Towards galaxy cluster models in Aether-Scalar-Tensor theory

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MINISTRY OF EDUCATION, YOUTH AND SPORTS

In the dark

From small to large scales

- Independent lines of evidence, on a range of scales, suggest that there is *either* more matter than expected, dark matter, *or* that gravity is different, modified gravity.
- After all, the evidence for dark matter is inferred through gravity, so second option is *still* a possibility.
- Disregarding for now the intriguing, but controversial, inconclusive case of discrepant velocities of widely separated (kAU) binary stars, higher than expected in Newtonian gravity, the first evidence is galactic.

In the dark

Galactic evidence of an unknown

- Rather than a Keplerian decline in the outskirts, expected when all matter has been encompassed, $v^2/r = GM/r^2 \Rightarrow v^2 = GM/r$, velocities of stars and gas in disk galaxies are found to asymptote to a constant $v \rightarrow v_\infty$.
- Unexpected unless there is more mass, the dark matter halo extending well beyond the disk, arranged such that $M(r) \rightarrow r$, cancelling the dependence of v^2 on r in the denominator, and hence $v^2 \rightarrow v_\infty^2$.

A dark matter halo profile

- Since $M(r) \propto \int \mathrm{d}r\, r^2 \rho(r)$, then $M(r) \sim r$ implies there should be a regime $\rho(r) \sim 1/r^2$.
- This is the case for the NFW profile

$$\rho(r) = \frac{\rho_0}{(r/r_s)(1 + r/r_s)^2}$$

- where for $r \ll r_s$, $\rho \sim 1/r$ and $\rho \sim 1/r^3$ for $r \gg r_s$ so $\rho \sim 1/r^2$ intermediately around $r \sim r_s$.
- Hence, *constant* rotation curves should not extend *indefinitely*.*

Clues

Galactic regularity: baryonic Tully-Fisher relation

- It turns out that v_∞ can be inferred from just the baryonic mass of the galaxy M_b, implying a non-trivial relation between the baryonic and dark matter distribution.
- There is evidence that $v_\infty^4 \propto M_b$.

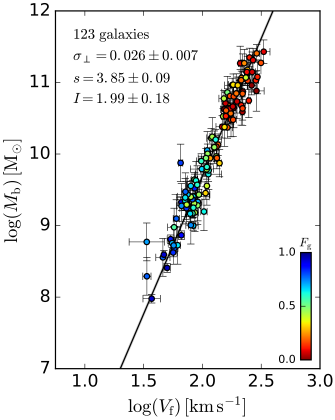


Figure: Power-law relation between v_∞ and M_b. Slope consistent with 4. (Lelli et al., 2019)

The MOND proposal

- This is natural in Modified Newtonian Dynamics (MOND).
- There is another way to cancel the r in the denominator of the gravitational acceleration $GM/r^2 = a_{\rm obs} = v^2/r$.
- To take a square root!
- So for very low accelerations $a_{obs} \propto \sqrt{a_{\rm N}}$. For units to match, must introduce a new scale a_0 and have $a_{\rm obs} = \sqrt{a_0 a_{\rm N}}$.
- a_0 sets the scale of transition to MOND behaviour ($a_0 \approx 1.2 \cdot 10^{-10} {\rm m} \cdot {\rm s}^{-2}$, $a_0 \sim cH_0$).
- Then $a_{\rm obs} = \sqrt{a_0}\sqrt{GM_b/r^2} = \sqrt{a_0 GM_b}/r = v_\infty^2/r \Rightarrow v_\infty^2 = \sqrt{a_0 GM_b}$.
- Squaring again find that $v_\infty^4 = (a_0 G)\, M_b$, the baryonic Tully-Fisher relation, with the constant now identified!

The non-relativistic field equation of MOND

- What could the non-relativistic field equation look like?
- $\nabla^2 \Phi_{\rm N} = \nabla \cdot (\nabla \Phi_{\rm N}) = -\nabla \cdot \vec{a}_{\rm N} = 4\pi G \rho_b$
- In MOND, we must have that $|\vec{a}_{\rm obs}| = \sqrt{a_0}\sqrt{|\vec{a}_{\rm N}|}$ or squaring, $|\vec{a}_{\rm obs}|\vec{a}_{\rm obs} = a_0 \vec{a}_{\rm N}$
- But want acceleration to come from a *potential*, so $\vec{a}_{\rm obs} = -\nabla \Phi$, so substituting
- $-|\nabla \Phi|\nabla \Phi = a_0 \vec{a}_{\rm N}$
- into $-\nabla \cdot \vec{a}_{\rm N} = 4\pi G \rho_b$
- $\nabla \cdot (|\nabla \Phi|/a_0 \nabla \Phi) = 4\pi G \rho_b$, a modified Poisson equation.
- To interpolate between MOND and Newtonian behaviour we introduce an interpolating function $\mathcal{M}$ so that generally
- $\nabla \cdot (\mathcal{M}(|\nabla \Phi|/a_0) \nabla \Phi) = 4\pi G \rho_b$
- where $\mathcal{M}(x) \to x$ for low accelerations $x \ll 1$ and $\mathcal{M}(x) \to 1$ for $x \gg 1$.

