UNIVERSITÉ DE GENÈVE

Wave optics lensing of GW :

Theory and phenomenology of triple systems in the LISA band

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In collaboration with Giulia Cusin, Cyril Pitrou, Jean-Philippe Uzan

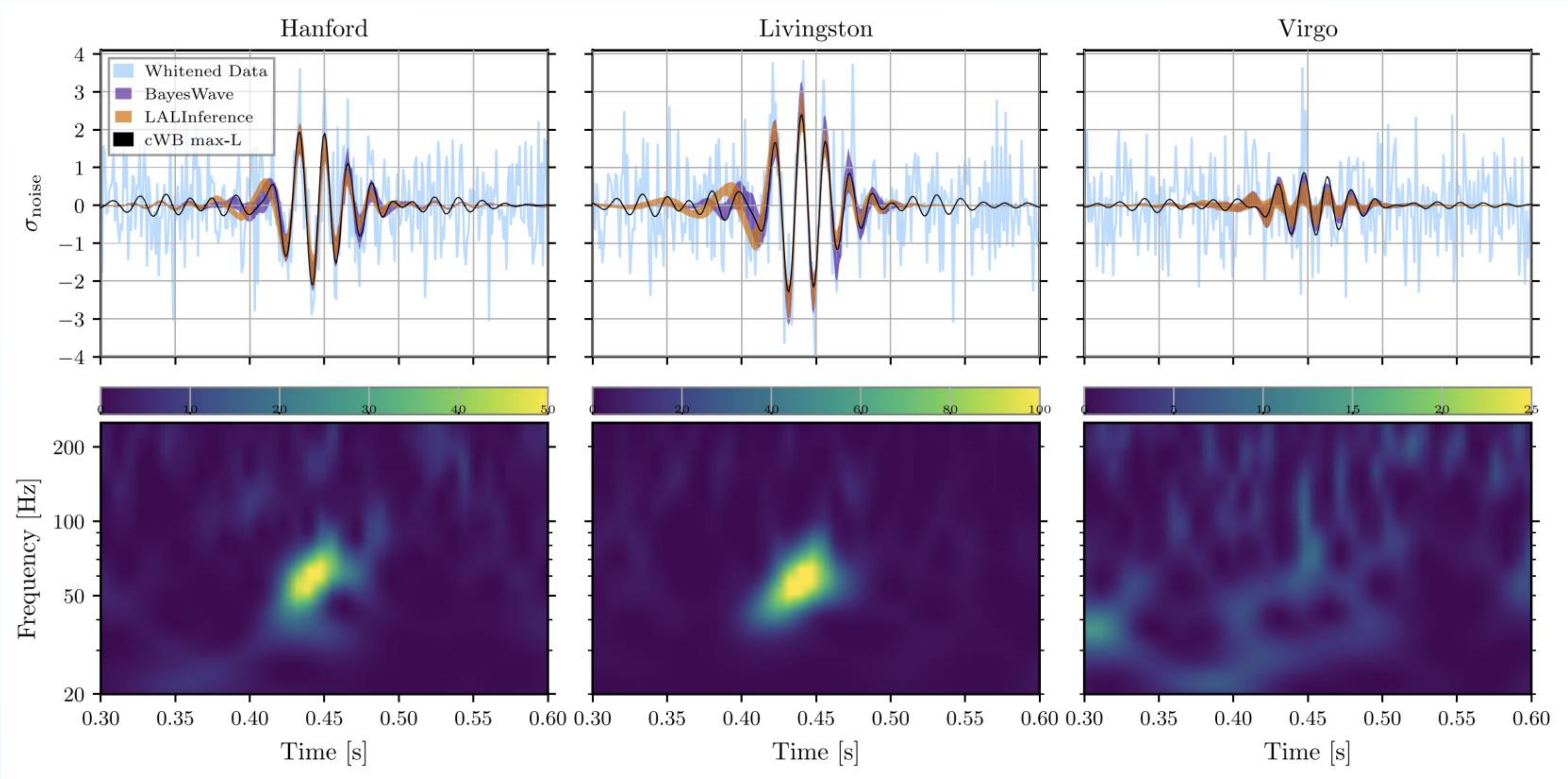
arXiv:2404.07186 , to appear in Phys. Rev. D

Outline

1. Introduction on triple systems & motivation
2. GW lensing, wave optics the *usual* way, quantum mechanical analogy
3. Our contribution : GW lensing considering the tensorial structure
4. Application to LISA-band triple systems

Hierarchical triple systems : GW190521 ?

Observational fact: 5 years ago ...



Best fit masses:

$$m_1 = 85 M_\odot,$$
$$m_2 = 66 M_\odot$$

(heavy !)

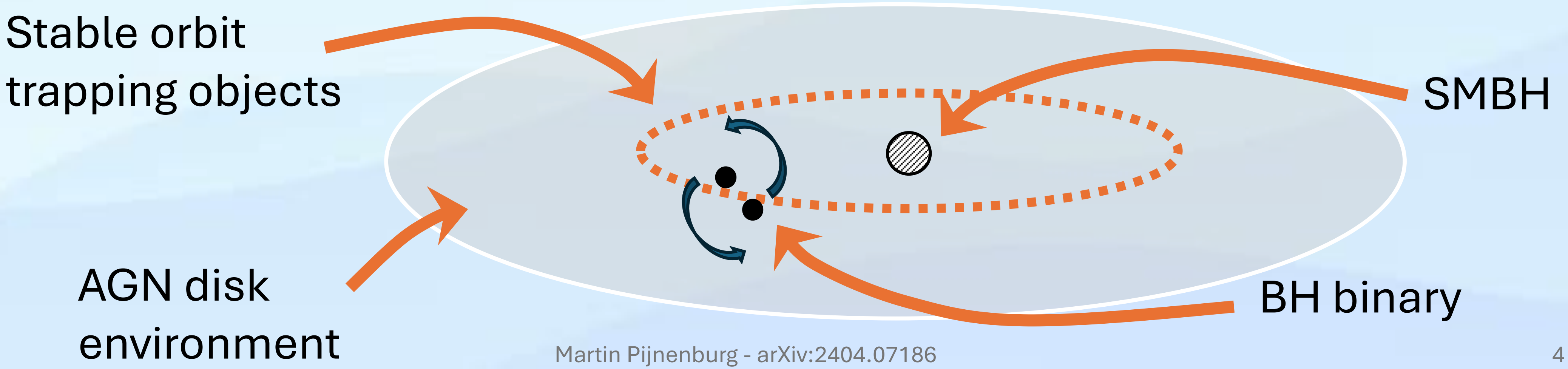
R. Abbott *et al.* (LIGO Scientific Collaboration and Virgo Collaboration),
GW190521: A Binary Black Hole Merger with a Total Mass of 150 $M_\odot$
Phys. Rev. Lett. **125**, 101102, 2020

Hierarchical triple systems : GW190521 ?

Multiple possible scenarii, inconclusive analysis looking at GW signal **alone**

Abbott *et al.* (LIGO Scientific Collaboration and Virgo Collaboration), 2020

One possibility: AGN disk migration traps   Bellovary *et al.*, 2016



Hierarchical triple systems : GW190521 ?

- 34 days later: AGN signal from the same location
- Suspect merger ocurred $\sim 350\, r_S$ away from AGN : triple system

Considering **both GW & EM** signals, **evidence** for hierarchical triple:

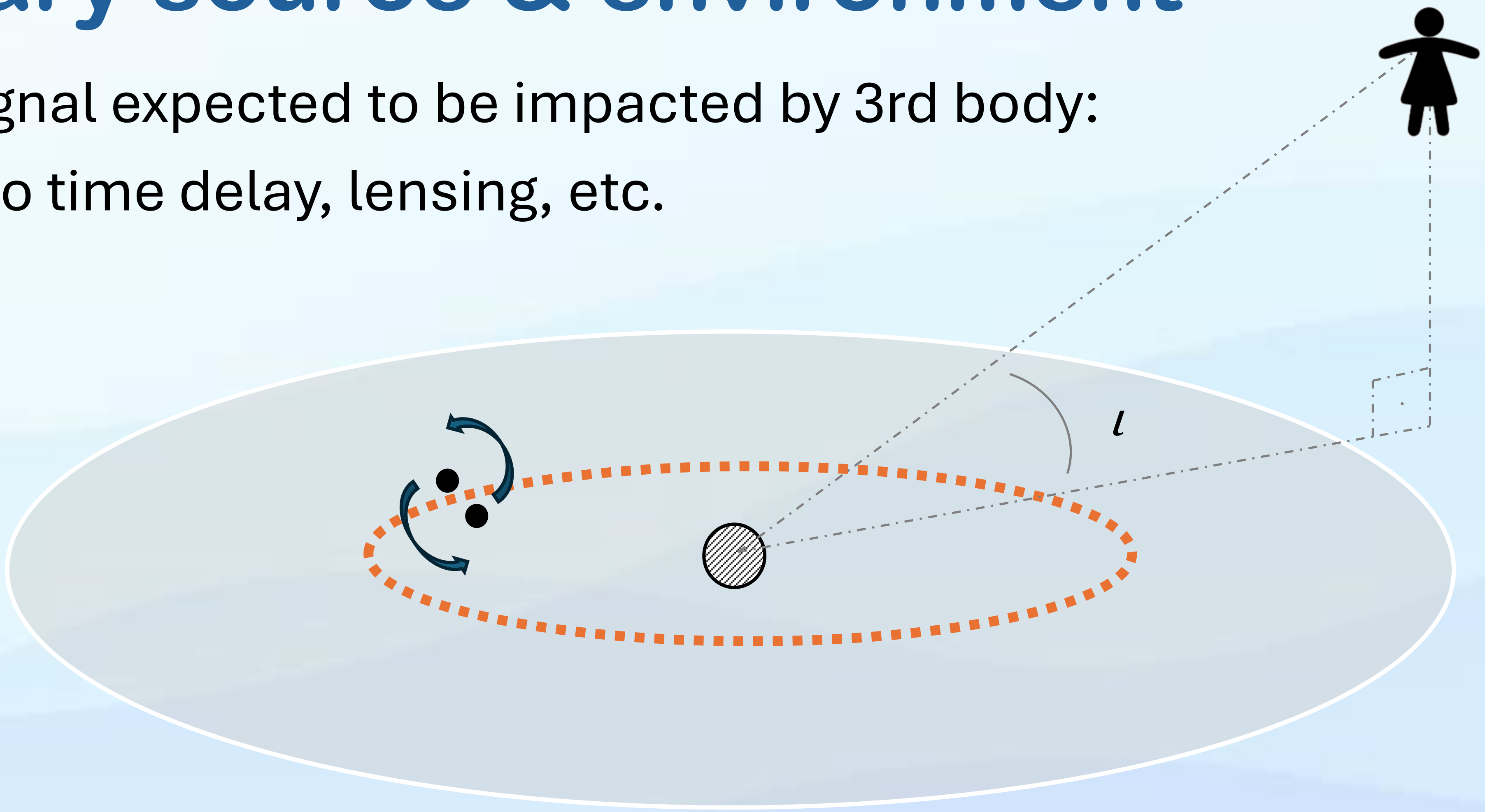
marginal Ashton *et al.* (2021), Palmese *et al.* (2021)

confident Graham *et al.* (2020), Morton *et al.* (2023)

