# Black holes with electroweak hair

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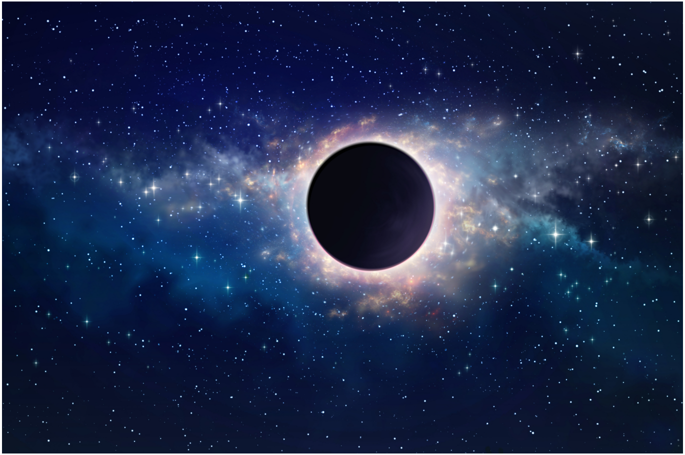
# Isolated black hole – Kerr-Newman geometry



3 parameters: mass $M$, charge $Q$, angular momentum $J$.

# Brief history: no-hair conjecture

No-hair conjecture /Ruffini and Wheeler, 1971/: black holes formed by gravitational collapse are characterized by their mass, angular momentum, and electric/magnetic charge. These are the only parameters that can survive during the gravitational collapse, all other information is lost. Black holes have no memory.

# No-hair conjecture

from the archives

From January 1971, pages 30–41

## Introducing the black hole

Remo Ruffini and John A. Wheeler

**According to present cosmology, certain stars end their careers in a total gravitational collapse that transcends the ordinary laws of physics.**

At the time of this article, **Remo Ruffini** and **John Wheeler** were both at Princeton University; Wheeler, on leave from Princeton, was spending a year at the California Institute of Technology and Moscow State University.

**The quasi-stellar object,** the pulsar, the neutron star have all come onto the scene of physics within the space of a few years. Is the next entrant destined to be the black hole? If so, it is difficult to think of any development that could be of greater significance. A black hole, whether of "ordinary size" (approximately one solar mass, 1 $M_\odot$) or much larger (around $10^6\ M_\odot$ to $10^{10}\ M_\odot$, as proposed in the nuclei of some galaxies), provides our "laboratory model" for the gravitational collapse, predicted by Einstein's theory, of the universe itself.

A black hole is what is left behind after an object has undergone complete gravitational collapse. Spacetime is so strongly curved that no light can come out, no matter can be ejected, and no measuring rod can ever survive being put in. Any kind of object that falls into the black hole loses its separate identity, preserving only its mass, charge, angular momentum, and linear momentum (see figure 1). No one has yet found a way to distinguish between two black holes constructed out of the most different kinds of matter if they have the same mass, charge, and angular momentum. Measurement of these three determinants is permitted by their effect on the Kepler orbits of test objects, charged and uncharged, in revolution about the black hole.

How the physics of a black hole looks depends more upon an act of choice by the observer himself than on anything else. Suppose he decides to follow the collapsing matter through its collapse down into the black hole. Then he will see it crushed to indefinitely high density, and he himself will be torn apart eventually by indefinitely increasing tidal forces. No restraining force whatsoever has the power to hold him away from this catastrophe, once he crossed a certain critical surface known as the "horizon." The final collapse occurs a finite time after the passage of this surface, but it is inevitable. Time and space are interchanged inside a black hole in

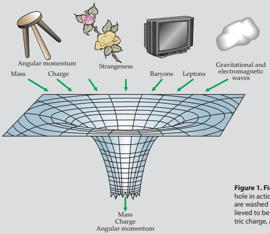



**Figure 1. Figurative representation** of a black hole in action. All details of the infalling matter are washed out. The final configuration is believed to be uniquely determined by mass, electric charge, and angular momentum.

# Uniqueness and no-hair theorems

- Uniqueness theorems /Israel, Robinson, Mazur/: All electrovacuum holes are described by the Kerr-Newman metrics. This confirms the conjecture.
- Are there other black holes, not described by Kerr-Newman metrics ?
- No-hair theorems /Bekenstein, 1972,.../ confirm the conjecture for a number of special cases. Considering

$$G_{\mu\nu} = T_{\mu\nu}(\Phi), \qquad \Box\Phi = U(\Phi),$$

  where $\Phi =$ scalar, spinor, massive vector field, etc., field, one can show that the only black hole solutions are of the Kerr-Newman type.
- However, if $\Phi = A^a_\mu$ is a pure Yang-Mills field then there are new black holes without new charges:

# Non-Abelian Einstein-Yang–Mills black holes

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(Submitted 7 September 1989)
Pis'ma Zh. Eksp. Teor. Fiz. **50**, No. 7, 312–315 (10 October 1989)

Solutions of the self-consistent system of Einstein-Yang–Mills equations with the SU(2) group are derived to describe black holes with a non-Abelian structure of gauge fields in the external region.

In the case of the electrovacuum, the most general family of solutions describing spherically symmetric black holes is the two-parameter Reissner–Nordström family, which is characterized by a mass $M$ and an electric charge $Q$. It was recently shown for the Einstein-Yang–Mills systems of equations with the SU(2) group that a corresponding assertion holds when the hold has a nonvanishing color-magnetic charge. In this case the structure of the Yang–Mills hair is effectively Abelian.[1] In the present letter we numerically construct a family of definitely non-Abelian solutions for Einstein-Yang–Mills black holes in the case of zero magnetic charge. These solutions are characterized by metrics which asymptotically approach the Schwarzschild metric far from the horizon but are otherwise distinct from metrics of the Reissner–Nordström family. In addition to the complete Schwarzschild metric, the family of solutions is parametrized by a discrete value of $n$: the number of nodes of the gauge function. For a

# Zoo of hairy black holes

- before 2000: Einstein-Yang-Mills black holes and their generalizations – higher gauge groups, additional fields (Higgs, dilaton), non-spherical solutions, stationary generalizations, Skyrme black holes, Gauss-Bonnet, ... /M.S.V.+Gal'tsov, Phys.Rep. 319 (1999) 1/

- after 2000: black holes via engineering the scalar field potential, Horndeski black holes, spontaneously scalarized black holes, black holes supporting spinning clouds of ultralight bosons /Herdeiro-Radu/, hairy black holes in higher dimensions, with stringy corrections, with massive gravitons /Gervalle+M.S.V., 2020/, etc, ... /M.S.V., 1601.0823/

- Which of these solutions are physical ? Unfortunately, one cannot be too optimistic in this respect.