The Lagrangian

- Is there a Lagrangian for

$$\nabla \cdot \left(\mathcal{M} \left(\frac{|\nabla \Phi|}{a_0} \right) \nabla \Phi \right) = 4\pi G \rho_b?$$

- Yes! The a-quadratic Lagrangian (AQUAL)

$$\mathcal{L} = \mathcal{J} \left((\nabla \Phi \cdot \nabla \Phi) / a_0^2 \right) + 4\pi G \rho_b \Phi$$

  leads to the field equation when $\mathcal{J}'(x^2) = \mathcal{M}(x)$, so in the deep-MOND regime $\mathcal{J}(x) \propto x^{3/2}$.
- Then one can rest assured that equations of motion will be consistent.

Another approach

QUMOND (Quasi-linear MOND)

- Another approach is to invert

$$\mathcal{M}(|\vec{a}_{\rm obs}|/a_0)\vec{a}_{\rm obs} = \vec{a}_{\rm N}$$

so that

$$\vec{a}_{\rm obs} = \nu(|\vec{a}_{\rm N}|/a_0)\vec{a}_{\rm N}$$

- and introduce a hierarchy such that $\Phi_{\rm N}$ satisfies the Poisson equation

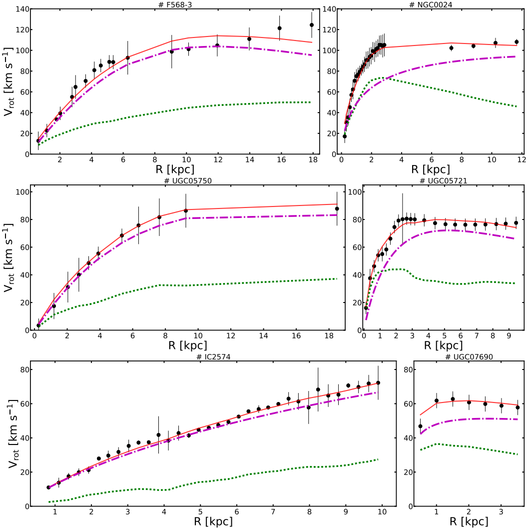
$$\nabla^2\Phi_{\rm N} = 4\pi G\rho_b$$

and enforce a relation between $\nabla\Phi$ and $\nabla\Phi_{\rm N}$, so that

$$\nabla\cdot(\overbrace{\nabla\Phi}^{\vec{a}_{\rm obs}}) = \underbrace{\nabla\cdot(\overbrace{\nu(|\nabla\Phi_{\rm N}|)\nabla\Phi_{\rm N}}^{\vec{a}_{\rm obs}})}_{4\pi G\rho_{\rm dyn}}.$$

- Amounts to solving Poisson equation twice.

$v_\infty^4 = a_0 G M_b$ is not all. Diversity of rotation curves.



- Three v_∞ twins (Ghari, Famaey et al., 2019).
- Same v_∞, slow/fast approach to v_∞.
- Rotation curve tracks trend of baryonic contribution.

Further evidence

Additional galactic regularity: the Radial Acceleration Relation (RAR)

- Observational support for MOND most clearly illustrated by the Radial Acceleration Relation.
- Accelerations in diverse galaxies land, with small scatter, on the same 1-1 relation with the acceleration expected from the baryons alone, i.e., there is an algebraic relation $g_{\rm obs} = \nu(g_{\rm N}) g_{\rm N}$.

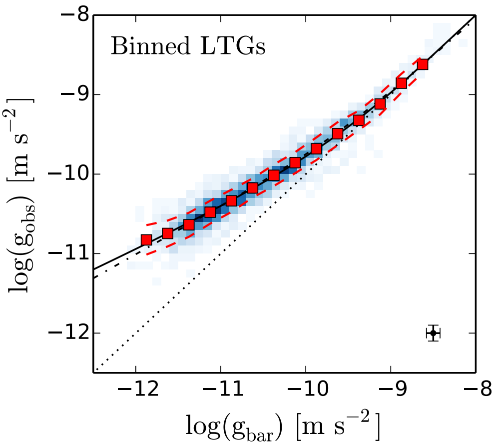


Figure: The baryons alone *predict* the dynamics (153 LTG) (Lelli et al., 2019).