Hierarchical triple systems : binary source & environment

GW signal expected to be impacted by 3rd body:

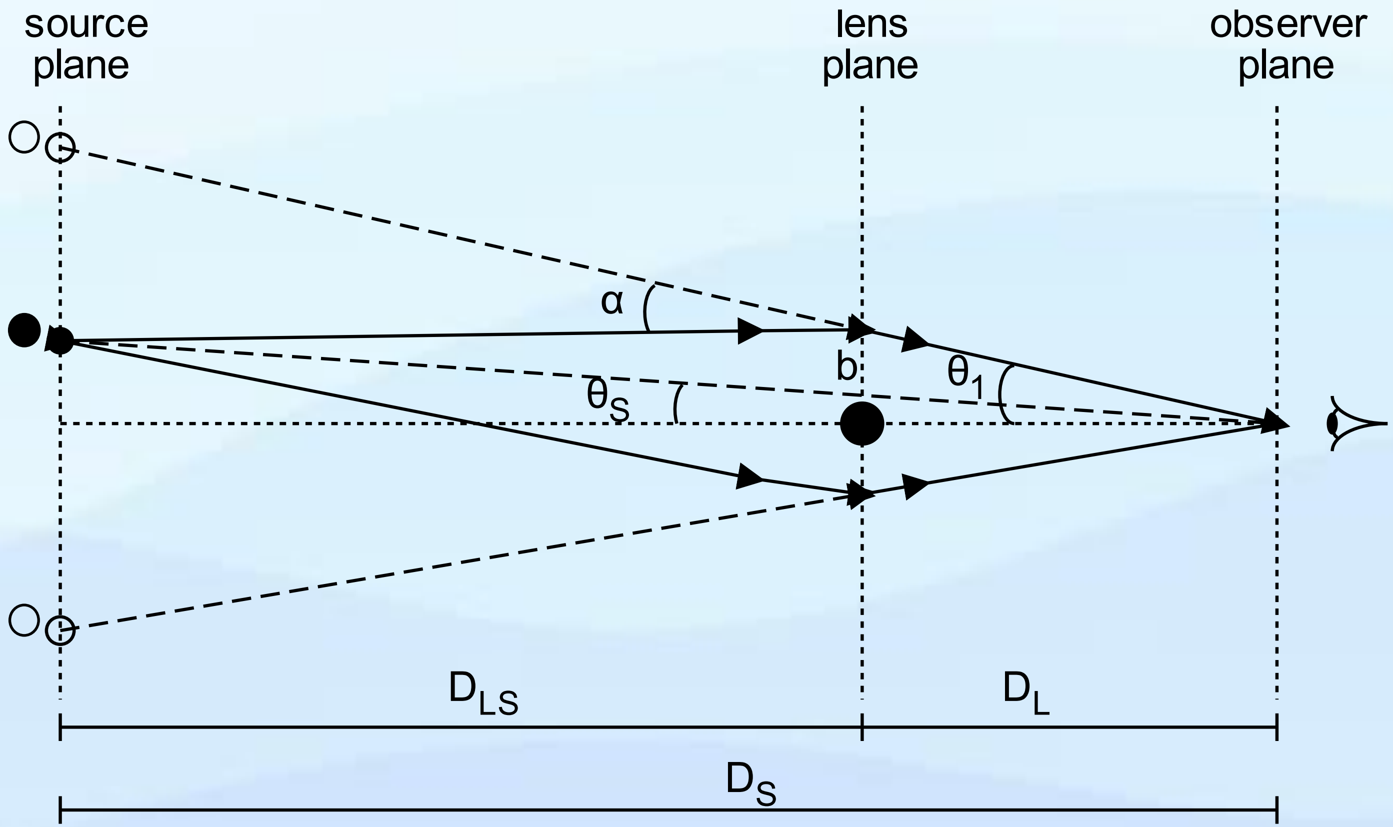
Shapiro time delay, lensing, etc.



GW lensing: geometrical optics

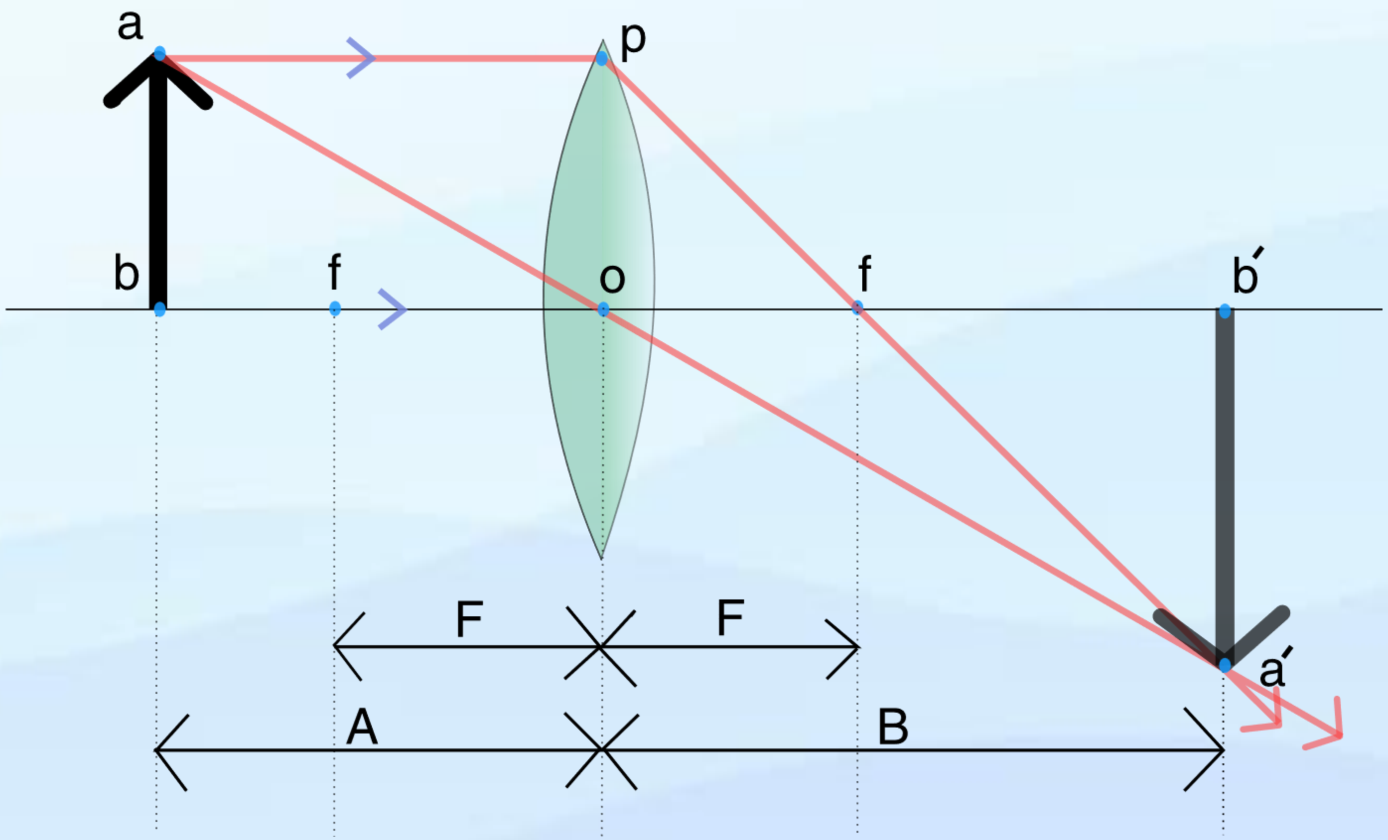
Assume you have a notion of ***ray*** :

→ Usual lensing picture (deflection angle, etc.)



GW lensing: geometrical optics

Conceptually similar to (my) undergraduate lab optics



GW lensing: wave optics

But just as light, GW are ... **waves** !

Geometrical optics is just a high frequency approximation, which **breaks down** when

$$\lambda_{\text{wave}} \gtrsim \text{(lens size)} \,.$$

At the fundamental level, signal obeys a wave equation

$\rightarrow$ allows for diffraction, interference, ...

GW lensing: wave optics

LISA has a best sensitivity around 10^{-3}Hz.

At this frequency, **low mass AGN** with $M \sim \mathcal{O}(10^6) M_\odot$ fulfil the wave optics requirement

$$\lambda_{GW} > \frac{2GM}{c^2}$$

Equivalently : $\omega M < 1$ (natural units)

GW lensing: wave optics

Start with : $\mathrm{d}s^2 = (\bar{g}_{\mu\nu} + h_{\mu\nu})\mathrm{d}x^\mu \mathrm{d}x^\nu$

E.g. gauge fixing : $h^{\nu}_{\mu;\nu} = 0, \quad h^{\mu}{}_{\mu} = 0$

→ Wave equation :

$$h_{\mu\nu;\alpha}{}^{;\alpha} + 2\bar{R}_{\alpha\mu\beta\nu}\, h^{\alpha\beta} = 0, \quad \text{with} \quad h_{\mu\nu;\alpha}{}^{;\alpha} \equiv \Box h_{\mu\nu}.$$

GW lensing: wave optics

Reference work for phenomenology :

Wave effects in gravitational lensing of gravitational waves from chirping binaries

Ryuichi Takahashi (Kyoto U.), Takashi Nakamura (Kyoto U.)

