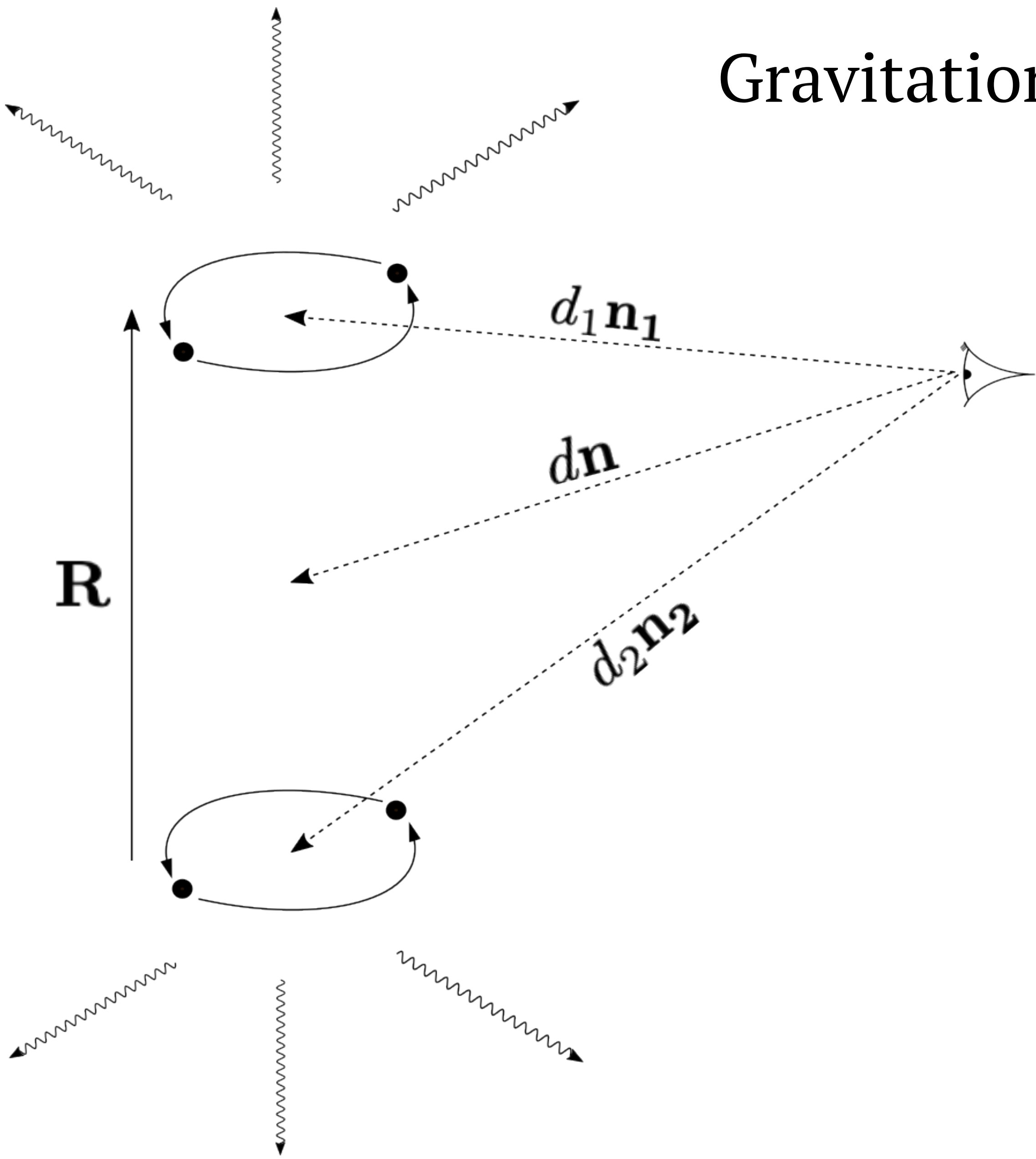
H2020-MSCA-RISE-2020 GRU 101007855

ERC-2018-COG GRAMS 815673

Interference of GW

Gravitational waves are… waves

Interference of GW



Gravitational waves are… waves

$$h_\times^{(1)} = \frac{\mathcal{A}}{d_1}\sin\left(2\omega(t-d_1)+2\phi_1\right)$$

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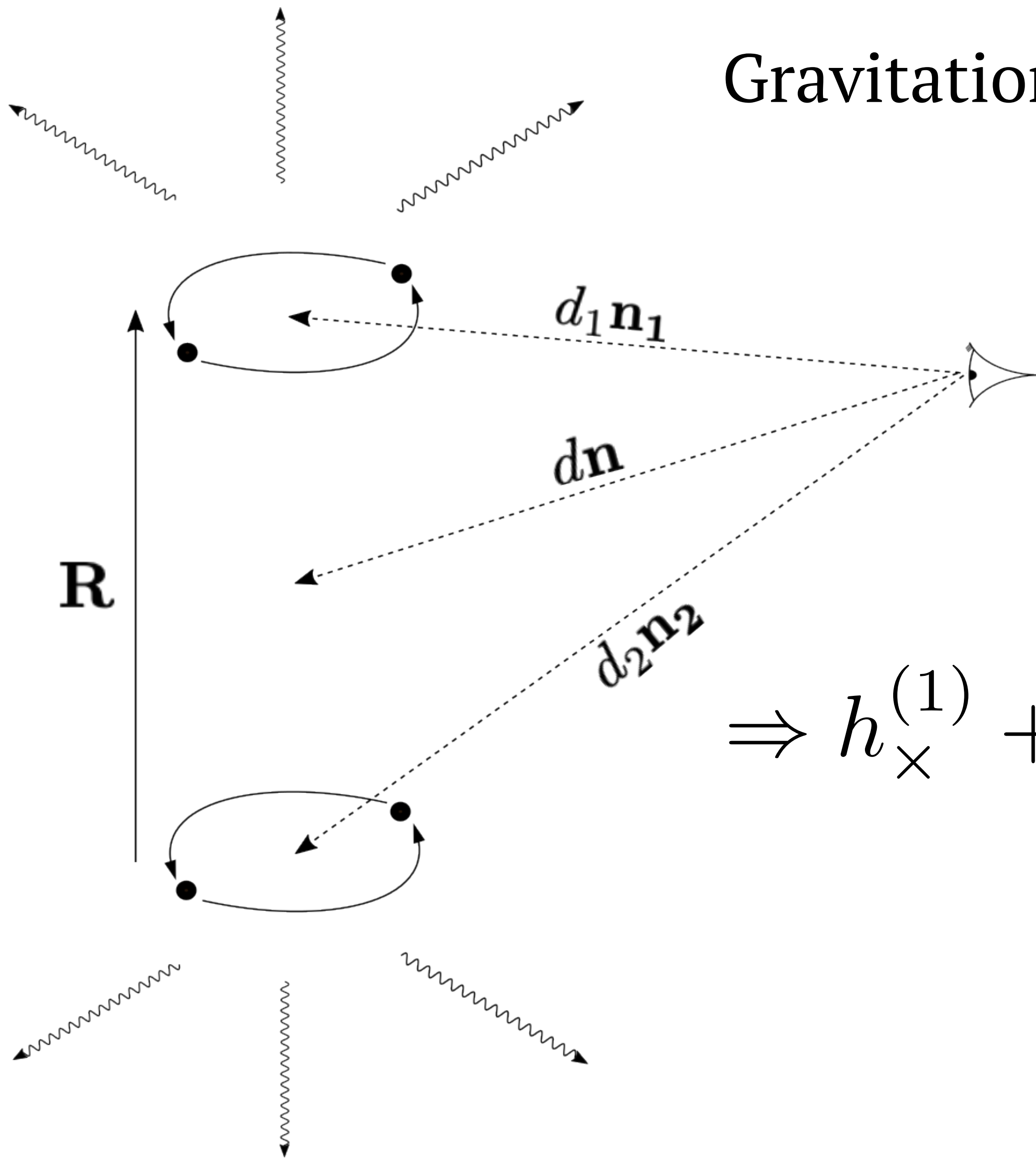