Recent underappreciated observations

Additional regularity: the radial acceleration relation (RAR) extended

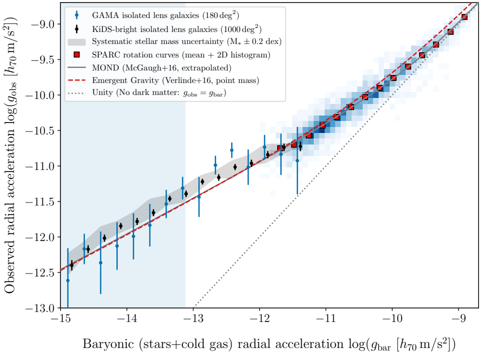


Figure: RAR extended by weak lensing agrees with deep MOND behaviour (slope 1/2) (Brouwer et al., 2021).

- Recently, the Radial Acceleration Relation has been extended by orders of magnitude using weak lensing.
- Signal of $\sim 10^5$ lenses (KiDS and GAMA) of isolated late-type and early-type galaxies, stacked.
- Consistent with MOND behaviour persisting!

Recent observations

The radial acceleration relation (RAR) extended and independently confirmed

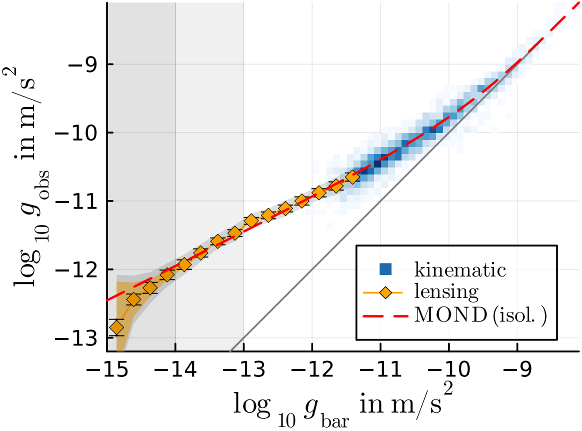


Figure: Recently independently confirmed and made more robust by Mistele et al (2023).

To larger scales: galaxy clusters

- Relaxed galaxy clusters are modelled as gaseous spherically symmetric configurations in hydrostatic equilibrium $\nabla p = \nabla \Phi$.
- Most of the baryonic mass is in X-ray emitting gas.
- Potential can be derived using
  - velocity dispersion of galactic population (via virial theorem),
  - weak and strong gravitational lensing,
  - thermal Sunyaev-Zeldovich effect (distortion of CMB proportional to line-of-sight integral of electron pressure, pressure related to potential by hydrostatic eq.),
  - X-ray bremsstrahlung luminosity (depending on density and temperature hence pressure, hence potential).

To larger scales: galaxy clusters

A Radial Acceleration Relation for galaxy clusters? Conflicts.

- Combining tSZ observations and X-ray observations for five nearby galaxy clusters, Eckert et al. have found a RAR for galaxy clusters in conflict with the galactic RAR.
- Accelerations mostly larger than the galactic RAR, hence stronger gravity or missing matter.

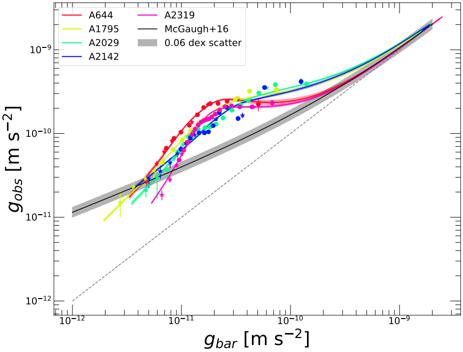


Figure: The RAR of galaxy clusters (tSZ and X-ray obs.) departs from the galactic RAR (Eckert et al., 2022).

To larger scales: galaxy clusters

A Radial Acceleration Relation for galaxy clusters? Conflicts.

- Using weak and strong lensing data Tian et al. have found that MOND could work, but with $a_0 \rightarrow 17a_0$ for galaxy clusters alone*.