# Present status of hairy black holes

- Almost all known hairy solutions have been obtained either within too much simplified models, or within exotic models relying on a new physics = yet undiscovered particles and fields. They are nice theoretically but their physical relevance is not obvious.

- New physics (stringy effects, SUSY, GUT fields, Horndeski fields, ultralight Dark Matter, massive gravitons, etc) may exist. However, its existence has not been confirmed yet.

- To be physically relevant, solutions should be obtained within General Relativity (GR) + Standard Model (SM) of fundamental interactions.

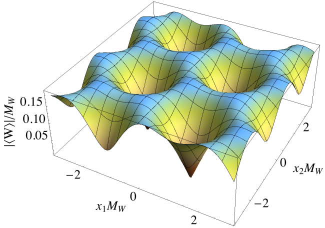
- The SM contains the QCD sector with pure Yang-Mills (gluons). Therefore, hairy black holes with Yang-Mills field may have some relevance. However, classical configurations in QCD are destroyed by large quantum corrections.

# Electroweak black holes ?

- The Standard Model contains also the electroweak (EW) sector where the quantum corrections are not very large. Therefore, it makes sense to study classical solutions of the Einstein-Weinberg-Salam theory. This theory contains the Einstein-Maxwell sector and hence describes the Kerr-Newman black holes.
- Does it describe something else ?
- Only unphysical limits of the electroweak theory (vanishing Weinberg angle) have been analyzed in the black hole context, since in the full theory the spherical symmetry is lost due to the electroweak condensation.

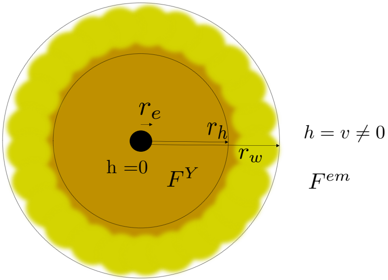
# Electroweak condensation /Ambjorn-Olesen 1989/

- Constant homogeneous magnetic field $\vec{\mathcal{B}} = (0, 0, \mathcal{B})$ may exist if only $\mathcal{B} < m_{\rm w}^2/e \approx 10^{20}$ Tesla.
- For $m_{\rm w}^2/e < \mathcal{B} < m_{\rm h}^2/e$ the vacuum structure changes leading to the appearance of a condensate of massive $W, Z, \Phi$ fields forming a lattice of vortices. Anti-Lenz: the magnetic field is maximal where the condensate is maximal.




- For $B > m_{\rm h}^2/e$ the vortices disappear and the Higgs field approaches zero – the full electroweak symmetry is restored.

# Magnetic electroweak black hole /Maldacena 2020/




Radial magnetic field near the horizon where Higgs=0, followed by electroweak "corona" made of vortex pieces, followed by radial magnetic field in the far field where Higgs is constant = magnetic Reissner-Nordstrom.

Nobody tried to confirm this

# Preliminary analyzis in flat space

- The electroweak corona should exist already in flat space around a pointlike magnetic charge. Therefore, one may start by studying condensation around the electroweak monopoles in flat space.

- What is known about magnetic monopoles in general and about magnetic monopoles in the electroweak theory ?

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# Pointlike magnetic monopole of Dirac in flat space

# Dirac monopole /1930/

Pointlike magnetic charge in Maxwell electrodynamics:

$$\vec{\mathcal{B}} = \frac{P\vec{r}}{r^3}, \quad \Rightarrow \quad \vec{\nabla}\cdot\vec{\mathcal{B}} = 4\pi P \delta^3(\vec{r}) \neq 0, \text{ nevertheless } \quad \vec{\mathcal{B}} = \vec{\nabla}\times\vec{\mathcal{A}}_\pm$$

where the vector potential contais the Dirac string singularity, which can be excluded by using two local gauges:

$$\begin{aligned} \mathcal{A}_- &= -P(\cos\vartheta - 1)d\varphi \ \text{ in northern hemisphere } \quad \vartheta &\in [0, \pi/2 + \epsilon) \\ \mathcal{A}_+ &= -P(\cos\vartheta + 1)d\varphi \ \text{ in southern hemisphere } \quad \vartheta &\in (\pi/2 - \epsilon, \pi] \end{aligned}$$

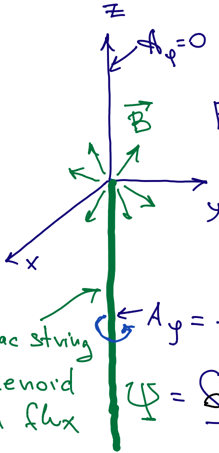
These two gauges are related in the equatorial region,

$$\mathcal{A}_- = \mathcal{A}_+ + d\,(2P\varphi)\,, \qquad \psi_+ = \exp\left(i\,2eP\varphi\right)\psi_-$$

hence $2eP = n \in \mathbb{Z} \ \Rightarrow \boxed{P = \frac{n}{2e}}$ /$P$, $n$ are called "magnetic charge"/

Energy is infinite.

# Dirac monopole as a solenoid




$$A = P(\cos\theta - 1)\, d\varphi$$

$$F = dA = -P \sin\theta \, d\theta \wedge d\varphi$$

$$\boxed{B = \ast F = -\frac{P}{r^2}\, dr}$$

radial magnetic field

$$\Psi = \oint A_\varphi \, d\varphi = -4\pi P$$

Dirac string = solenoid with flux $\Psi = -4\pi P$

# Dirac monopole in the electroweak theory

$$B = W^3 \;\; = \;\; -\frac{n}{2}\,(\cos\vartheta \pm 1)\,d\varphi, \qquad W^1 = W^2 = 0. \qquad \Phi = \begin{pmatrix} 0 \\ 1 \end{pmatrix}$$

Electromagnetic field is a linear combination:

$$\mathcal{A} \;\; = \;\; \frac{1}{e}\,\left(\sin^2\theta_{\rm W}\,B + \cos^2\theta_{\rm W}\,W^3\right) = \frac{1}{e}\,B, \qquad \vec{\mathcal{B}} = \frac{P\vec{r}}{r^3}, \quad P = \frac{n}{2e}.$$

The energy is infinite.