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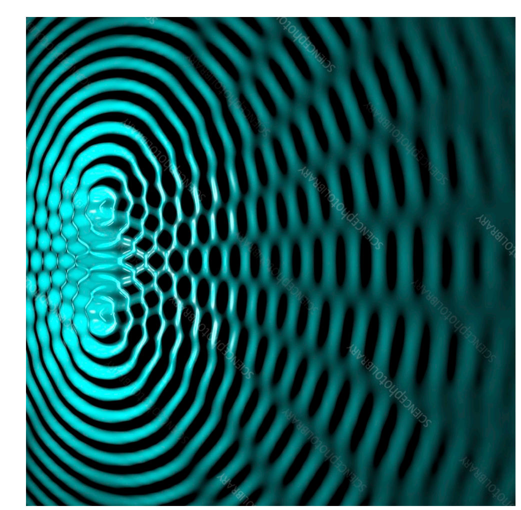
$$\Rightarrow h_\times^{(1)} + h_\times^{(2)} \simeq \frac{2\mathcal{A}}{d}\sin\left(2\omega(t-d)+\phi_1+\phi_2\right)$$

$$\times\cos\left(\phi_2-\phi_1+\omega\mathbf{n}\cdot\mathbf{R}\right)$$

Interference pattern!



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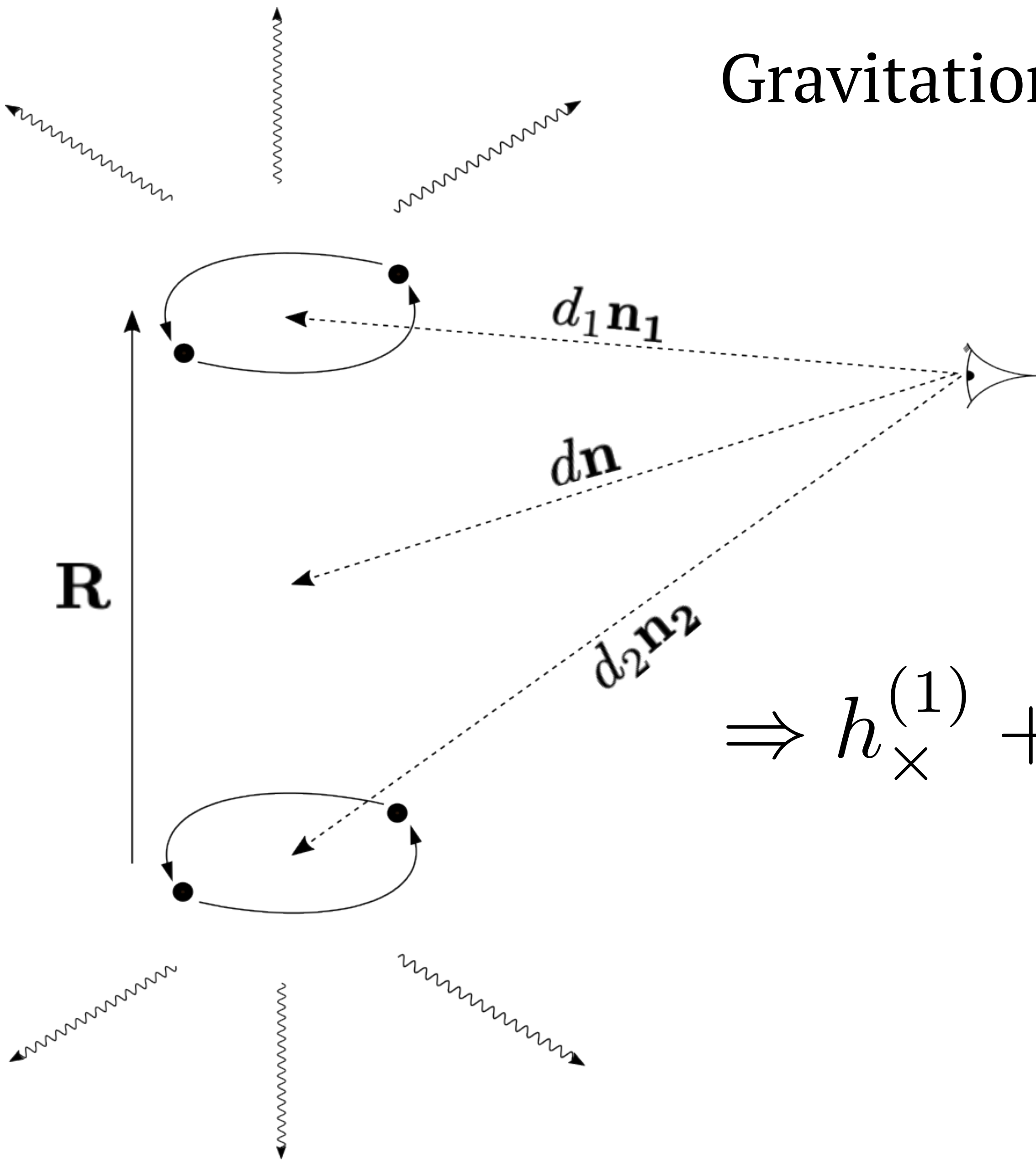

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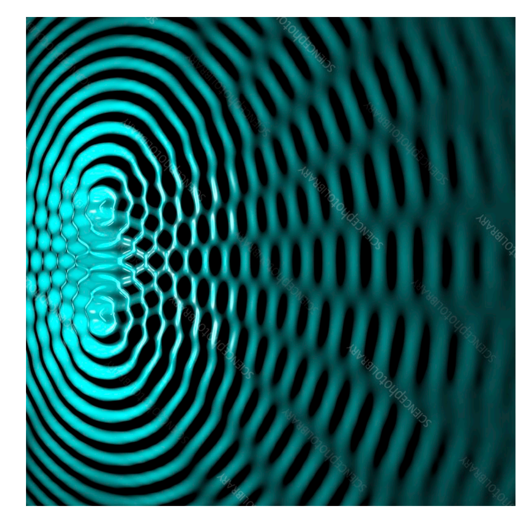

If $\phi_2 = \phi_1 + \dfrac{\pi}{2}$ and $\omega R \ll 1 \Leftrightarrow R \ll \lambda_{\mathrm{GW}}$:

Destructive interference everywhere!