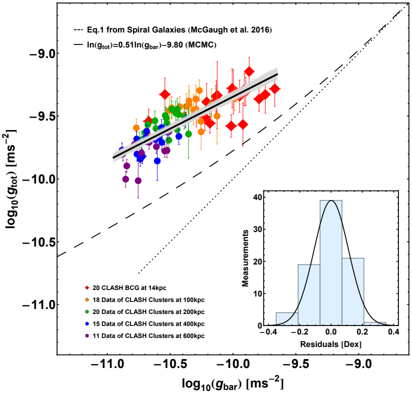


Figure: RAR of galaxy clusters inferred from lensing also departs from galactic RAR. (Tian et al., 2020)

The cosmological challenge

Largest scales, cosmology, CMB and LSS

- Best fit model is flat ΛCDM model with $\Omega_{\mathrm{CDM}} \approx 5\Omega_b$.
- Tightly constrained dust-like (pressureless) behaviour: energy density decays as a^{-3}, negligible speed of sound c_s.
- Need relativistic theory to address.

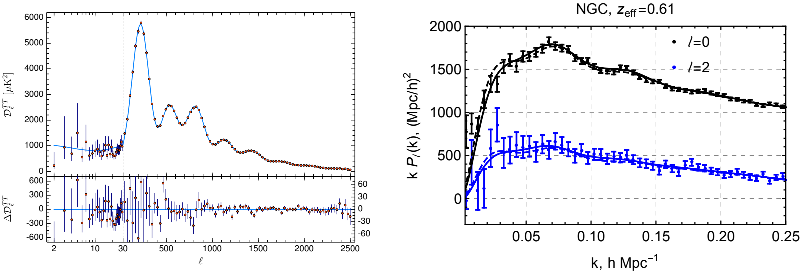


Figure: Left: Planck angular power spectrum. (Aghanim et al., 2018). Right: Matter power spectrum from BOSS (SDSS), monopole and quadrupole. (Ivanov et al., 2021)

Relativistic extensions of Modified Newtonian Dynamics

- Modifying the Poisson equation $\nabla^2\Phi = 4\pi G\rho_b$ to have a MOND limit was in retrospect straightforward: $\nabla \cdot (\mathcal{M}(|\nabla\Phi|/a_0)\nabla\Phi) = 4\pi G\rho_b$.
- This theory is clearly non-relativistic: It has only spatial derivatives $\nabla\Phi$. A relativistic theory would necessarily involve time derivatives $\partial\Phi/\partial t$ (symmetrically).
- A natural starting point, to not spoil all the successes of general relativity, is to have a metric theory, involving $g_{\mu\nu}$.
- Just as general relativity reduces to Newtonian gravity in the weak-field, slow-motion ($v \ll c$) regime, so we need to find a theory whose weak-field, slow motion *and low acceleration regime* $a < a_0$ is governed by MOND.

Relativistic extensions of Modified Newtonian Dynamics

- The road to general relativity was not a simple promotion of gradients ∇ to four-derivatives ∂_i and Laplacians ∇^2 to d'Alembertians $\Box = \eta^{\mu\nu}\partial_\mu\partial_\nu$ involving only the potential Φ.
- In general relativity, the potential Φ is only the diagonal part of the larger metric tensor $g_{\mu\nu}$: $g_{00} = -1 + 2\Phi$, $g_{ii} = 1 + 2\Phi$.

Relativistic extensions of Modified Newtonian Dynamics

Enter Aether-Scalar-Tensor theory (AeST)

- Motivated by the need to have a theory that
  - has a MOND limit for $|\nabla\Phi| < a_0$,
  - is *GR-like* for large accelerations $|\nabla\Phi| \gg a_0$, strong field-regime,
  - is consistent with observations of CMB anisotropies and of large scale structure,
  - has gravitational waves that travel at light speed,

- Skordis and Złośnik (2020) proposed such a theory (called Aether-Scalar-Tensor theory) with a unit time-like vector field A_μ, a scalar field ϕ, and a metric/tensor $g_{\mu\nu}$.
- Defining kinetic terms for the scalar field along the direction of A^μ: $\mathcal{Q} \equiv A^\mu \nabla_\mu \phi$, perpendicular to A^μ: $\mathcal{Y} \equiv \nabla_\mu \phi \nabla_\nu \phi (g^{\mu\nu} + A^\mu A^\nu)$ and the projected vector field gradient $J^\mu = A^\alpha (\nabla_\alpha A^\mu)$ it reads

$$\frac{16\pi\tilde{G}}{\sqrt{-g}}\mathcal{L}_{\mathrm{AeST}} = R - \frac{K_B}{2}F^{\mu\nu}F_{\mu\nu} + 2(2-K_B)J^\mu\nabla_\mu\phi - (2-K_B)\mathcal{Y} - \mathcal{F}(\mathcal{Y},\mathcal{Q}) - \lambda(A^\mu A_\mu + 1) \quad (1)$$

where R is the Ricci scalar, K_B is a coupling constant, $F_{\mu\nu} = \nabla_\mu A_\nu - \nabla_\nu A_\mu$ is the field strength, $\mathcal{F}$ is a free function and λ is a Lagrange multipler that imposes the unit time-like constraint: $A^\mu A_\mu = g_{\mu\nu}A^\mu A^\nu = -1$.