May, 2003

28 pages

Published in: *Astrophys.J.* 595 (2003) 1039-1051

e-Print: astro-ph/0305055 [astro-ph]

DOI: 10.1086/377430

View in: ADS Abstract Service

Scalar wave optics

Usual process : **scalar** wave Ansatz $h_{\mu\nu} = \phi \cdot e_{\mu\nu}$

Assume known behaviour for $e_{\mu\nu}$ (parallel transport)

Specify the lens background $\bar{g}_{\mu\nu}$ (in our case : Schwarzschild)

Decompose ϕ in multipoles : $\phi = e^{-i\omega t} \sum_{\ell} \frac{u_\ell(r)}{r\,(1 - 2M/r)^{1/2}} \, P_\ell(\cos(\theta))$

Scalar wave optics

Differential equation is

$$\left[\frac{\mathrm{d}^2}{\mathrm{d}r^2} + \omega^2 + \frac{4M\omega^2}{r} - \frac{\ell(\ell+1)}{r^2} + \frac{12M^2\omega^2}{r^2} + \mathcal{O}(r^{-3})\right] u_\ell = 0$$

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Schrödinger in 1/r Coulomb potential (Rutherford) : $\Psi = e^{-i\omega t}\sum_\ell \frac{u_\ell}{r} P_\ell$

$$\left[\frac{\mathrm{d}^2}{\mathrm{d}r^2} + \omega^2 - \frac{2\omega\gamma}{r} - \frac{\ell(\ell+1)}{r^2}\right] u_\ell = 0 \qquad , \ \gamma \propto \text{charges}$$

Scalar wave optics

deep wave optics

Differential equation is

$$\left[\frac{\mathrm{d}^2}{\mathrm{d}r^2} + \omega^2 + \frac{4M\omega^2}{r} - \frac{\ell(\ell+1)}{r^2} + \frac{12M^2\omega^2}{r^2} + \mathcal{O}(r^{-3})\right] u_\ell = 0$$

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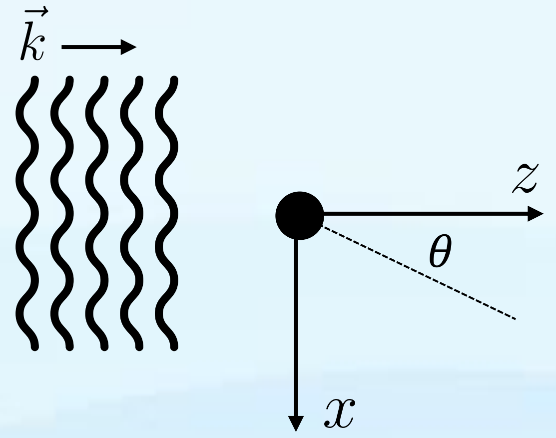
Insights from quantum mechanics

QM problem has exact solution:



$$\Psi \propto e^{-i\omega t} e^{ikz} {}_1F_1[-i\gamma, 1; ikr(1-\cos(\theta))]$$

Solution decouples for $kr(1-\cos(\theta)) \gg 1$:

$$e^{i\omega t}\Psi \sim e^{ikz+i\gamma \ln kr(1-\cos(\theta))} - \frac{\tilde{\gamma}}{kr(1-\cos(\theta))} e^{ikr-i\gamma \ln kr(1-\cos(\theta))}$$

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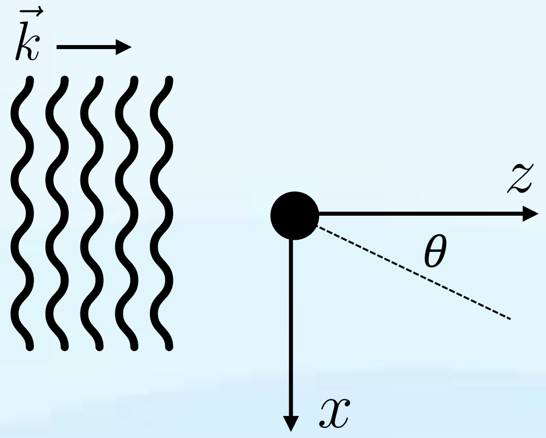

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Plane wave

Spherical wave

Insights from quantum mechanics

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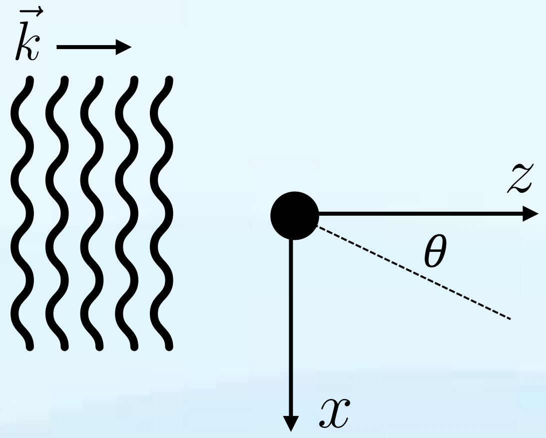

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apparent divergence as $\theta \to 0$

Insights from quantum mechanics

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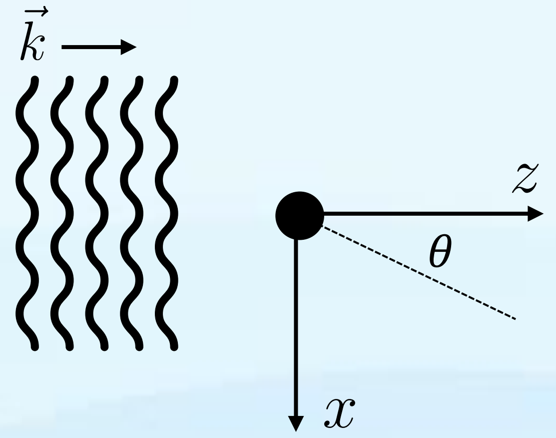

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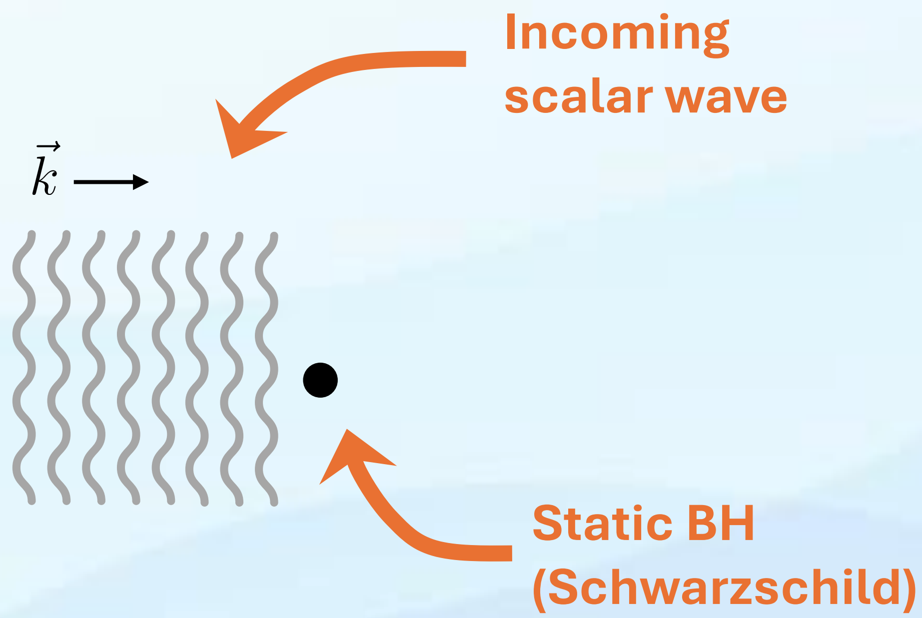
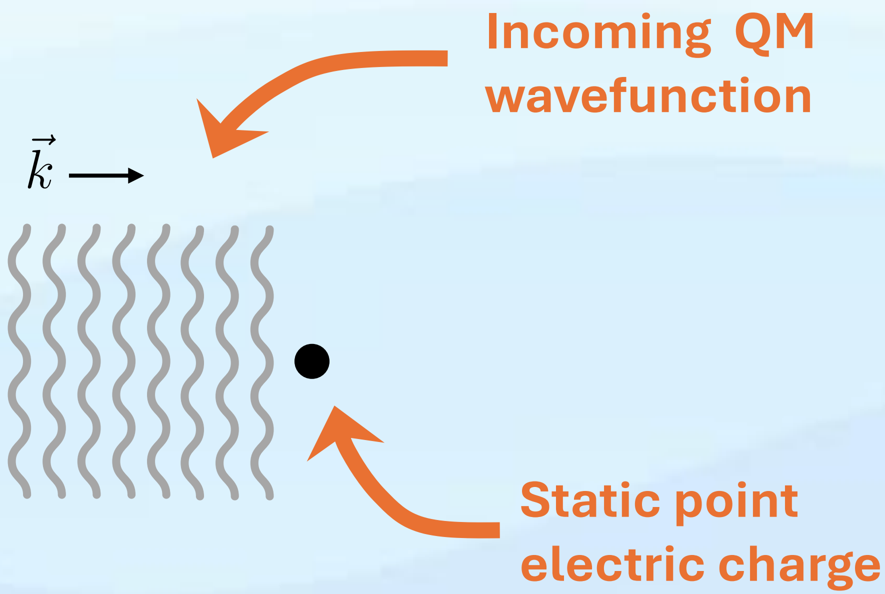
log phase corrections