- New result: Dirac monopole within the electroweak theory is perturbatively unstable because the magnetic field is unbounded for $r \to 0$ and triggers the condensation.

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# Non-Abelian monopoles in flat space

# t'Hooft-Polyakov monopole /1974/

Gauge fiel theory with a triplet Higgs field

$$\mathcal{L} = -\frac{1}{4e^2}\, F^a_{\mu\nu} F^{a\mu\nu} - \frac{1}{2} D_\mu \Phi^a D^\mu \Phi^a - \frac{\lambda}{4}\left(\Phi^a\Phi^a - \Phi_0^2\right)^2$$

with $D_\mu \Phi^a = \partial_\mu \Phi^a + \epsilon_{abc} A^b_\mu \Phi^c$.

A globally regular solution with a *finite energy* and magnetic charge $P = 1/e$. Extremely popular.

Does not belong to the Standard Model, requires a new physics (GUT) which may or may not exist.

In the electroweak theory there is other non-Abelian monopole:

# Cho-Maison monopole /1996/

U(1) hypercharge field $B = (1 - \cos\vartheta)\, d\varphi$ as for the Dirac monopole with $n = 2$, combined with non-Abelian

$$W^a_\mu\, dx^\mu \quad = \quad (1 - f(r))\, \epsilon_{aik} \frac{x^i dx^k}{r^2}\,, \quad \Phi = \phi(r) \begin{pmatrix} \sin\frac{\vartheta}{2}\, e^{-i\varphi} \\ -\cos\frac{\vartheta}{2} \end{pmatrix}$$

$\Rightarrow$ *non-linear superposition of Dirac and t'Hooft-Polyakov.*
The total magnetic charge

$$P = \frac{1}{e} = \frac{\cos^2\theta_{\rm W}}{e} + \frac{\sin^2\theta_{\rm W}}{e} \equiv P_{\rm U(1)} + P_{\rm SU(2)}$$

where $P_{\rm U(1)}$ is pointlike and $P_{SU(2)}$ is distributed over the space. The energy is infinite due to the pointlike charge.

New result: solution is perturbatively stable. Can be considered as the pointlike Dirac monopole dressed with the condensate.

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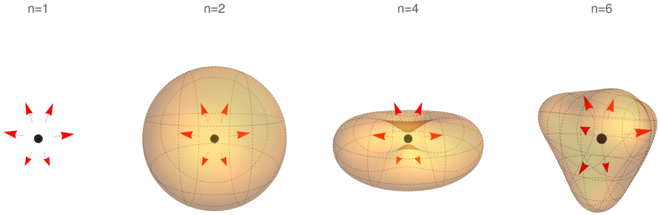
New result: this solution can be generalized to $|n| > 2$:

# Electroweak monopoles in flat space




New result: we constructed numerically monopoles with axial symmetry up to $|n| = 200$. They contain a pointlike magnetic charge surrounded by a condensate. The energy is infinite due to the central singularity.

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When gravity is taken into account, the singularity should be shielded by a horizon and the energy will become finite.

# Including gravity

/R.Gervalle and M.S.V.,
PRL 133 (2024) 171402/

# Einstein-Weinberg-Salam theory

$$\mathcal{L} = \frac{1}{2\kappa}\, R + \mathcal{L}_{\rm WS}$$

$$\mathcal{L}_{\rm WS} = -\frac{1}{4g^2}\, \mathrm{W}^a_{\mu\nu}\mathrm{W}^{a\mu\nu} - \frac{1}{4g'^2}\, B_{\mu\nu}B^{\mu\nu} - (D_\mu\Phi)^\dagger D^\mu\Phi - \frac{\beta}{8}\left(\Phi^\dagger\Phi - 1\right)^2$$

where Higgs is a complex doublet, $\Phi^{\rm tr} = (\phi_1, \phi_2)$,

$$\begin{aligned} \mathrm{W}^a_{\mu\nu} &= \partial_\mu \mathrm{W}^a_\nu - \partial_\nu \mathrm{W}^a_\mu + \epsilon_{abc}\mathrm{W}^b_\mu \mathrm{W}^c_\nu\,, \qquad B_{\mu\nu} = \partial_\mu B_\nu - \partial_\nu B_\mu\,, \\ D_\mu\Phi &= \left(\partial_\mu - \frac{i}{2}\, B_\mu - \frac{i}{2}\,\tau^a \mathrm{W}^a_\mu\right)\Phi\,. \end{aligned}$$

The length scale and mass scale are $\boldsymbol{l}_0 = 1.5\times 10^{-16}$ cm and $\boldsymbol{m}_0 = 128.6$ GeV. The couplings $g' = \sin\theta_{\rm W}$, $\quad g = \cos\theta_{\rm W}$,

$$g^2 = 0.78, \quad g'^2 = 0.22, \quad \beta = 1.88, \quad \kappa = \frac{4e^2}{\alpha}\frac{m_z^2}{M_{\rm pl}^2} = 5.30\times 10^{-33}.$$

Electron charge $e = gg'$, $\alpha = 1/137$. The $Z, W$, Higgs masses in unites of $\boldsymbol{m}_0$ are $m_z = 1/\sqrt{2}$, $m_w = g m_z$, $m_h = \sqrt{\beta}\, m_z$.