RADIATION-REACTION FORCES

In the case of coherent emission $\omega R \ll 1$:

Energy loss is modified!

$$\frac{\mathrm{d}a_1}{\mathrm{d}t} = \left.\frac{\mathrm{d}a_1}{\mathrm{d}t}\right|_{\mathrm{Peters}} \left(1 + \cos(\phi_1 - \phi_2)\right)$$

Even if the quadrupole emission is suppressed, there will be octupolar waves

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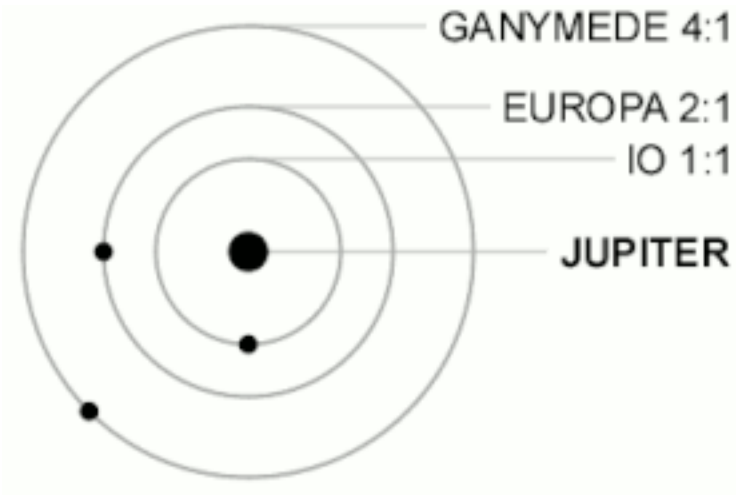
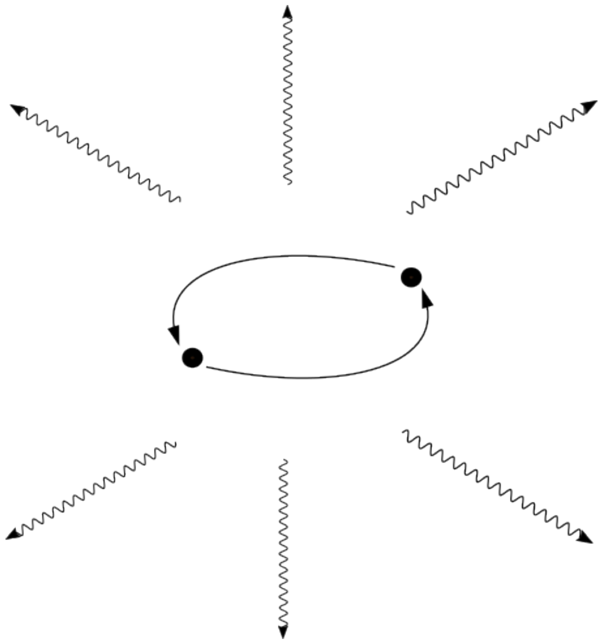


Can such a system exist in Nature? Is the resonance stable?

$\Rightarrow$ Backreaction of GW

Backreaction of GW

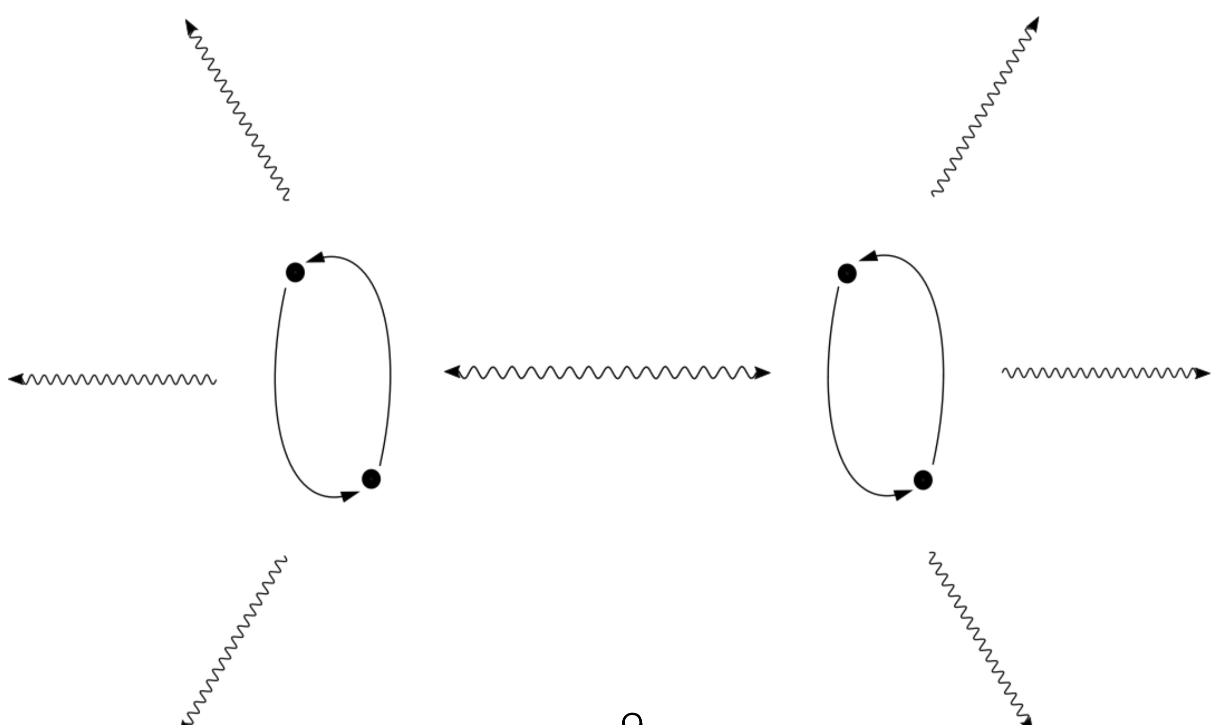
For a two-body system it's easy:



$$\frac{\mathrm{d}E}{\mathrm{d}t} = -\dot{E}_{\mathrm{GW}} \qquad \frac{\mathrm{d}L}{\mathrm{d}t} = -\dot{L}_{\mathrm{GW}}$$