Insights from quantum mechanics

GR scalar wave, NR simulation

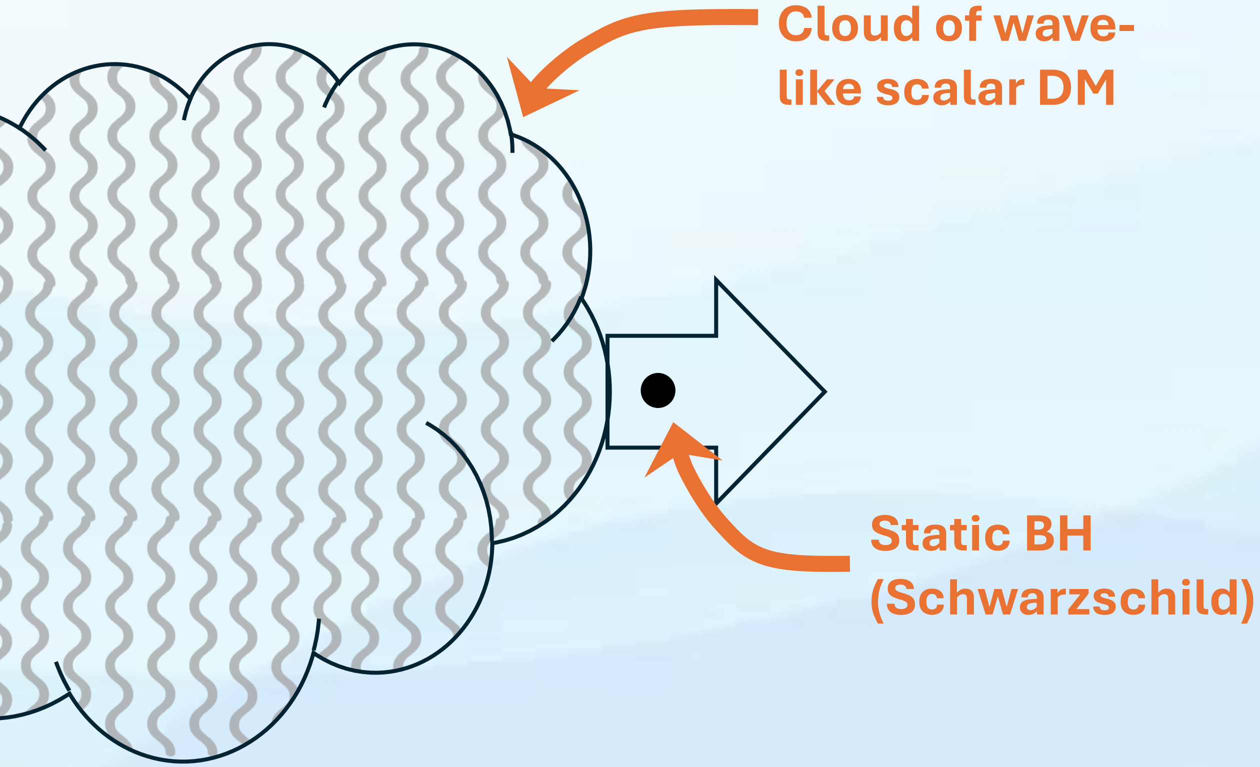


QM, analytical solution



Insights from quantum mechanics

GR scalar wave, NR simulation



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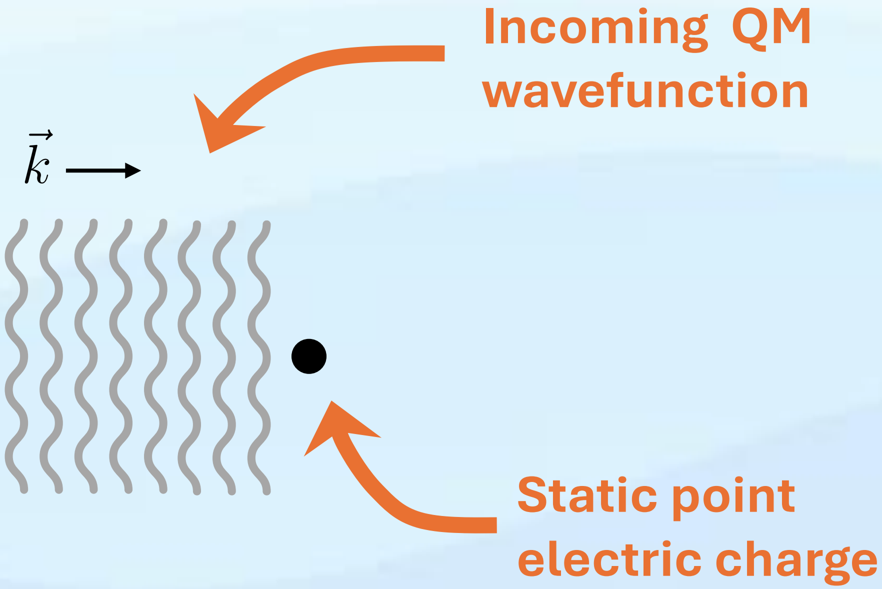

Insights from quantum mechanics

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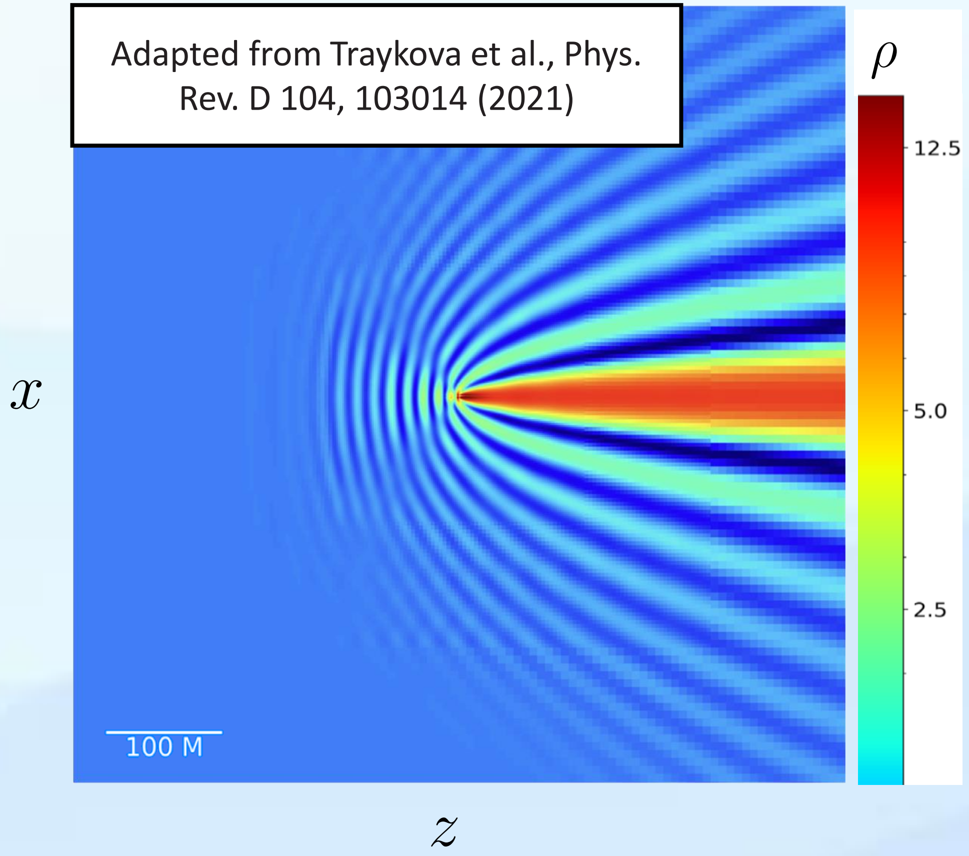

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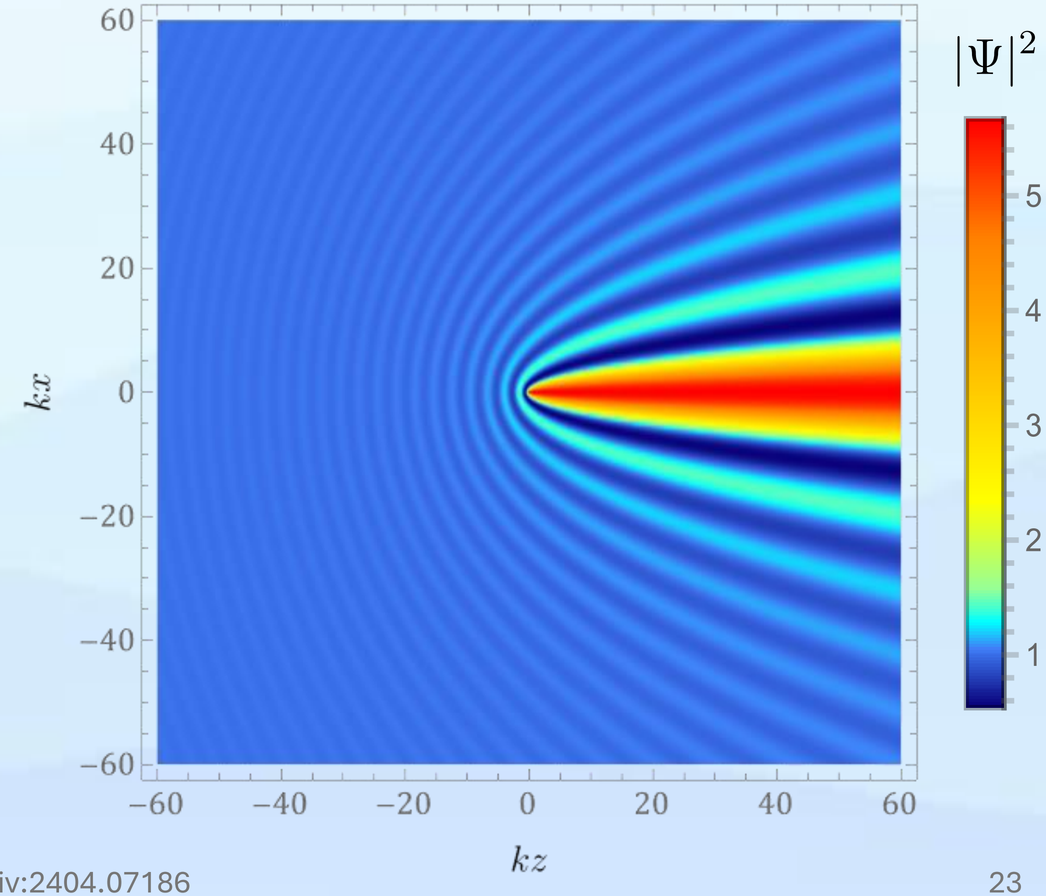


Insights from quantum mechanics

If one doesn't know the exact solution :

Possible to solve the differential equation in multipole space, by

- Taking $kr \gg 1$ limit
- Requiring $\Psi \sim \Psi_{\text{plane}} + \Psi_{\text{spherical}}$

$\rightarrow$ **Correctly** recover $\Psi_{\text{spherical}} = -\dfrac{\tilde{\gamma}}{kr(1-\cos(\theta))} e^{ikr - i\gamma \ln kr(1-\cos(\theta))}$

Tensorial wave optics

Start again with : $$\mathrm{d}s^2 = (\bar{g}_{\mu\nu} + h_{\mu\nu})\mathrm{d}x^\mu \mathrm{d}x^\nu$$

... but **avoid** extra assumption $h_{\mu\nu} = \phi \cdot e_{\mu\nu}$

Rather treat $h_{\mu\nu}$ with tools of black hole perturbation theory (BHPT),

to keep track of the full polarisation structure

Tensorial wave optics : BHPT

Project $h_{\mu\nu}$ on basis functions on the sphere with even (Y) and odd (X) parity :

$$h_{rr} = \sum_{\ell m} h_{rr}^{\ell m} \, Y^{\ell m},$$ (radial)

$$h_{rA} = \sum_{\ell m} h_{r}^{\ell m} \, X_{A}^{\ell m} + j_{r}^{\ell m} \, Y_{A}^{\ell m}, \quad A = \theta, \phi,$$ (radial/angular)

$$h_{AB} = \sum_{\ell m} h_{2}^{\ell m} X_{AB}^{\ell m} + r^2 G^{\ell m} Y_{AB}^{\ell m} + r^2 K^{\ell m} \Omega_{AB} Y^{\ell m}, \quad A, B = \theta, \phi,$$ (angular)

Tensorial wave optics : BHPT

From metric multipoles, define two **gauge invariant** master functions:

$$\Psi_{\text{odd}}^{\ell m} = \frac{2r}{(\ell-1)(\ell+2)}\left(\frac{\partial}{\partial r}\hat{h}_t^{\ell m} - \frac{\partial}{\partial t}\hat{h}_r^{\ell m} - \frac{2}{r}\hat{h}_t^{\ell m}\right)$$

$$r^{-1}\Psi_{\text{even}}^{\ell m} \propto \hat{K}^{\ell m} + \frac{2(1-2M/r)}{(\ell-1)(\ell+2)+6M/r}\left((1-2M/r)\hat{h}_{rr}^{\ell m} - r\frac{\partial}{\partial r}\hat{K}^{\ell m}\right)$$

Martel, Poisson. *Physical Review. D* 71.10 (2005)