# Electromagnetic field (no unique definition if $\Phi \neq const$):

Nambu: $e\mathcal{F}_{\mu\nu} = g^2 B_{\mu\nu} - g'^2 n_a \mathrm{W}^a_{\mu\nu}, \quad n_a = (\Phi^\dagger \tau_a \Phi)/(\Phi^\dagger \Phi)$

defines conserved electric and magnetic currents

$$4\pi \mathcal{J}^\mu = \nabla_\nu \mathcal{F}^{\mu\nu}, \qquad 4\pi \tilde{\mathcal{J}}^\mu = \nabla_\nu \tilde{\mathcal{F}}^{\mu\nu},$$

magnetic charge

$$\boxed{P = \int \tilde{\mathcal{J}}^0 \sqrt{-\mathrm{g}} d^3x.}$$

t'Hooft: $F_{\mu\nu} = \mathcal{F}_{\mu\nu} + \epsilon_{abc} n^a \mathcal{D}_\mu n^b \mathcal{D}_\nu n^c = \partial_\mu A_\nu - \partial_\nu A_\mu,$

electric current

$$\boxed{4\pi J^\mu = \nabla_\nu F^{\mu\nu}}.$$

Using Nambu for magnetic charge and t'Hooft for electric current.

# 30 coupled equations to solve:

Weinberg-Salam:

$$\nabla^\mu B_{\mu\nu} = g'^2 \frac{i}{2}(\Phi^\dagger D_\nu \Phi - (D_\nu \Phi)^\dagger \Phi),$$
$$\mathcal{D}^\mu \mathrm{W}^a_{\mu\nu} = g^2 \frac{i}{2}(\Phi^\dagger \tau^a D_\nu \Phi - (D_\nu \Phi)^\dagger \tau^a \Phi),$$
$$D_\mu D^\mu \Phi - \frac{\beta}{4}(\Phi^\dagger \Phi - 1)\Phi = 0,$$

Einstein:

$$G_{\mu\nu} = \kappa T_{\mu\nu} \qquad \text{where } \kappa \sim 10^{-33} \text{ is very small and}$$
$$T_{\mu\nu} = \frac{1}{g^2}\mathrm{W}^a{}_{\mu\sigma}\mathrm{W}^a{}_\nu{}^\sigma + \frac{1}{g'^2}B_{\mu\sigma}B_\nu{}^\sigma + 2D_{(\mu}\Phi^\dagger D_{\nu)}\Phi + \mathrm{g}_{\mu\nu}\mathcal{L}_{\mathrm{WS}}$$

=30 coupled equations. Vacuum solution:

$$\mathrm{g}_{\mu\nu} = \eta_{\mu\nu}, \quad \mathrm{B} = \mathrm{W} = 0, \quad \Phi = \begin{pmatrix} 0 \\ 1 \end{pmatrix}$$

# Simplest solution: Reissner-Nordstrom

Same electroweak fields as for Dirac monopole,

$$
\begin{aligned}
B &= W^3 = -\frac{n}{2}\cos\vartheta\, d\varphi, \quad W^1 = W^2 = 0, \quad \Phi = \begin{pmatrix} 0 \\ 1 \end{pmatrix}, \\
\vec{\mathcal{B}} &= \frac{P\vec{r}}{r^3} \quad \text{where} \quad P = \frac{n}{2e}, \quad n \in \mathbb{Z} = \text{magnetic charge}
\end{aligned}
$$

*Higgs field is in vacuum.* Reissner-Nordstrom (RN) geometry

$$
\begin{aligned}
ds^2 &= -N(r)\, dt^2 + \frac{dr^2}{N(r)} + r^2 \left(d\vartheta^2 + \sin^2\vartheta\, d\varphi^2\right), \\
N(r) &= 1 - \frac{2M}{r} + \frac{Q^2}{r^2}, \qquad Q^2 = \frac{\kappa n^2}{8e^2}, \qquad r_h = M + \sqrt{M^2 - Q^2}
\end{aligned}
$$

Describes the $r \to \infty$ limit of the hairy black holes.

# Another simple solution: RN-de Sitter

$$B = -\frac{n}{2}\cos\vartheta\, d\varphi, \quad W = 0, \quad \Phi = \begin{pmatrix} 0 \\ 0 \end{pmatrix}$$

$$\Rightarrow \quad \text{Higgs is in } \underline{\text{false vacuum}}, \qquad \vec{\mathcal{B}} = g^2 \frac{P\vec{r}}{r^3},$$

$$ds^2 = -N(r)\,dt^2 + \frac{dr^2}{N(r)} + r^2\left(d\vartheta^2 + \sin^2\vartheta\, d\varphi^2\right),$$

$$N(r) = 1 - \frac{2M}{r} + \frac{Q^2}{r^2} - \frac{\Lambda}{3}r^2, \qquad \Lambda = \frac{\kappa\beta}{8}.$$

In the extremal limit one has

$$N(r) = \left(1 - \frac{r_{\rm ex}}{r}\right)^2 \left(1 - \frac{\Lambda}{3}\left[r^2 + 2rr_{\rm ex} + 3r_{\rm ex}^2\right]\right),$$

$$r_{\rm ex} = \sqrt{\frac{1 - \sqrt{1 - 4g^2\Lambda Q^2}}{2\Lambda}} = g|Q| + \mathcal{O}(n^3\kappa^{5/2}).$$

Describes the horizon geometry of extremal hairy black holes.

# Perturbations around Reissner-Nordström

$$g_{\mu\nu} \to g_{\mu\nu} + \delta g_{\mu\nu}, \quad \mathrm{W}^{a}_{\mu} \to \mathrm{W}^{a}_{\mu} + \delta \mathrm{W}^{a}_{\mu}$$

$$B_{\mu} \to B_{\mu} + \delta B_{\mu}, \quad \Phi \to \Phi + \delta \Phi$$

# Perturbations $w_\mu = \delta \mathrm{W}^1_\mu + i\delta \mathrm{W}^2_\mu$ fulfil

charged Proca: $\boxed{D^\mu w_{\mu\nu} + ieF_{\nu\sigma} w^\sigma = m_{\rm w}^2 w_\nu}$ whose solution is

$$
\begin{aligned}
w_\mu dx^\mu &= \sum_{\mathrm{m}\in[-j,j]} c_{\rm m} w_{\rm m}(t,r,\vartheta,\varphi), \qquad /\text{arbitrary } c_{\rm m}/, \\
w_{\rm m}(t,r,\vartheta,\varphi) &= e^{i\omega t}\psi(r)(\sin\vartheta)^j \left(\tan\frac{\vartheta}{2}\right)^{\rm m} e^{i\mathrm{m}\varphi}(d\vartheta + i\sin\vartheta d\varphi), \\
j &= |n|/2 - 1 \;\Rightarrow\; \boxed{j = 0 \text{ only if } |n| = 2}.
\end{aligned}
$$