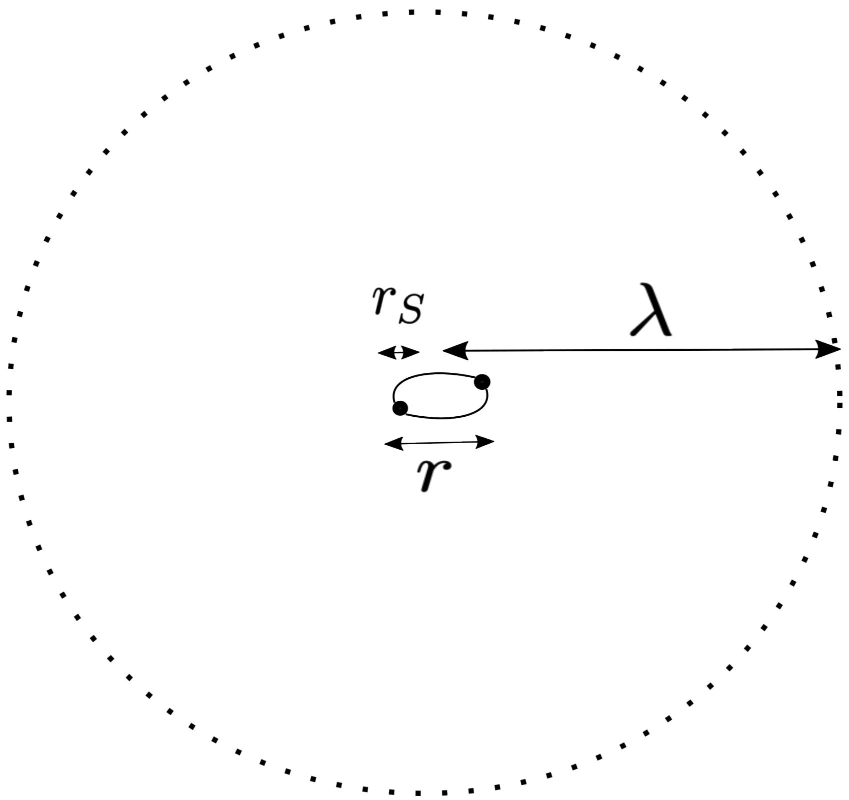
$$\Rightarrow \dot{a} \text{ and } \dot{e}$$

In a four-body system this is far from obvious!



Effective field theory tools

Setup: tools for 2-body problem



Full theory:

$$\mathrm{d}s^2 = -f(r)\mathrm{d}t^2 + f(r)^{-1}\mathrm{d}r^2 + r^2\mathrm{d}\Omega^2$$

Integrate out scale r_S: point-particle model

$$S_{\mathrm{pp}} = -m\int \mathrm{d}\tau = -m\int \mathrm{d}t\sqrt{-g_{\mu\nu}v^\mu v^\nu}$$

EFFECTIVE FIELD THEORY TOOLS

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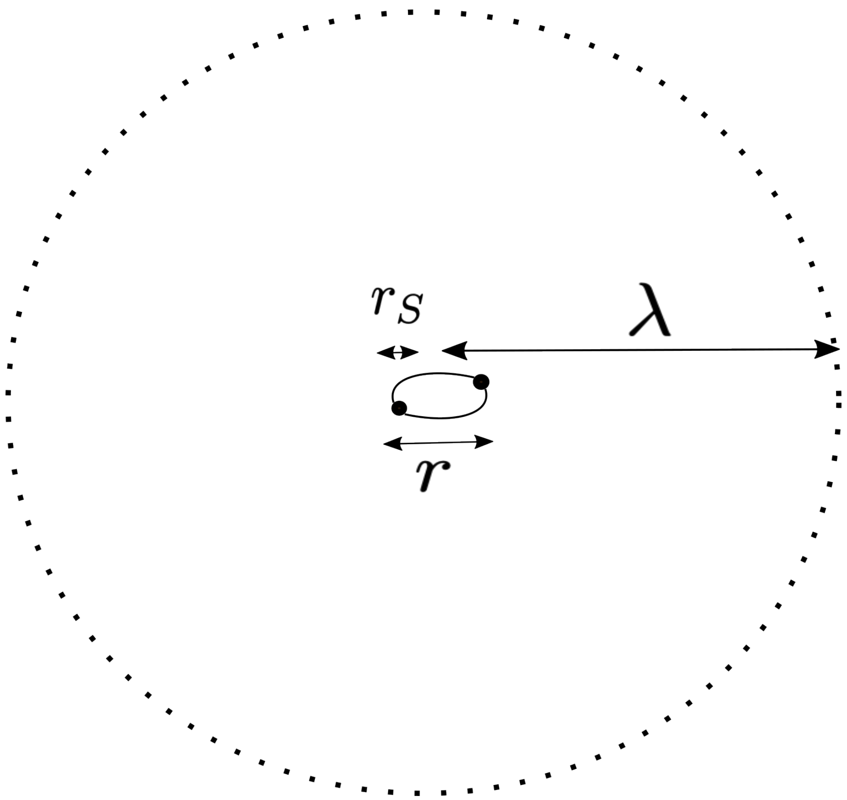

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Integrate out scale r_S: point-particle model

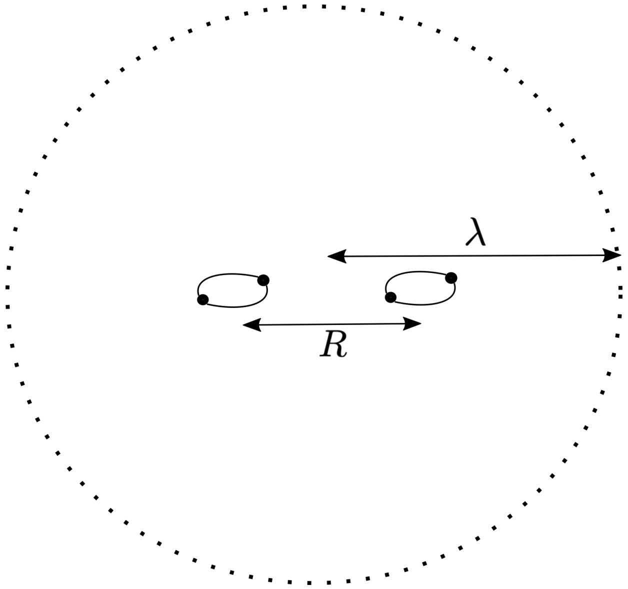
$$S_{\rm pp} = -m\int \mathrm{d}\tau = -m\int \mathrm{d}t\sqrt{-g_{\mu\nu}v^\mu v^\nu}$$

Integrate out scale r : binary as a point-particle coupled to radiative modes

$$S_{\rm binary} = \int \mathrm{d}\tilde{\tau}\left[-\mathcal{E} + \frac{1}{2}J_{\mu\nu}\Omega^{\mu\nu} + \frac{1}{2}E_{ij}Q^{ij} + \ldots\right]$$

EFT WITH 4 BODIES

PHYSICAL SCALES



All bodies lie in the Near Zone: « Standard » PN tools

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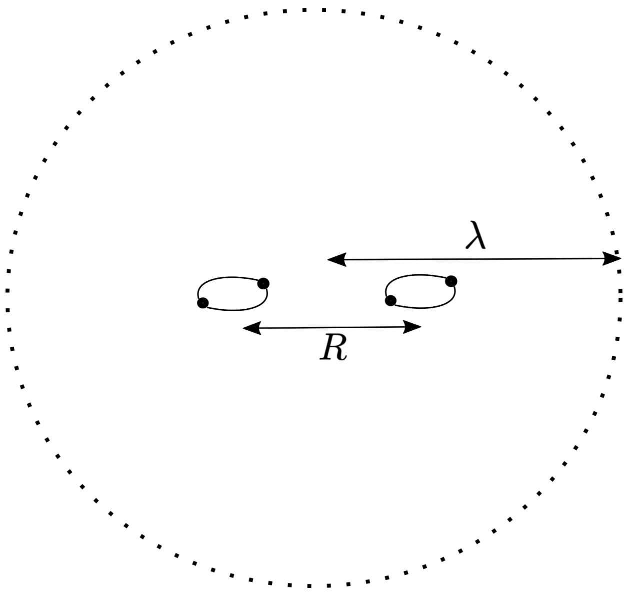