Tensorial wave optics : BHPT

$\Psi_\bullet^{\ell m}$ obey Zerilli & Regge-Wheeler equations, $\bullet = \text{even, odd}$

$$\frac{\mathrm{d}^2\Psi_\bullet}{\mathrm{d}r_*^2} + \left(\omega^2 - V_\bullet\right)\Psi_\bullet = 0, \qquad \text{with } r_*(r) = r - 2M\ln\left(\frac{r}{2M} - 1\right)$$

Schrödinger-like, for given potentials $V_\bullet(\ell, r, M)$

For the scattering problem :

Asymptotic solutions **for $\boldsymbol{\omega M \ll 1}$** are known, expect $\Psi_\bullet^{\ell m} \sim \Psi_\bullet^{\text{plane}} + \Psi_\bullet^{\text{sph}}$

Poisson, Sasaki. *Physical Review D* 51.10 (1995)

Tensorial wave optics : BHPT

« Observables »

Assume initial

$h_{\mu\nu}^{\text{source}}$

(absence of lens)

Derive corresponding

$\Psi_{\bullet}^{\text{source}}$

Solve differential equ. such that

$\Psi_{\bullet}^{\ell m} \sim \Psi_{\bullet}^{\text{source}} + \Psi_{\bullet}^{\text{lensed}}$

Derive final

$h_{\mu\nu}$

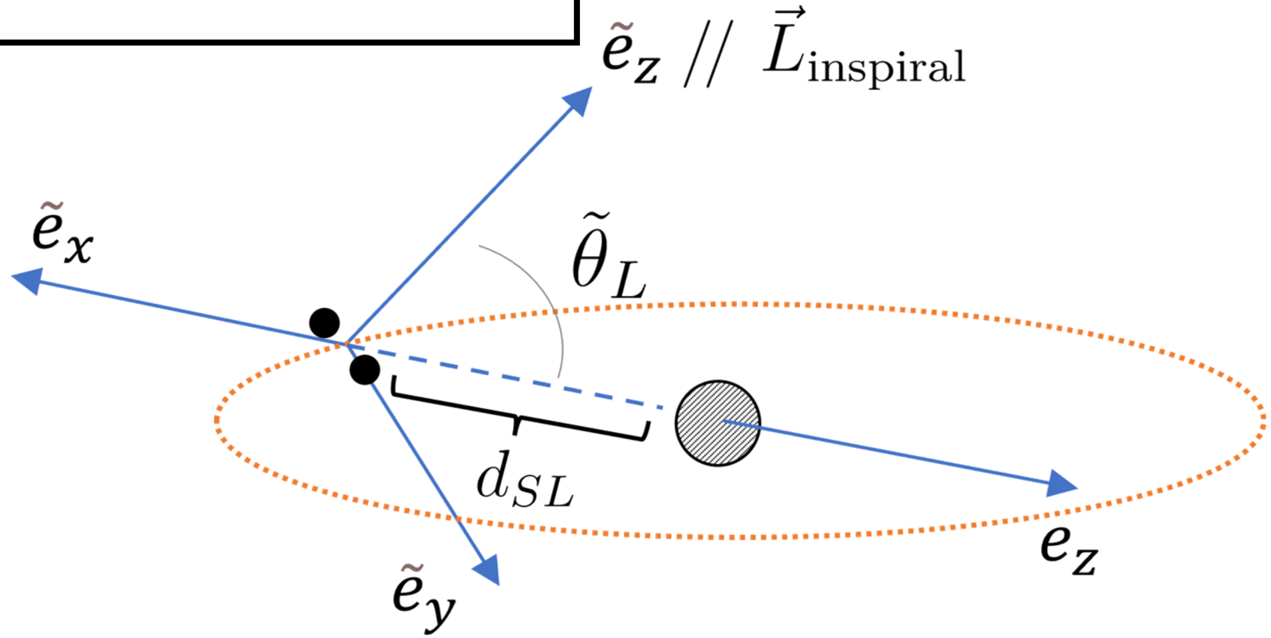
(incl. lensing)

Tensorial wave optics : BHPT

Assume initial

$$h_{\mu\nu}^{\text{source}}$$

(absence of lens)



TT gauge, propagation along e_z :

$$h_{ij}^{\text{source}} = \begin{pmatrix} h_+ & h_\times & 0 \\ h_\times & -h_+ & 0 \\ 0 & 0 & 0 \end{pmatrix}_{ij}$$

$$h_+ = \frac{A_{\text{in}}}{\tilde{r}} \frac{1+\cos^2\tilde{\theta}_L}{2} \cos[\omega(t-\tilde{r}) - 2\tilde{\phi}_L]$$

$$h_\times = \frac{A_{\text{in}}}{\tilde{r}} \cos\tilde{\theta}_L \sin[\omega(t-\tilde{r}) - 2\tilde{\phi}_L]$$

(locally plane wave)

Tensorial wave optics : BHPT

Assume initial

$h_{\mu\nu}^{\text{source}}$

(absence of lens)

→

Derive corresponding

$\Psi_\bullet^{\text{source}}$

Using scattering phase shifts

↓

Solve differential equ. such that

$\Psi_\bullet^{\ell m} \sim \Psi_\bullet^{\text{source}} + \Psi_\bullet^{\text{lensed}}$

←

Derive final

$h_{\mu\nu}$

(incl. lensing)

Tensorial wave optics : BHPT

Assume initial

$$h_{\mu\nu}^{\text{source}}$$

(absence of lens)

→

Derive corresponding

$$\Psi_\bullet^{\text{source}}$$

↓

Derive final

$$h_{\mu\nu}$$

(incl. lensing)

←

Solve differential equ. such that

$$\Psi_\bullet^{\ell m} \sim \Psi_\bullet^{\text{source}} + \Psi_\bullet^{\text{lensed}}$$

Tensorial wave optics : BHPT

Technicality : in principle, should sum $\lim_{r\to\infty} \sum_{\ell m} \Psi_{\bullet}^{\ell m}$

In practice : $\sum_{\ell m} \lim_{r\to\infty} \Psi_{\bullet}^{\ell m}$ … **diverges** analytically & numerically

Derive final $h_{\mu\nu}$ (incl. lensing) ← **non standard summation methods** ← Solve differential equ. such that $\Psi_{\bullet}^{\ell m} \sim \Psi_{\bullet}^{\text{source}} + \Psi_{\bullet}^{\text{lensed}}$

Tensorial wave optics : results

Recovering familiar features:

$$h_+ - h_+^{\text{source}} - \simeq \frac{A_{\text{in}}}{r} \frac{2M}{d_{\text{SL}}} \frac{1+\cos^2\theta}{2} \frac{1}{1-\cos\theta} \left(\cos^4\left(\frac{\tilde{\theta}_L}{2}\right) \cos\left(\varphi - 2\phi\right) + \sin^4\left(\frac{\tilde{\theta}_L}{2}\right) \cos\left(\varphi + 2\phi\right) \right)$$

$$h_\times - h_\times^{\text{source}} - \simeq \frac{A_{\text{in}}}{r} \frac{2M}{d_{\text{SL}}} \cos\theta \frac{1}{1-\cos\theta} \left(\cos^4\left(\frac{\tilde{\theta}_L}{2}\right) \sin\left(\varphi - 2\phi\right) - \sin^4\left(\frac{\tilde{\theta}_L}{2}\right) \sin\left(\varphi + 2\phi\right) \right)$$

TT gauge projection
of a quadrupole
(cf source)

Tensorial wave optics : results

Recovering familiar features:

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lensing features

Natural *expected* validity range : $kr(1-\cos(\theta)) \gg 1$

Tensorial wave optics : results

Recovering familiar features:

$$h_+ - h_+^{\text{source}} - \simeq \frac{A_{\text{in}}}{r}\frac{2M}{d_{\text{SL}}}\frac{1+\cos^2\theta}{2}\frac{1}{1-\cos\theta}\left(\cos^4\left(\frac{\tilde{\theta}_L}{2}\right)\cos(\varphi - 2\phi) + \sin^4\left(\frac{\tilde{\theta}_L}{2}\right)\cos(\varphi + 2\phi)\right)$$

$$h_\times - h_\times^{\text{source}} - \simeq \frac{A_{\text{in}}}{r}\frac{2M}{d_{\text{SL}}}\cos\theta\frac{1}{1-\cos\theta}\left(\cos^4\left(\frac{\tilde{\theta}_L}{2}\right)\sin(\varphi - 2\phi) - \sin^4\left(\frac{\tilde{\theta}_L}{2}\right)\sin(\varphi + 2\phi)\right)$$

$$\varphi(r,t) \equiv \omega(t - d_{\text{SL}*} - r_*) - 2\tilde{\phi}_L + \Phi - 2M\omega\left(\ln\left(1-\cos\theta\right) - 1 - \ln 2\right)$$

log phase corrections

Polarisation

Quantifying the signal polarisation content $\mathcal{V} \in [-1, 1]$:

$$\mathcal{V} \equiv \frac{2\mathrm{Im}[\tilde{h}_+ \tilde{h}_\times^*]}{|\tilde{h}_+|^2 + |\tilde{h}_\times|^2} = V/I \qquad \text{in terms of the Stokes parameters } V, I\,.$$

$$= \frac{|\tilde{h}^{(2)}|^2 - |\tilde{h}^{(-2)}|^2}{|\tilde{h}^{(2)}|^2 + |\tilde{h}^{(-2)}|^2}$$

constant in geometric optics and scalar wave optics

in general **not constant** tensorial wave optics

Polarisation

Quantifying the signal polarisation content $\mathcal{V} \in [-1, 1]$:

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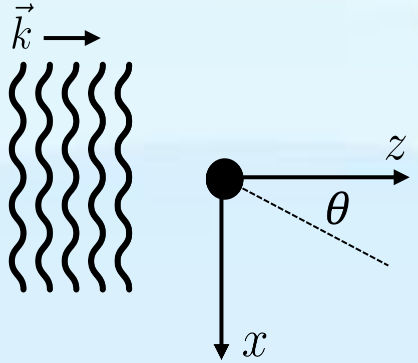
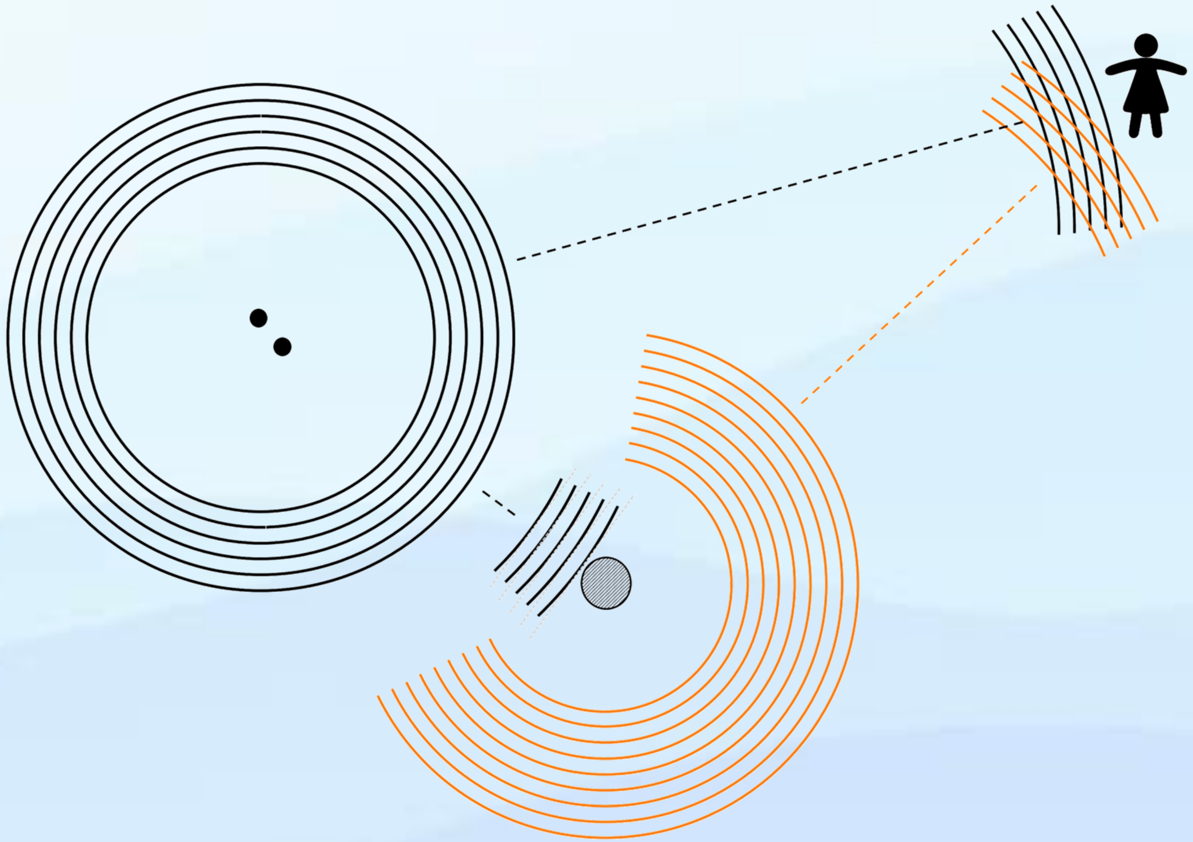


Wave optics lensing is **polarisation dependent**, e.g. :

$$\frac{\mathrm{d}\sigma}{\mathrm{d}\Omega} = M^2 \frac{\cos^8\left(\frac{\theta}{2}\right) + \sin^8\left(\frac{\theta}{2}\right)}{\sin^4\left(\frac{\theta}{2}\right)} + 2M^2\sqrt{1 - \mathcal{V}^2_{\mathrm{incident}}}\cos^4\left(\frac{\theta}{2}\right)\cos(4\phi)$$

Tensorial wave optics : interference

Full wave solution is a **superposition** of lensed and original wave

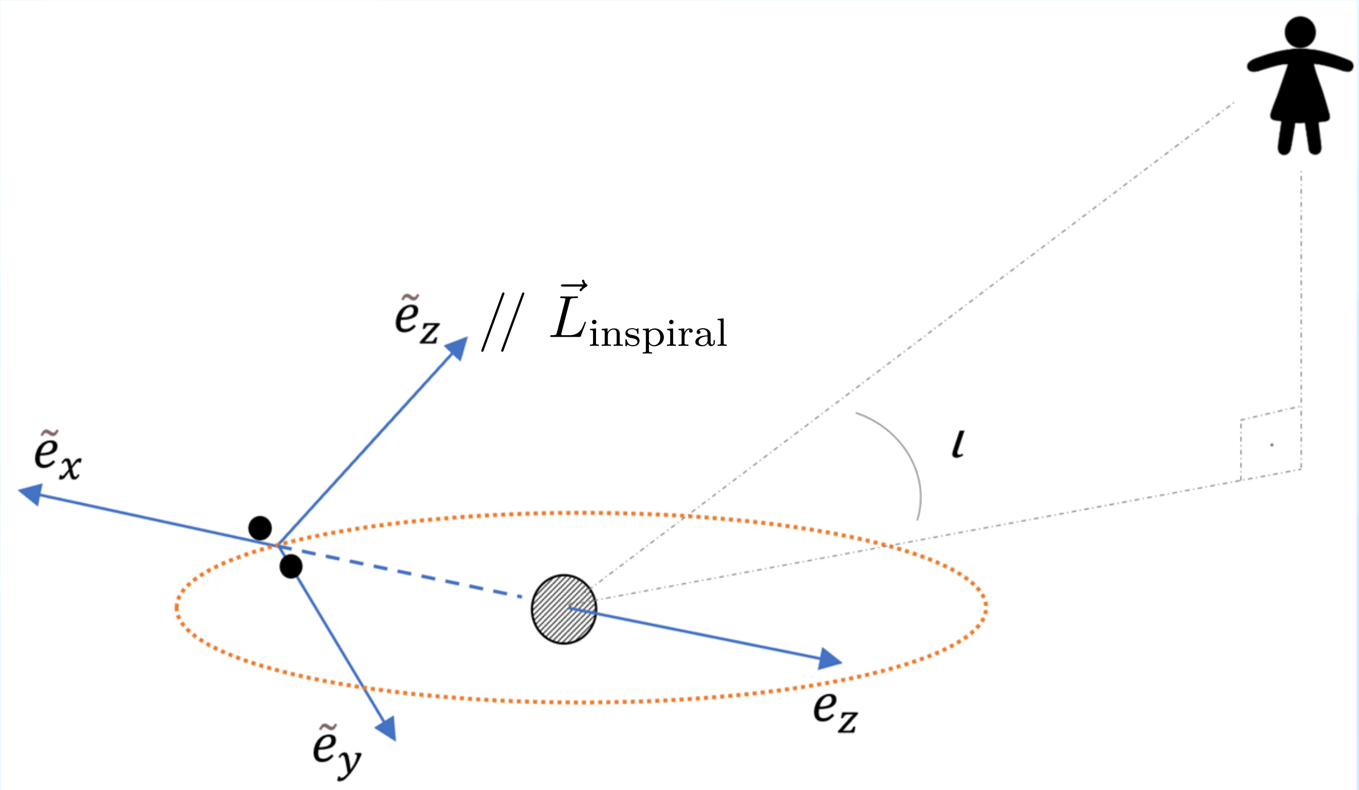


Wave optics lensing in triple systems: towards a phenomenology

Illustration:

- For simple circular orbits (disk migration traps)

- Considering a time-varying aligment (*explicitly ignoring other velocity dependent terms, e.g. Doppler*)



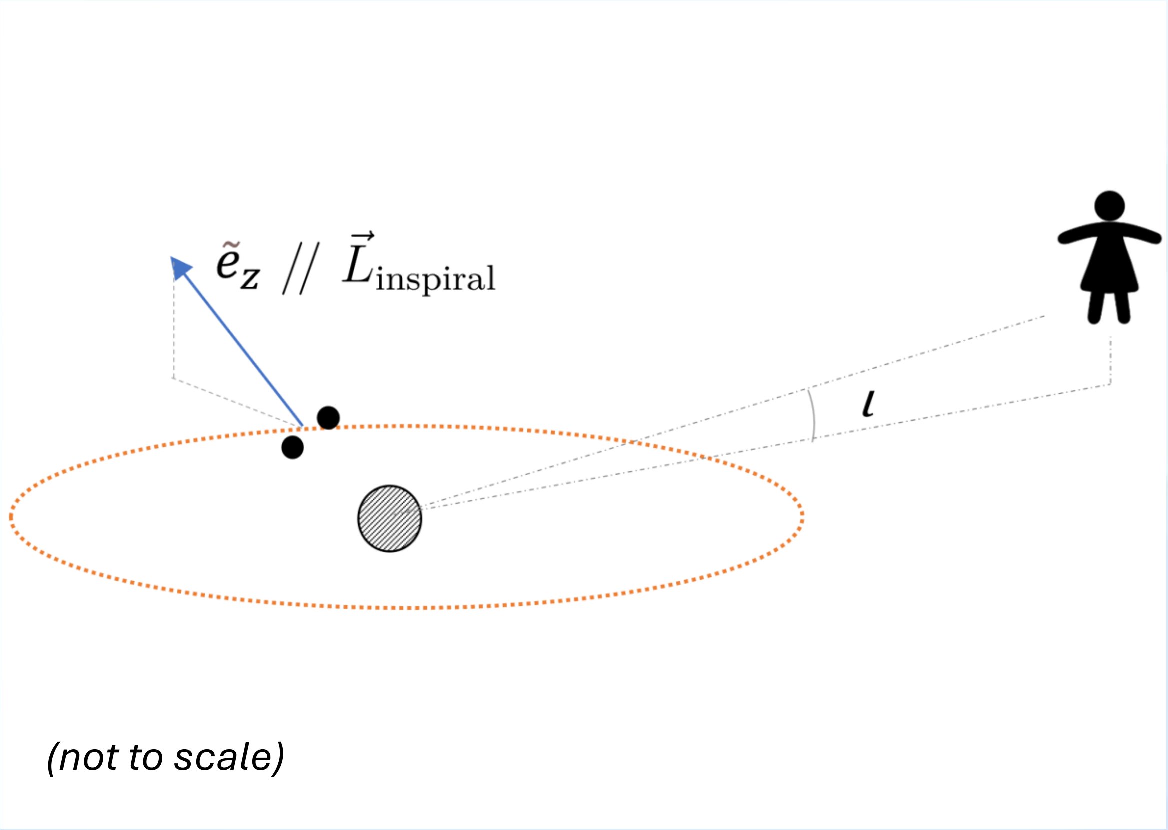
Wave optics lensing in triple systems: towards a phenomenology