For $|n| > 2$ perturbations are not spherically symmetric. One has

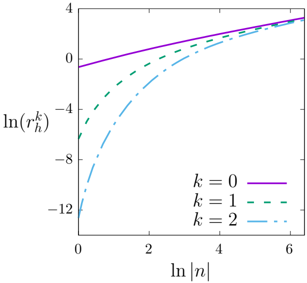
$$
\left(-\frac{d^2}{dr_\star^2} + N(r)\left[m_{\rm w}^2 - \frac{|n|}{2r^2}\right]\right)\psi(r) = \omega^2\psi(r). \qquad (\star)
$$

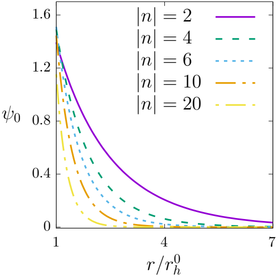
$\omega^2 > 0$ is positive if $r_h$ is large, $\omega^2 < 0$ if $r_h$ is small, for an intermediate value $r_h = r_h^0(n)$ there is a bound state $\psi_0(r)$ with $\omega^2 = 0$, which describes the condensate which starts to appear.

# Condensation threshold

| $n$ | 2 | 4 | 6 | 10 | 20 | 40 | 100 | 200 |
|---|---|---|---|---|---|---|---|---|
| $r_H^0$ | 0.89 | 1.47 | 1.93 | 2.69 | 4.12 | 6.19 | 10.33 | 15.03 |

One has $r_h^0(n) \approx \sqrt{|n|}/g$ for $n \gg 1$ hence $B(r_h^0) \approx m_{\rm w}^2$, which is the condition for the condensate to appear. The condensate is maximal at the horizon.



# Horizon distribution of vortices

The condensate field $w = \sum c_{\rm m} w_{\rm m}$ depends on coefficients $c_{\rm m}$ and produces a current $J^\mu = \nabla_\sigma \Im(\bar{w}^\sigma w^\mu)$ tangent to the horizon. The current sources second order corrections for the $F, Z, \Phi$ fields forming vortices orthogonal to the horizon. To determine the coefficients $c_{\rm m}$ we minimize the condensate energy. This amounts to minimizing

$$\langle |w_\mu|^4 \rangle \equiv \int |w_\mu|^4 \sqrt{-\mathrm{g}}\, d^3x \,,$$

by keeping fixed the norm

$$\langle |w_\mu|^2 \rangle \equiv \int |w_\mu|^2 \sqrt{-\mathrm{g}}\, d^3x = const.$$

This leads to the following prescription

# Minimization procedure

Minimize with respect to $c_{\mathrm{m}}$ and Lagrange multiplier $\mu$ the function

$$
\begin{aligned}
E[c_{\mathrm{m}}] &= E_4[c_{\mathrm{m}}] + \mu\,(E_2[c_{\mathrm{m}}] - 1),\\
E_4[c_{\mathrm{m}}] &= \sum_{\mathrm{k,m,l}\in[-\mathrm{j,j}]} A_{2j,\mathrm{k+l}}\, c_{\mathrm{m}}\, c_{\mathrm{k}}\, c_{\mathrm{l}}\, c_{\mathrm{k+l-m}}\\
E_2[c_{\mathrm{m}}] &= \sum_{\mathrm{m}\in[-j,j]} A_{j,\mathrm{m}}\, c_{\mathrm{m}}^2
\end{aligned}
$$

with $j = |n|/2 - 1$ and

$$
A_{j,\mathrm{m}} = \int_0^{\pi} (\sin\vartheta)^{2j+1} \left(\tan\frac{\vartheta}{2}\right)^{2\mathrm{m}} d\vartheta = 2^{2j+1}\frac{\Gamma(j+1+\mathrm{m})\Gamma(j+1-\mathrm{m})}{\Gamma(2j+2)}\,.
$$

This gives values of $c_{\mathrm{m}}$ determining positions of $|n| - 2$ vortices homogeneously distributed over the horizon.

# Lattice of vortices – corona

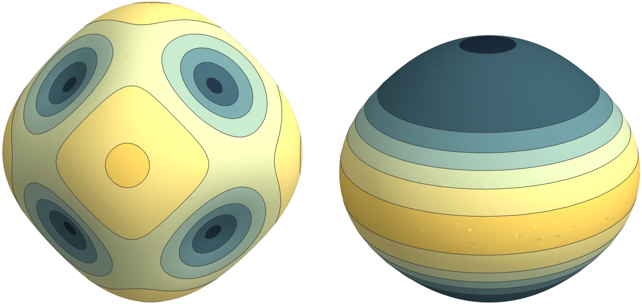


Figure: Left: the horizon distribution of the W-condensate $\bar{w}^{\mu} w_{\mu}$ corresponding to the global energy minimum for $n = 10$. The level lines coincide with the electric current flow forming loops around 8 radial vortices (dark spots) repelling each other and forming a lattice.
Right: the same when all vortices merge into two oppositely directed multi-vortices with axial symmetry, $c_{\mathrm{m}} \sim \delta_{0\mathrm{m}}$, also a stationary point.