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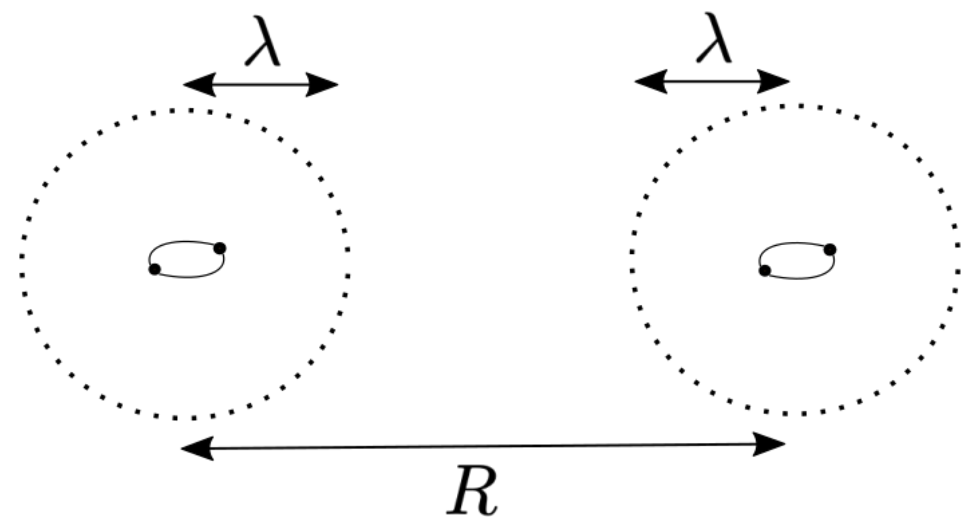
Scattering of GW



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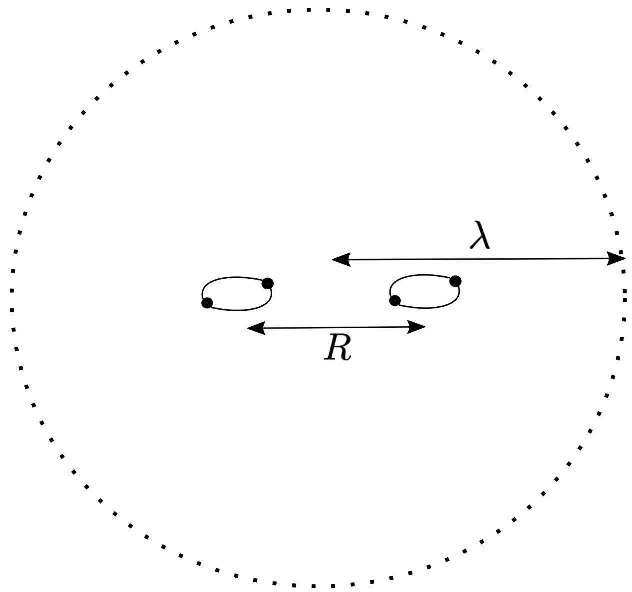

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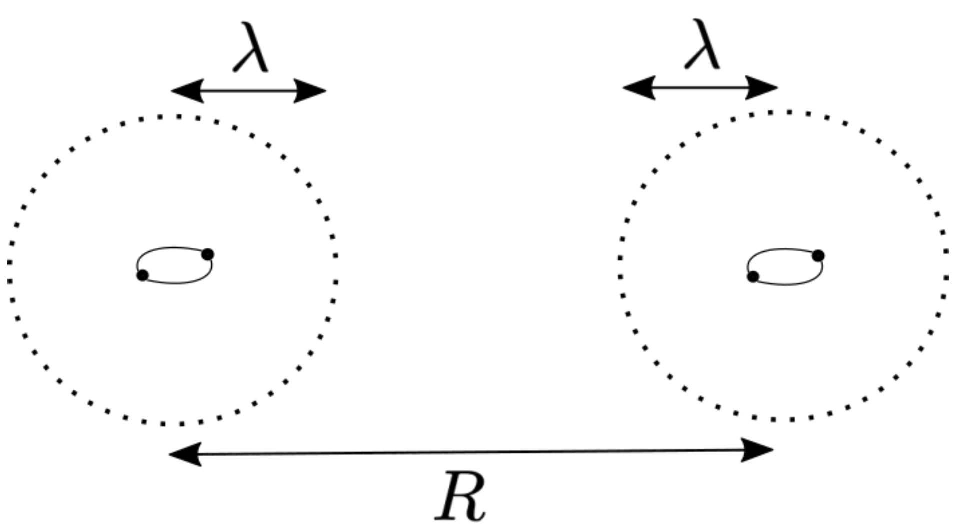

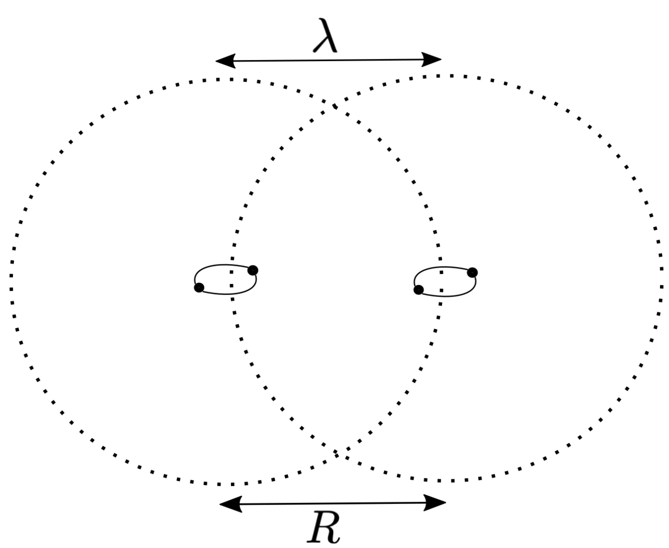

??