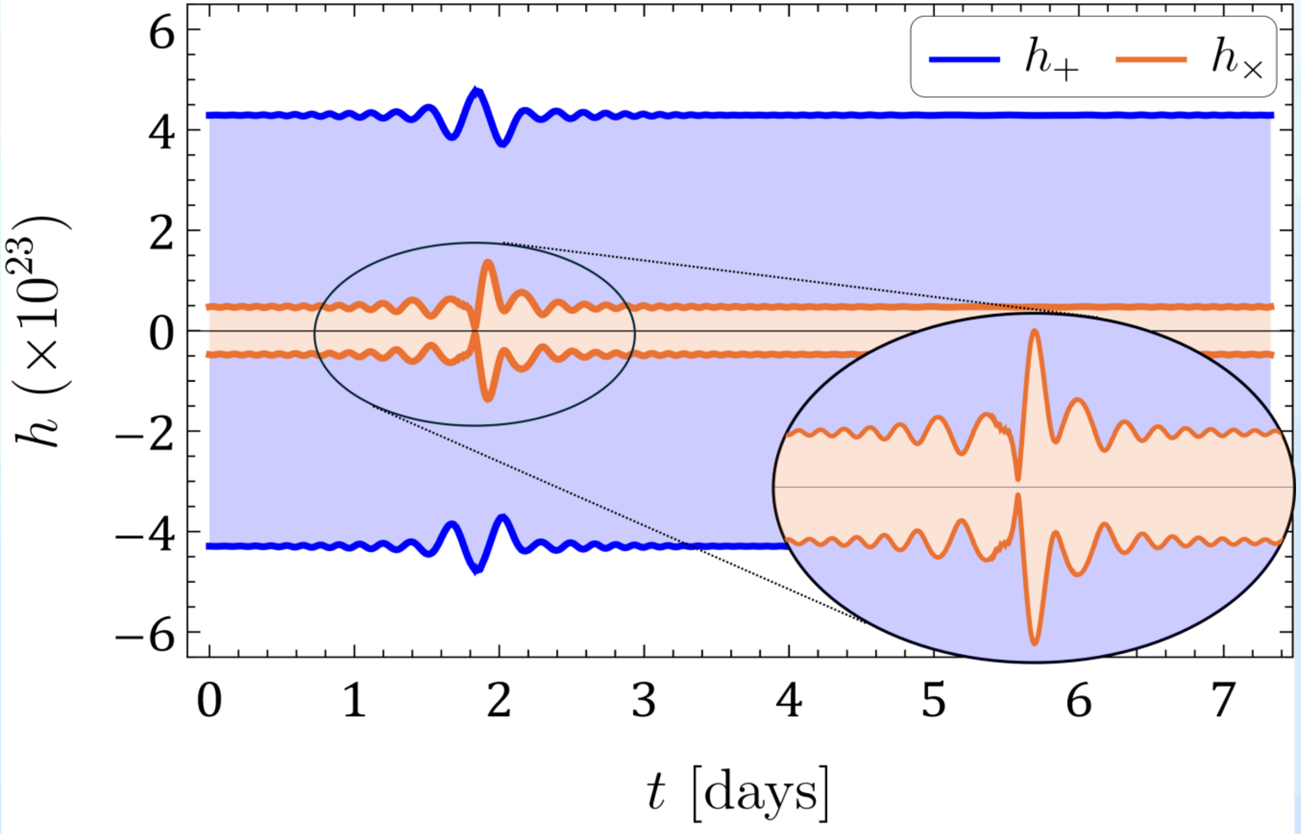
- LISA-band system : $\omega = 2\pi f = 2\pi \times 3 \times 10^{-3}\mathrm{Hz}$
- AGN lens mass : $M = 1.2 \times 10^6 M_\odot \implies \boxed{\omega M = 0.11}$
- Source-Lens distance (disk migration trap) : $d_{SL} = 700M$
- GW190521-inspired heavy source : $m_1 = 120 M_\odot, m_2 = 71 M_\odot$

LISA **detectable** with SNR > 100 if at $z \sim 0.01$

Wave optics lensing in triple systems: towards a phenomenology

Toy GW190521-inspired source, in LISA- « optimal » wave optics





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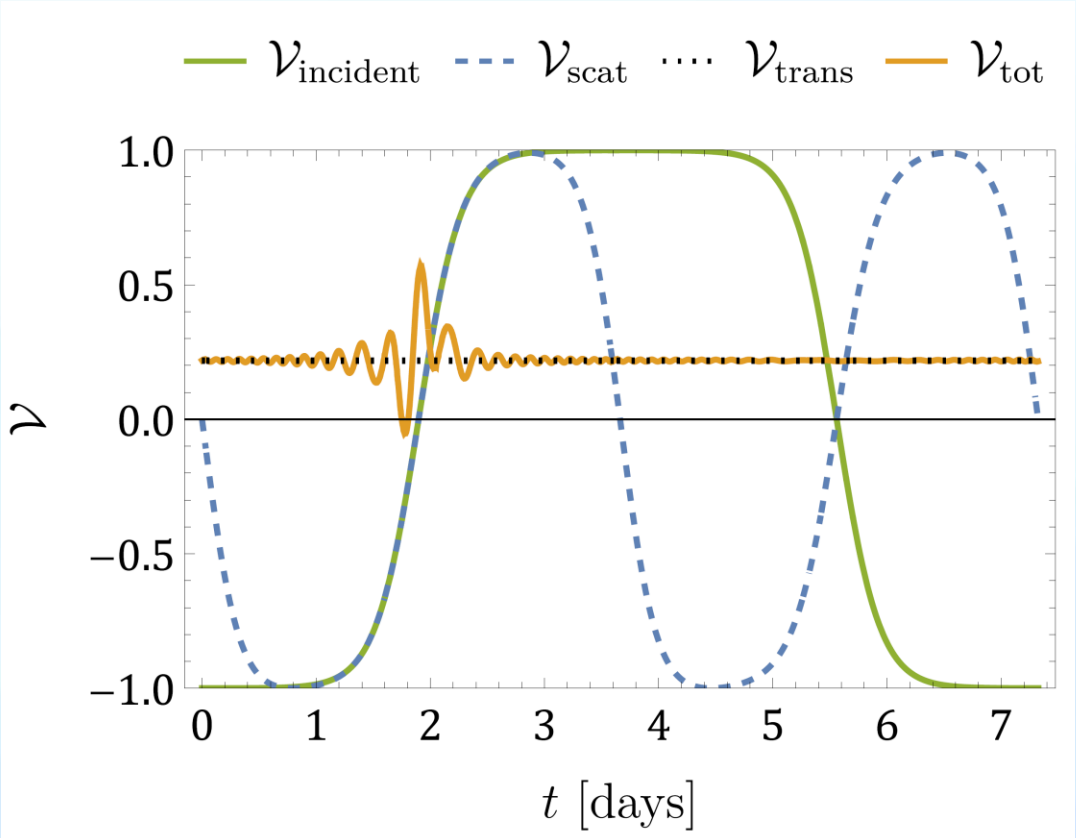