# Non-perturbative analysis

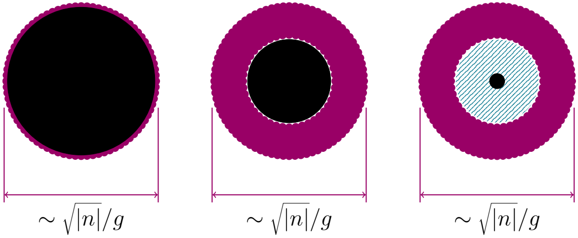
# Axial symmetry

$$
\begin{aligned}
ds^2 &= -e^{2\mathrm{U}}\mathrm{N}(\mathrm{r})\,dt^2 + e^{-2\mathrm{U}}\,dl^2,\\
dl^2 &= e^{2\mathrm{K}}\left[\frac{d\mathrm{r}^2}{\mathrm{N}(\mathrm{r})} + \mathrm{r}^2 d\vartheta^2\right] + e^{2\mathrm{S}}\,\mathrm{r}^2\sin^2\vartheta d\varphi^2,\\
\mathrm{W} &= \mathrm{T}_2\,(F_1\,d\mathrm{r} + F_2\,d\vartheta) - \frac{n}{2}\,(\mathrm{T}_3\,F_3 - \mathrm{T}_1 F_4\,)\,d\varphi\,,\\
B &= -(n/2)\,Y\,d\varphi\,,\quad \Phi^{\mathrm{tr}} = (\phi_1,\phi_2).
\end{aligned}
$$

Here $\mathrm{U},\mathrm{K},\mathrm{S},F_1,F_2,F_3,F_4,Y,\phi_1,\phi_2$ are 10 real functions of $\mathrm{r},\vartheta$. For non-extremal solutions $\mathrm{N}(\mathrm{r}) = 1 - \mathrm{r}_\mathrm{H}/\mathrm{r}$ where $\mathrm{r}_\mathrm{H}$ labels the solutions. We require $Z_2$ invariance under $\vartheta \to \pi - \vartheta$, obtain 10 elliptic equations fer the 10 functions, and solve them numerically with the FreeFem++ numericatl solver. We solve for values of the gravity coupling $10^{-10} < \kappa < 10^{-2}$ and then extrapolate to the physical value $\kappa \sim 10^{-33}$.

# Hairy black holes

Same magnetic charge $P = n/(2e)$ as for the RN black hole. We start at $\mathrm{r_H} = \mathrm{r_H^0}(n)$ when hairy solutions just start deviating from RN and then we decrease $\mathrm{r_H}$. The massive hair appears and gets longer as the horizon shrinks. When the field at the horizon increases up to $B(\mathrm{r_H}) = m_{\mathrm{h}}^2$, the hair stops growing and a bubble of symmetric phase appears. This bubble expands as the horizon shrinks further till reaching the minimal value when it becomes degenerate, surface gravity vanishes, but the area remain finite. The black hole then becomes extremal.



# Charge contained in the hair

The total magnetic charge $P$ of the black hole splits as

$$P_{\rm h} = \int_{\rm r>r_H} \tilde{J}^0 \sqrt{-{\rm g}}\, d^3x, \quad P_{\rm H} = P - P_{\rm h},$$

where $P_{\rm h}$ is contained in the hair outside the horizon and $P_{\rm H}$ remains inside. The hair charge $P_{\rm h}$ grows when the horizon shrinks and in the extremal limit one has

$$P_{\rm h} = g'^2 P = 0.22\, P$$

hence 22% of the charge moves to the hair.

# ADM mass

is determined from the asymptotic $\mathrm{g}_{00} = -1 + 2M/\mathrm{r} + \ldots$ or from the formula (same result)

$$M = \frac{\mathrm{k_H A_H}}{4\pi} \quad + \quad \frac{\kappa}{8\pi}\int_{\mathrm{r}>\mathrm{r_H}} \left(-T^0_{\ 0} + T^k_{\ k}\right) \sqrt{-\mathrm{g}}\, d^3x,$$

$$\text{surface gravity}: \quad \mathrm{k_H} = (1/2)\, \mathrm{N}' \mathrm{e}^{2\mathrm{U}-\mathrm{K}}\Big|_{\mathrm{r}=\mathrm{r_H}}$$

$$\text{horizon area}: \quad \mathrm{A_H} = 2\pi \mathrm{r}_\mathrm{H}^2 \int_0^\pi e^{\mathrm{K}+\mathrm{S}-2\mathrm{U}} \sin\vartheta\, d\vartheta\Big|_{\mathrm{r}=\mathrm{r_H}}$$

This can be split as

$$M = M_\mathrm{H} + M_\mathrm{h}$$

where the "horizon mass" $M_\mathrm{H}$ is the mass of the RN black hole with the same area $\mathrm{A_H}$ and with the charge $P_\mathrm{H}$. The rest is the "hair mass" $M_\mathrm{h} = M - M_\mathrm{H}$. When the horizon gets smaller, the hair mass $M_\mathrm{h}$ and hair charge $P_\mathrm{h}$ increase.

# Horizon oblatness

The configurations are not spherical, one can define

$$\begin{aligned}
\text{horizon radius :} \quad r_h &= \sqrt{\mathrm{A_H}/(4\pi)} \\
\text{equatorial radius :} \quad r_{\mathrm{H}}^{\mathrm{eq}} &= \sqrt{\mathrm{g}_{\varphi\varphi}(\mathrm{r_H}, \pi/2)} \\
\text{polar radius :} \quad r_{\mathrm{H}}^{\mathrm{pl}} &= (1/\pi) \int_0^{\pi} \sqrt{g_{\vartheta\vartheta}(\mathrm{r_H}, \vartheta)}\, d\vartheta \\
\text{horizon oblateness :} \quad \delta &= r_{\mathrm{H}}^{\mathrm{eq}}/r_{\mathrm{H}}^{\mathrm{pl}} - 1
\end{aligned}$$

As the horizon radius decreases, the oblateness $\delta$ stars from zero and increases, then reaches a maximum, starts decreasing and approaches zero in the extreme limit. The extremal horizon is perfectly spherical, although the hair is squashed.