« Potential » (conservative) gravitons at scale R mix with radiative gravitons at scale λ

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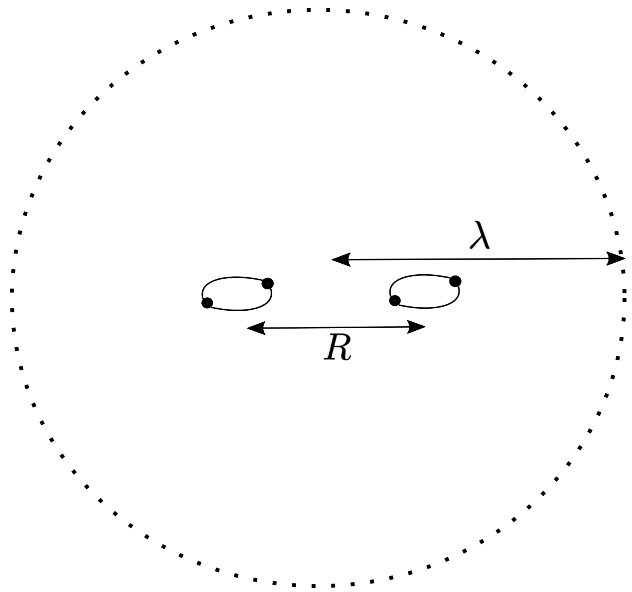

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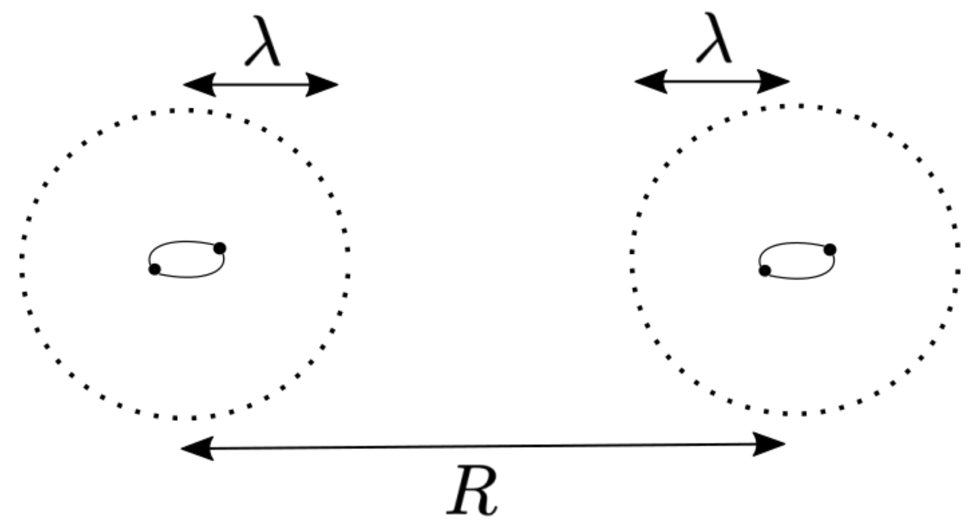
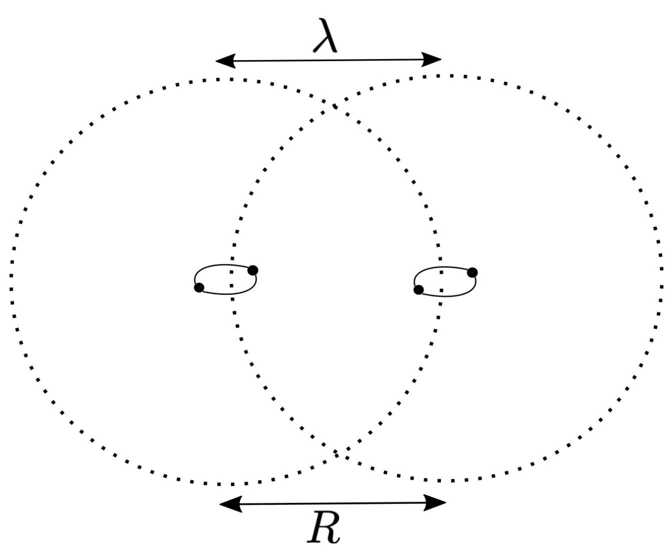




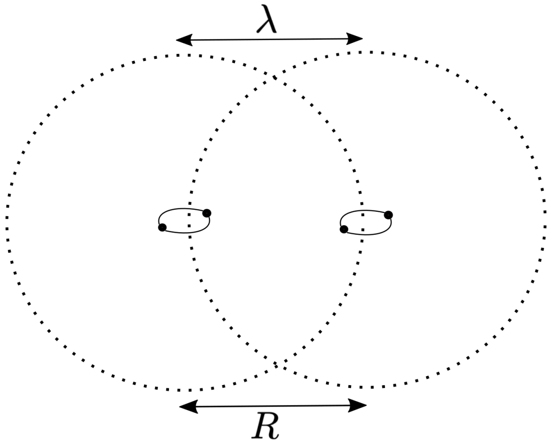
?? « Potential » (conservative) gravitons at scale R mix with radiative gravitons at scale λ

Seems an abstract discussion?

Globular clusters: average distance between 2 binaries: 0.2 Pc

$\lambda \sim 0.2\ \mathrm{Pc} \Rightarrow$ Binary merge in 1 Hubble time (with KL mechanism)

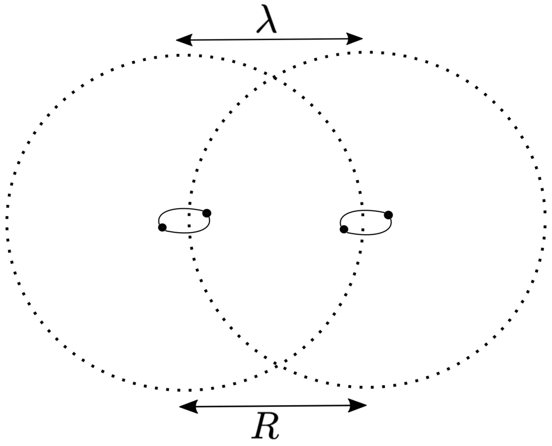
EFT WITH 4 BODIES



ACTION FOR A 4-BODY SYSTEM

$$S_{4-\mathrm{body}} = -\frac{1}{2}\int \mathrm{d}\tilde{\tau}\left[E_{ij}(\mathbf{X}_1)Q_1^{ij} + E_{ij}(\mathbf{X}_2)Q_2^{ij}\right]$$

EFT WITH 4 BODIES



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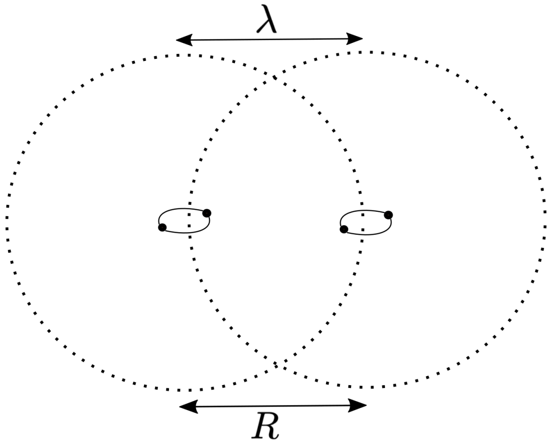
PN expansion:

$$Q_1^{ij}(t-\frac{r}{c}) \simeq Q_1^{ij}(t) - \frac{r}{c}\partial_t Q_1^{ij} + \dots$$