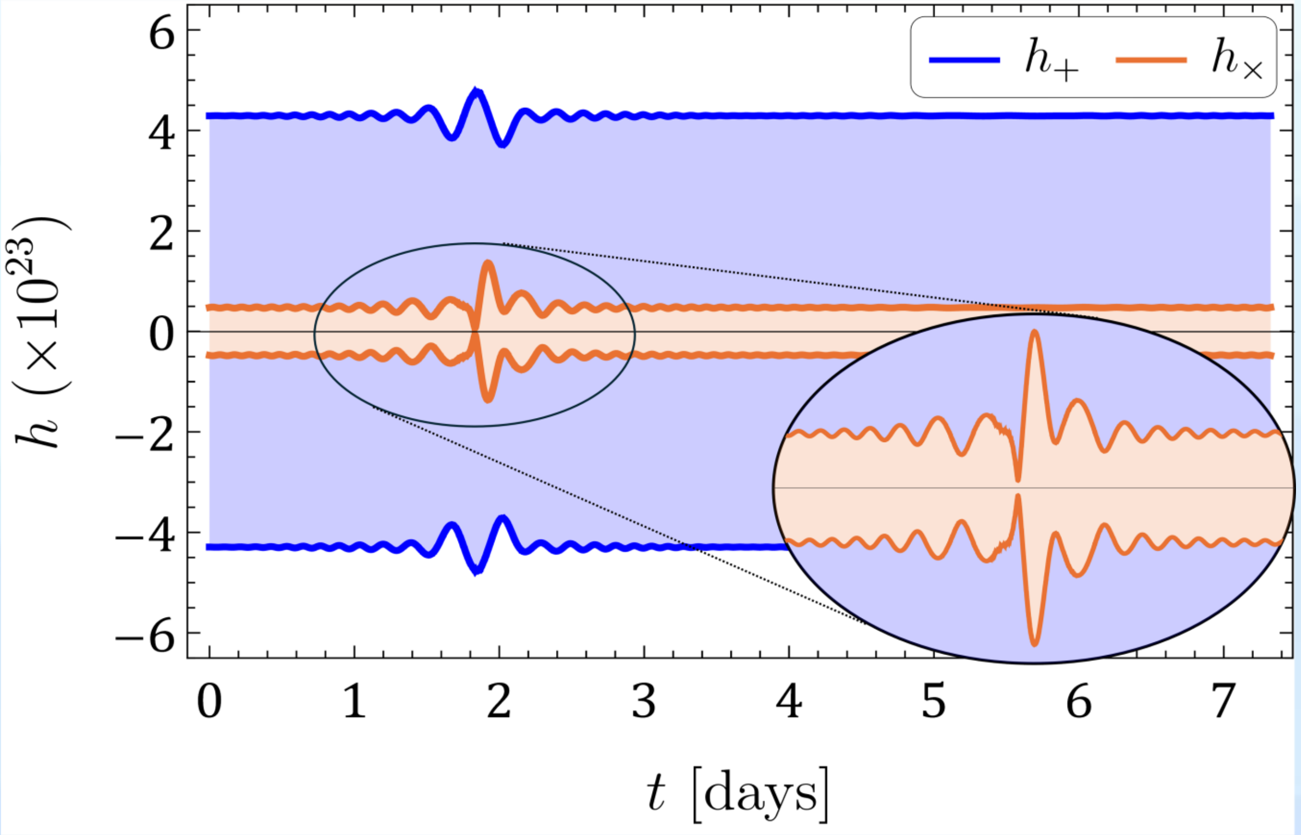

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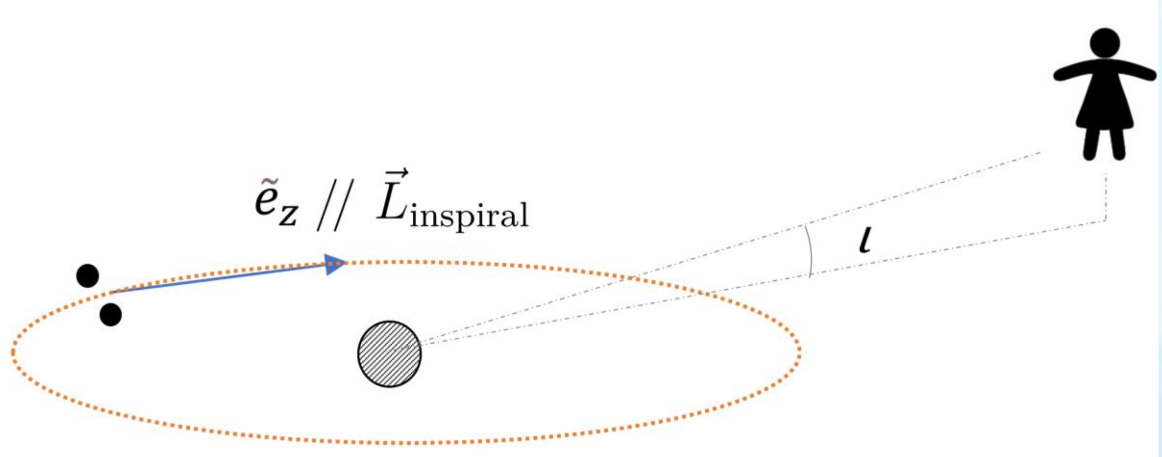
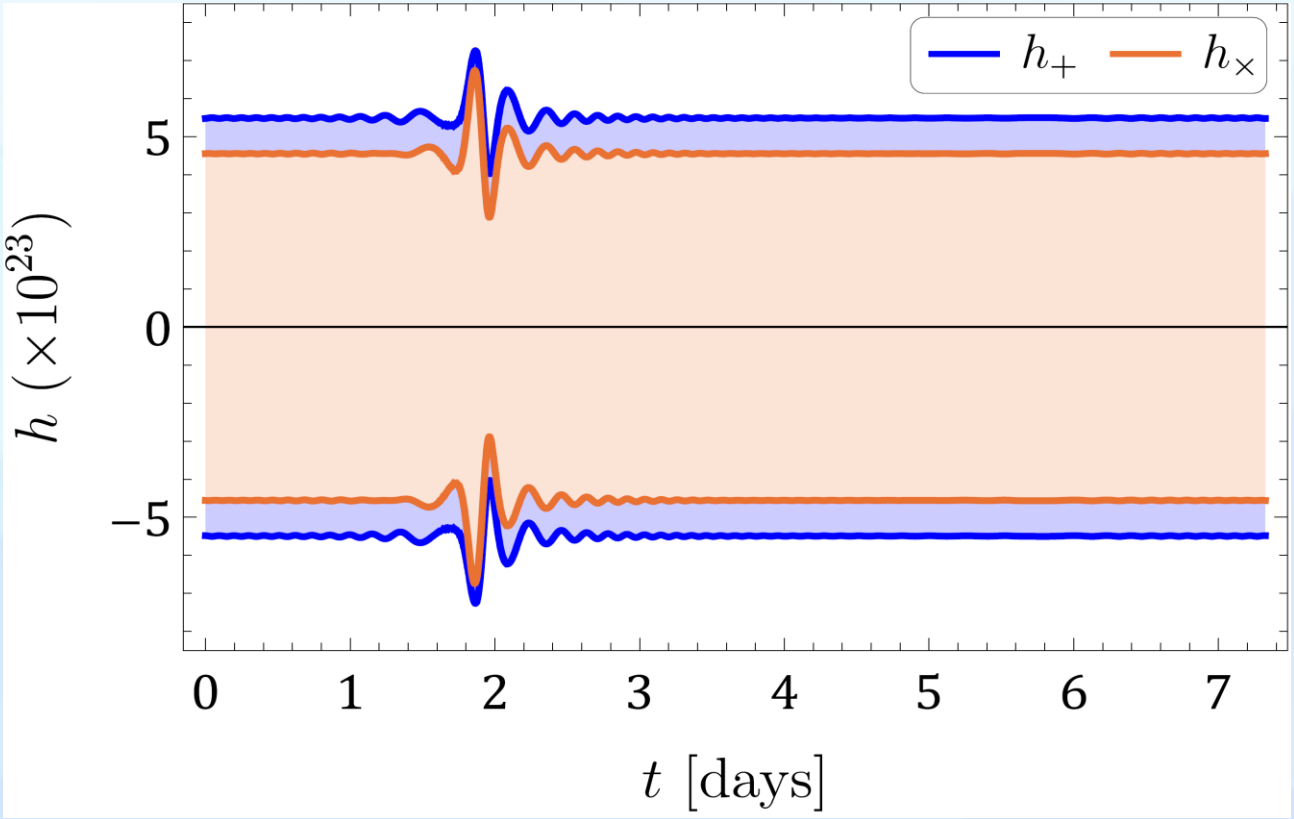
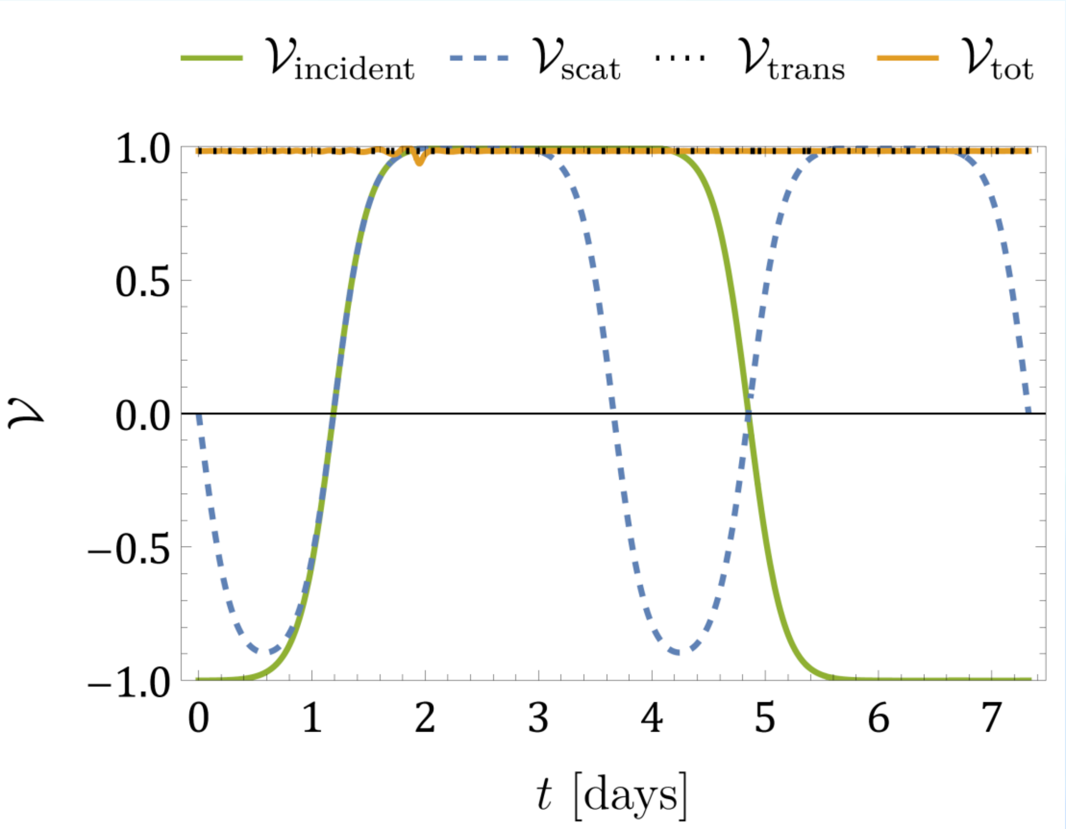


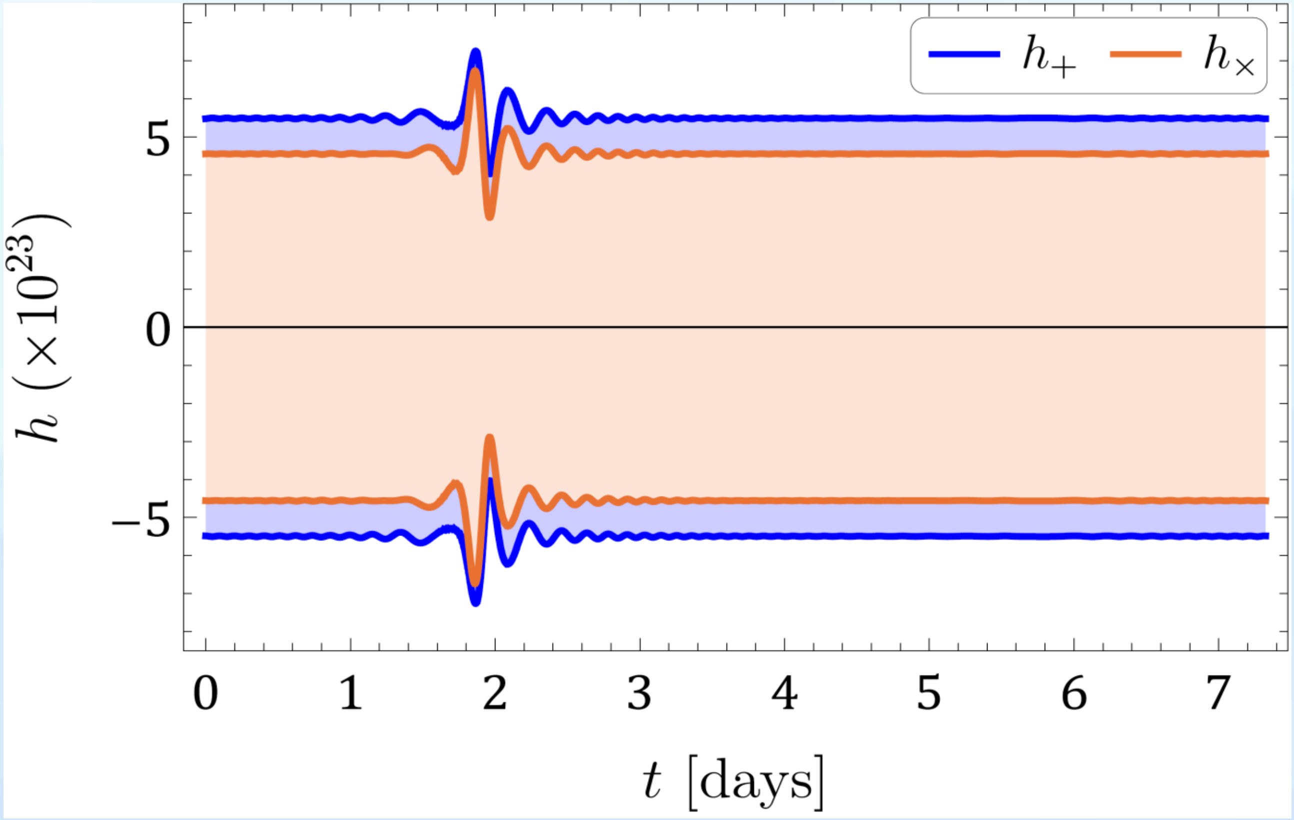
(not to scale)



Wave optics lensing in triple systems: towards a phenomenology

Toy GW190521-inspired source, in LISA- « optimal » wave optics





Conclusions

- **Wave optics** is a regime in the range of future observations (LISA)
- **Triple systems** exhibit a rich dynamical lensing phenomenology
- In wave optics, GW lensing is a **fully tensorial** process:
  - Results go beyond a simple scalar amplification factor
  - **The polarisation/helicity structure is not preserved by lensing**