# Quadrupole moments

Far away from the horizon the theory reduces to electrovacuum,

$$\mathcal{L} = \frac{1}{2\kappa} R - \frac{1}{4} F_{\mu\nu} F^{\mu\nu}.$$

Writing the metric as

$$\begin{aligned} ds^2 &= -e^{2\mathrm{U}} dt^2 + e^{-2\mathrm{U}} dl^2, \\ dl^2 &= e^{2\mathrm{K}} (d\mathrm{r}^2 + \mathrm{r}^2 d\vartheta^2) + \mathrm{r}^2 e^{2\mathrm{S}} \sin^2 \vartheta \, d\varphi^2 \, , \end{aligned}$$

dualizing the magnetic field, $\sqrt{\frac{\kappa}{2}} F_{ik} = e^{-2\mathrm{U}} \sqrt{h} \, \epsilon_{iks} \, \partial^s \Psi$ , the Ernst potential $\mathcal{E} = e^{2\mathrm{U}} - \Psi^2$. Passing to the Weyl coordinates where $dl^2 = e^{2K(\rho,z)}(d\rho^2 + dz^2) + \rho^2 d\varphi^2$ and considering the asymptotic expansions at the symmetry axis of

$$\xi = \frac{\mathcal{E} - 1}{1 + \mathcal{E}} = \sum_{k \geq 0} \frac{a_k}{z^{k+1}}, \qquad q = -\frac{2\Psi}{1 + \mathcal{E}} = \sum_{k \geq 0} \frac{b_k}{z^{k+1}},$$

gravitational and magnetic quadrupoles are $Q_{\mathrm{G}} = a_2$, $Q_{\mathrm{M}} = b_2$.

# Non-extremal hairy solutions

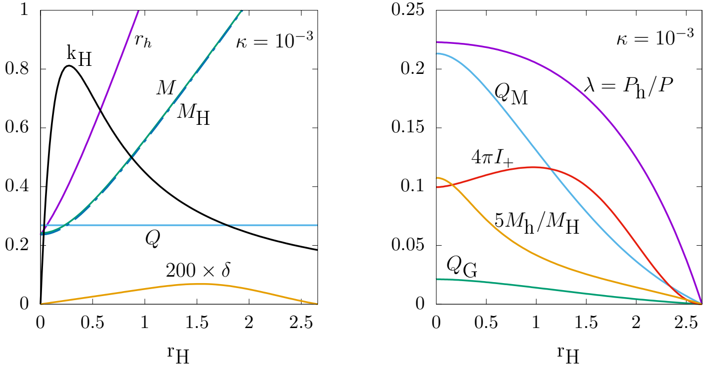



Figure: Parameters of non-extremal solutions with $n = 10$, $\kappa = 10^{-3}$. The $M$ and $M_\mathrm{H}$ curves are very close to each other. For $\mathrm{r_H} \to 0$ they become extremal, for $\mathrm{r_H} \to 2.66$ they loose hair and become RN.

# Extremal hairy solutions

They have zero surface gravity and are the most hairy. Depending on the value of their charge parameter

$$Q = \sqrt{\frac{\kappa}{2}}\, P = \sqrt{\frac{\kappa}{8}}\, \frac{n}{e}$$

there are two phases,

$$\text{phase I}: \quad Q < Q_\star, \qquad\qquad \text{phase II}: \quad Q > Q_\star,$$

where

$$Q_\star \approx \frac{0.3}{g\sqrt{\Lambda}}, \qquad \Lambda = \frac{\kappa\beta}{8}.$$

In phase I one has $B(r_h) > m_\mathrm{h}^2$ and the Higgs field vanishes at the horizon. In phase II one has $B(r_h) < m_\mathrm{h}^2$ and the Higgs field deviates from zero at the horizon.

# Extremal hairy solutions in phase I ($n = 40$)

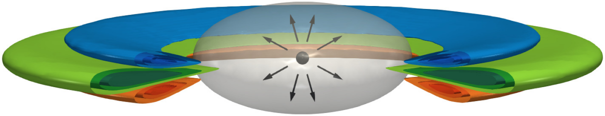


Figure: The extremal solutions contain a small charged black hole inside a bubble of symmetric phase, surrounded by a ring-shaped EW condensate supporting 22 % of the total magnetic charge and two opposite superconducting W-currents. This creates pieces of two magnetic multi-vortices along the positive and negative $z$-directions. Farther away the condensate disappears and the magnetic field becomes radial.

# Asymptotics

The Higgs vanishes at the horizon, the horizon geometry coincides with the extreme RN-de Sitter with $r_{\rm ex} \approx g|Q|$:

$$
\begin{aligned}
ds^2 &= -N\,dt^2 + \frac{dr^2}{N} + r^2(d\vartheta^2 + \sin^2\vartheta\, d\varphi^2) \qquad (\star)\\
N &= \left(1 - \frac{r_{\rm ex}}{r}\right)^2 \left(1 - \frac{\Lambda}{3}\left[r^2 + 2rr_{\rm ex} + 3r_{\rm ex}^2\right]\right)
\end{aligned}
$$

Far away from the horizon the geometry approaches RN described by $(\star)$ with

$$N = 1 - \frac{2M}{r} + \frac{Q^2}{r^2} + \mathcal{O}(1/r^3)$$

where (!!!)

$$\boxed{M < |Q|}$$

Condensate lowers the mass.

# Weak gravity

One has $M = M_{\rm H} + M_{\rm h}$. The hair carries 22% of the total charge, $Q_{\rm h} = 0.22 \times Q$, but hair mass $M_{\rm h}$ is very small due to the negative Zeeman energy of the condensate interacting with the magnetic field of the black hole, which shifts the W-mass as

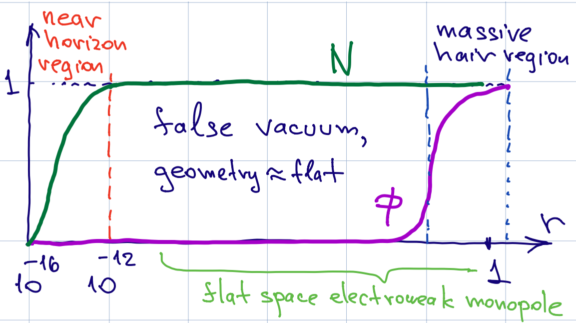
$$m_{\rm w}^2 \to m_{\rm w}^2 - |{\rm B}| \approx 0$$

As a result, the mass-to-charge ratio for the hair is very small, $M_{\rm h}/|Q_{\rm h}| \sim \sqrt{\kappa} \ll 1$. This can be viewed as a manifestation of the weak gravity conjecture. The condensate is magnetically repelled by the black hole stronger than attracted gravitationally, but it cannot fly away because it has to follow the Yukawa law. Since the hair mass is small, one has (if $Q \ll Q_\star$)

$$M = M_{\rm H} + M_{\rm h} \approx M_{\rm H} = \frac{r_{\rm ex}}{2} + \frac{g^2 Q^2}{2 r_{\rm ex}} \approx g|Q| = 0.88\,|Q| < |Q|$$

Hairy black hole is less energetic than RN for which $M \geq Q$
$\Rightarrow$ they cannot loose the hair and become RN.