$$\frac{r}{c}\partial_t \sim \frac{v}{c} \sim \frac{r}{\lambda} \ll 1$$

EFT WITH 4 BODIES



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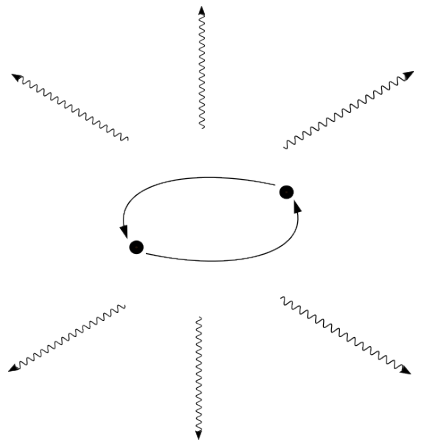
In the 4-body force: $$Q_1^{ij}(t-\frac{R}{c}) \neq Q_1^{ij}(t) - \frac{R}{c}\partial_t Q_1^{ij} + \dots$$

$$\frac{R}{c}\partial_t \sim \frac{R}{\lambda}$$

RR FORCE FOR A TWO-BODY SYSTEM

Dissipation of energy: needs Keldysh formalism

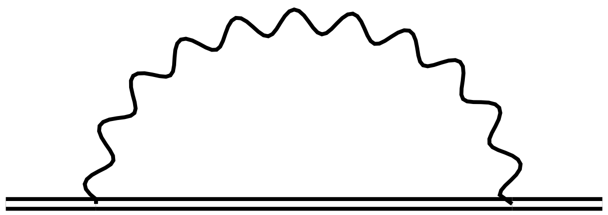


$$S = -\frac{1}{2}\int \mathrm{d}\tilde{\tau} E^{a}{}_{ij} Q^{ij}_{a} \qquad a = \pm$$

Propagator: $\langle h_{a,\mu\nu} h_{b,\alpha\beta} \rangle = P_{\mu\nu;\alpha\beta} D_{ab}(x-y)$

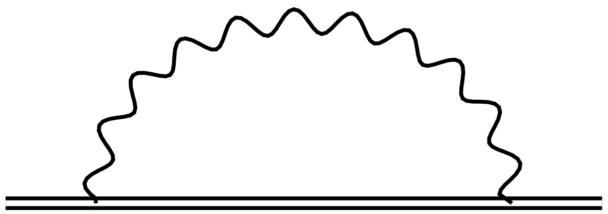
$$D_{ab}(x-y) = \begin{pmatrix} 0 & -iD^{\mathrm{adv}}(x-y) \\ -iD^{\mathrm{ret}}(x-y) & 0 \end{pmatrix}$$

Feynman diagram:

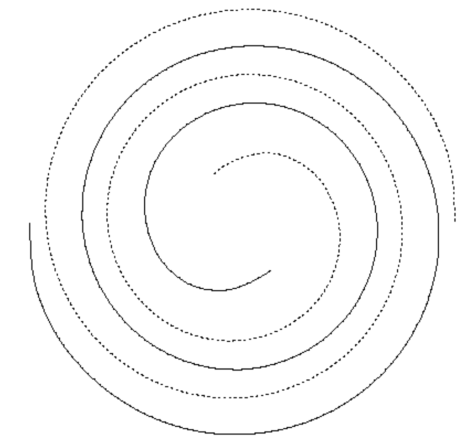


$$\frac{1}{2}\frac{i^2}{4}\int \mathrm{d}t_1 \mathrm{d}t_2 \; Q^{ij,a}(t_1) Q^{kl,b}(t_2) \langle R_{i0j0,a}(t_1, \mathbf{X}(t_1)) R_{k0l0,b}(t_2, \mathbf{X}(t_2)) \rangle$$

RR force for a two-body system



$$= -\frac{iG}{5}\int \mathrm{d}t\; Q^{-}_{ij}\frac{\mathrm{d}^5}{\mathrm{d}t^5}Q^{ij,+}$$

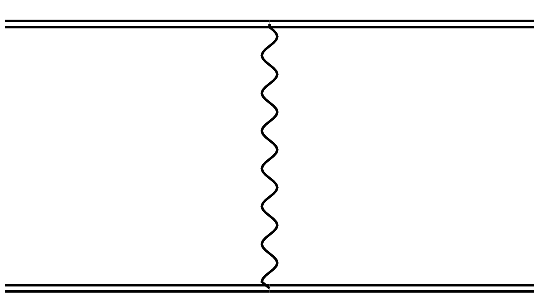


RR force: vary wrt -

$$\Rightarrow F^i_{\mathrm{A}} = -\frac{2\mu G}{5}\frac{\mathrm{d}^5 Q^{ij}}{\mathrm{d}t^5}r_j$$

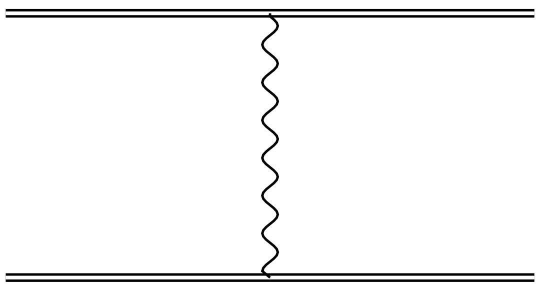
(Burke-Thorne)

Four bodies



$$\frac{i^2}{4}\int \mathrm{d}t_1\mathrm{d}t_2\; Q_1^{ij,a}(t_1)Q_2^{kl,b}(t_2)\big\langle R_{i0j0,a}\big(t_1,\mathbf{X}_1(t_1)\big)R_{k0l0,b}\big(t_2,\mathbf{X}_2(t_2)\big)\big\rangle$$

RR FORCE FOR A FOUR-BODY SYSTEM



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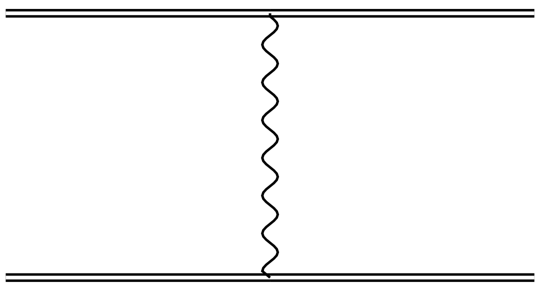
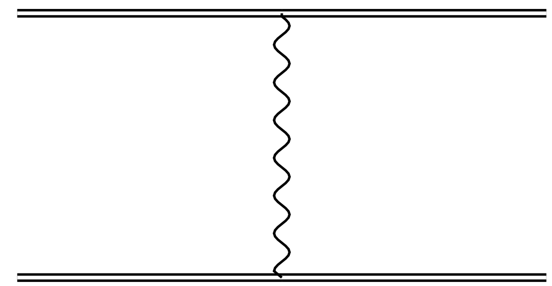


$$
\begin{aligned}
S_{\rm BB} = \frac{G}{2}\int \mathrm{d}t\, \frac{Q_1^{ij,-}}{R}\, &\left[\frac{\mathrm{d}^4}{\mathrm{d}t^4}Q^+_{2,ij} + \frac{2}{R}\frac{\mathrm{d}^3}{\mathrm{d}t^3}Q^+_{2,ij} + \frac{3}{R^2}\frac{\mathrm{d}^2}{\mathrm{d}t^2}Q^+_{2,ij} + \frac{3}{R^3}\frac{\mathrm{d}}{\mathrm{d}t}Q^+_{2,ij} + \frac{3}{R^4}Q^+_{2,ij}\right]\Bigg|_{t-R} \\
- Q_1^{ij,-}\frac{R_iR^k}{R^3}\, &\left[2\frac{\mathrm{d}^4}{\mathrm{d}t^4}Q^+_{2,kj} + \frac{8}{R}\frac{\mathrm{d}^3}{\mathrm{d}t^3}Q^+_{2,kj} + \frac{18}{R^2}\frac{\mathrm{d}^2}{\mathrm{d}t^2}Q^+_{2,kj} + \frac{30}{R^3}\frac{\mathrm{d}}{\mathrm{d}t}Q^+_{2,kj} + \frac{30}{R^4}Q^+_{2,kj}\right]\Bigg|_{t-R} \\
+ Q_1^{ij,-}\frac{R_iR_jR^kR^l}{2R^5}\, &\left[\frac{\mathrm{d}^4}{\mathrm{d}t^4}Q^+_{2,kl} + \frac{10}{R}\frac{\mathrm{d}^3}{\mathrm{d}t^3}Q^+_{2,kl} + \frac{45}{R^2}\frac{\mathrm{d}^2}{\mathrm{d}t^2}Q^+_{2,kl} + \frac{105}{R^3}\frac{\mathrm{d}}{\mathrm{d}t}Q^+_{2,kl} + \frac{105}{R^4}Q^+_{2,kl}\right]\Bigg|_{t-R} + (1\leftrightarrow 2)\,.
\end{aligned}
$$

RR FORCE FOR A FOUR-BODY SYSTEM



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\end{aligned}
$$

- It mixes different PN orders $(\mathrm{d}/\mathrm{d}t \to \mathrm{d}/c\mathrm{d}t$ and $t - R \to t - R/c)$
  - But one cannot cleanly separate PN orders because $c \to \infty$ is not under control!
- Not invariant under time-reversal $\Longrightarrow$ Dissipation
  - But also contains a conservative piece!

Limits of the BB force

Newtonian limit $c \to \infty$

$\mathcal{O}(c^0)$ piece:

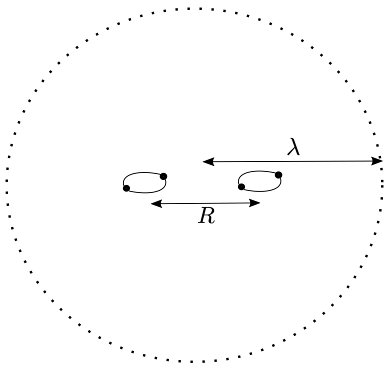
$$\frac{G}{2}\int \mathrm{d}t \left[\frac{3}{R^5} Q_1^{ij,-} Q^+_{2,ij} - \frac{30 R_i R^k}{R^7} Q_1^{ij,-} Q^+_{2,kj} + \frac{105 R_i R_j R^k R^l}{2R^9} Q_1^{ij,-} Q^+_{2,kl}\right] + (1 \leftrightarrow 2)$$

What does it mean? Quadrupole-Quadrupole interaction of

$$S_{\mathrm{Newt}} = \frac{G}{2}\int \mathrm{d}t \sum_{N,M} \frac{m_N m_M}{|\mathbf{x}_N - \mathbf{x}_M|}$$ (Expanded in CM frames)

- Time-symmetric
- Enhanced wrt usual RR force $\mathcal{O}(c^{-5})$
- But $\frac{S_{\mathrm{BB}}}{S_{\mathrm{KL}}} \sim \left(\frac{r}{R}\right)^2$: suppressed wrt Kozai-Lidov

LIMITS OF THE BB FORCE



$$\lambda \gg R$$

$$R\partial_t \sim \frac{R}{\lambda} \ll 1$$: usual PN expansions apply

$$S_{\mathrm{BB},\lambda\gg R} = -\frac{G}{5}\int \mathrm{d}t\; Q^{-}_{1,ij}\frac{\mathrm{d}^5}{\mathrm{d}t^5}Q^{ij,+}_{2} + (1\leftrightarrow 2)$$

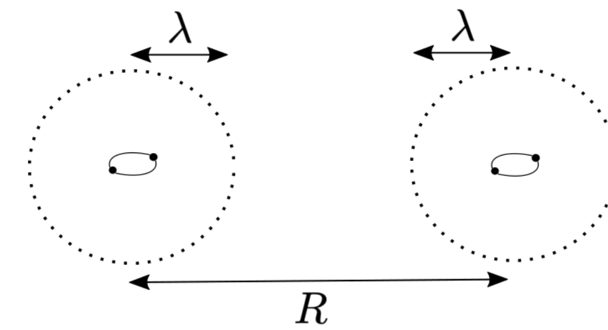
(Dissipative piece only)

- All the coefficients conspire to give five time-derivatives of Q! (2.5PN)
- Meaning of the force: radiation-reaction induced by total quadrupole moment

$$S_{\mathrm{eff}} = -\frac{G}{5}\int \mathrm{d}t\; Q^{-}_{1,ij}\frac{\mathrm{d}^5}{\mathrm{d}t^5}\left[\underbrace{Q^{ij,+}_{1}}_{\text{Burke-Thorne}} + \underbrace{Q^{ij,+}_{2}}_{\text{Binary-Binary}}\right] + (1\leftrightarrow 2)$$

Limits of the BB force

$$\lambda \ll R$$



$$S_{\mathrm{BB},\lambda\ll R} = \frac{G}{2}\int \mathrm{d}t\, \frac{Q_1^{ij,-}}{R}\left[\frac{\mathrm{d}^4}{\mathrm{d}t^4}Q^+_{2,ij} - 2\frac{R_iR^k}{R^2}\frac{\mathrm{d}^4}{\mathrm{d}t^4}Q^+_{2,kj} + \frac{R_iR_jR^kR^l}{2R^4}\frac{\mathrm{d}^4}{\mathrm{d}t^4}Q^+_{2,kl}\right]\Bigg|_{t-R} + (1\leftrightarrow 2)$$

Meaning: scattering of GW

$$S_{\mathrm{BB}} = -\frac{1}{2}\int \mathrm{d}\tilde{\tau} E_{ij}(\mathbf{X}_2)Q_2^{ij} \quad \text{with} \quad h^{\mathrm{TT}}_{2,ij} = \frac{2G}{R}\ddot{Q}^{\mathrm{TT}}_{1,ij}(t-R)$$

The force is subdominant wrt usual radiation-reaction:

$$\frac{S_{\mathrm{BB},\lambda\ll R}}{S_{\mathrm{Burke-Thorne}}} \sim \frac{\lambda}{R} \ll 1$$

EFFECT OF THE BB FORCE?

Want to find the effect on long timescales:

$\langle Q_1 Q_2 \rangle = 0$ apart in a resonant configuration

When in resonance:

$\lambda \gg R$: EOM in vacuum do not seem to lead to resonance capture

N. Seto (2018)

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CONCLUSIONS:

- Radiative effects for N-body systems are subtle!
- Is someone expert of delay-differential equations in the room?