# Extrapolating toward $\kappa \sim 10^{-33}$ if $Q \ll Q_\star$




$$ds^2 \approx -N\,dt^2 + \frac{dr^2}{N} + r^2\left(d\theta^2 + \sin^2\theta\, d\varphi^2\right)$$

near-horizon region is parametrically small as compared to the hair region. The hair lives in flat geometry and does not see the black hole, but the latter renders the mass finite by putting the cutoff at the horizon.

# Hairy black holes as magnetic monopoles

- Most popular magnetic monopoles of t'Hooft-Polyakov are not described by the Standard Model. They are described by GUT theories, which may or may not exist.
- Standard Model definitely exists and admits solutions describing electroweak monopoles, but in flat space their energy diverges because $B \sim n/(2r^2)$. This divergence might be cured by renormalization, but so far nobody has confirmed this.
- Gravity converts electroweak monopoles to hairy black holes and renders their mass finite:

$$M \approx 5.1\,|n|\,M_{\rm Pl}$$

Therefore, if there is no GUT physics, then these black holes are the only magnetic monopoles which may exist in Nature. They are heavy $\Rightarrow$ should be observed at very high energies

# Increasing the charge

- As the charge $Q \propto n$ increases, the hair length $\propto \sqrt{|Q|}$ grows till macroscopic size of order 1 cm. The horizon size $\propto |Q|$ grows faster and the black hole absorbs the bubble.

- The horizon value $B \sim 1/|Q|$ decreases and when $B < m_{\mathrm{h}}^2$ the Higgs deviates from zero at the horizon and the system enters phase II where the horizon becomes squashed. Near the transition point one has (with $s \approx 10.8$ if $\kappa = 10^{-2}$)

$$\delta \propto (|Q| - Q_\star)^s$$

- The fraction of the hair charge starts decreasing, the black hole starts loosing hair, the geometry approaches extreme RN and finally merges with it for

$$Q_{\max} = 2.15\, Q_\star = \frac{1.29}{2g\sqrt{\Lambda}}$$

No hairy solutions for $Q > Q_{\max}$.

# Existence diagram

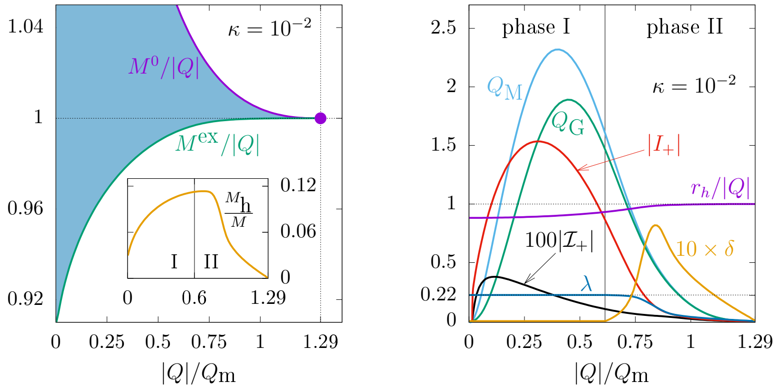



Figure: The parameters of extremal solutions (right) and the existence diagram for hairy solutions (left) for $\kappa = 10^{-2}$; $Q_{\mathrm{m}} = 1/(g\sqrt{\Lambda})$.

# Maximally hairy extremal black hole

The black hole is maximally hairy around the phase transition point when the fraction of the hair mass $M_{\mathrm{h}}/M$ is maximal. Then

$$|n| \approx 1.5 \times 10^{32}, \qquad r_h \approx 1.37 \text{ cm},$$

the black hole mass has a planetary value,

$$M \approx 2 \times 10^{25} \;\; \text{kg}$$

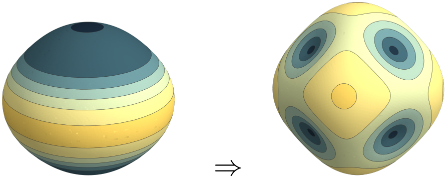
of which $\approx 11\%$ is contained in the hair condensate.

# Stability

According to Maldacena, the corona greatly enhances the Hawking evaporation rate, hence non-extremal black holes should quickly relax to the extremal state when their temperature is zero and

$$M < |Q|$$

Therefore, they cannot decay into RN black holes. However, axially symmetric black holes can further reduce their mass by splitting their hair into a hedgehog of vortices – "spreading the corona".
Then the condensate energy achieves an absolute minimum and the hairy black holes seem to become absolutely stable. The corresponding solutions have not yet been obtained.



# Phenomenology

Since they are described by well-tested theories, the hairy EW black holes are expected to be physically relevant. They could probably originate from primordial black holes. If the fluctuating magnetic field in the ambiant EW plasma becomes at some moment mostly orthogonal to the black hole horizon, or a piece of a magnetic vortex gets attached to the horizon, this creates a flux through the horizon = charge. This flux should be compensated by the opposite flux created on other black hole(s). The oppositely charged black holes will not necessarily annihilate, being pushed apart by the cosmic expansion, or maybe they form bound systems stablized by the scalar repulsion.

Such black holes should catalize the proton decay. They can be detected when captured by a neutron star, causing a sudden change of the star's rotation period. Estimates based on proton decay and Parker bound show that their contribution in Dark Matter is small, unless they form neutral bound systems.

# Conclusions

- We constructed for the first time hairy black holes described by well-tested theories, GR and SM. This suggests that they may really exist in Nature. Perhaps they could have been created by fluctuations in primordial electroweak plasma.
- The can be as large as $\approx 1$ cm with approximately Terrestrial mass $M \sim 10^{25}$ kg of which $\approx 10\%$ is stored in the electroweak condensate hair.
- They have $M < |Q|$ hence they cannot get rid of the hair and evolve into RN. When they spread their corona, they lower the energy to an absolute minimum and seem to become absolutely stable.
- If there is no GUT physics beyond the Standard Model, then they should be the only magnetic monopoles which may exist.