Features of Aether-Scalar-Tensor theory

$$\frac{16\pi\tilde{G}}{\sqrt{-g}}\mathcal{L}_{\rm AeST} = R - \frac{K_B}{2}F^{\mu\nu}F_{\mu\nu} + 2(2-K_B)J^\mu\nabla_\mu\phi - (2-K_B)\mathcal{Y} - \mathcal{F}(\mathcal{Y},\mathcal{Q}) - \lambda(A^\mu A_\mu + 1) \quad (2)$$

- Function $\mathcal{F}$ of $\mathcal{Q} = A^\mu\nabla_\mu\phi$ (time, cosmology) and $\mathcal{Y} = \nabla_\mu\phi\nabla_\nu\phi(g^{\mu\nu} + A^\mu A^\nu)$ (space) is undetermined.
- Different choices lead to different behaviour.
- MOND behaviour: $\frac{2}{3a_0}\mathcal{Y}^{3/2}$ in $\mathcal{F}$.
- CDM-like behaviour: $\sim \mathcal{K}_2(\mathcal{Q}-\mathcal{Q}_0)^2$ in $\mathcal{F}$, minimum at a non-zero value $\mathcal{Q}_0$.
- Turns out evolution of $\mathcal{Q}$ ($\sim \dot{\phi}$) towards $\mathcal{Q}_0$ mimicks a homogeneous dust component.
- The (cosmological) DM density $\Omega_{\rm CDM}$ is set by the amount of displacement of $\mathcal{Q}$ from $\mathcal{Q}_0$.

Static weak-field solutions of AeST

- That was the full relativistic action.
- To get the (quasi-)static weak-field equations, only quadratic terms of the fields were kept in the action, after expanding the metric as

$$ds^2 = -(1 + 2\Psi)\mathrm{d}t^2 + (1 - 2\Phi)\,\gamma_{ij}\mathrm{d}x^i\mathrm{d}x^j,$$

the scalar field about the minimum

$$\phi = \mathcal{Q}_0 t + \varphi,$$

time derivatives were neglected $\dot{\Phi} = \dot{\Psi} = \dot{\varphi} = 0$, and variational derivatives were taken.

Static weak-field solutions of AeST

- The weak-field (tensor-scalar) equations are

$$\nabla^2\Phi - \nabla^2\chi + \mu^2\Phi = 4\pi G\rho_b \tag{3}$$

$$\nabla\cdot\left(\beta\left(|\nabla\chi|/a_0\right)\nabla\chi\right) = \nabla^2\Phi \tag{4}$$

  where $\chi \equiv \varphi + \mathcal{Q}_0\alpha$ (piece of $\phi = \mathcal{Q}_0 t + \varphi$, and α scalar part of vector field), β is a function derived from $\mathcal{Y}$-dependent part of $\mathcal{F}$ and $\mu \equiv 2\mathcal{K}_2\mathcal{Q}_0^2/(2-K_B)$.
- This can be reduced to one equation* in only the gravitational potential

$$\nabla\cdot\left(\mathcal{M}\left(|\nabla\Phi|/a_0\right)\nabla\Phi\right) + \underbrace{\mu^2\Phi}_{\text{novel}} = 4\pi G\rho_b. \tag{5}$$

- Note that there is now explicit dependence of the potential: the absolute value of the potential matters. Can distinguish large from small potential. (Distinguish galaxy cluster from galaxy?)

Subtleties of the reduction

- In the Newtonian limit, the *reduced* equation

$$\nabla \cdot \left(\mathcal{M}\left(|\nabla\Phi|\right)\nabla\Phi\right) + \mu^2\Phi = 4\pi G\rho$$

  reduces to the Helmholtz equation

$$\nabla^2\Phi + \mu^2\Phi = 4\pi G\rho$$

  which has oscillatory solutions.

- In the Newtonian regime this is not a problem, but in the MOND regime we have

$$\nabla \cdot \left(|\nabla\Phi|\nabla\Phi\right) + \mu^2\Phi = 4\pi G\rho$$

  where it is a problem as a divergence (derivative) is acting on $|\nabla\Phi|$ where $\nabla\Phi \to 0$ (nodes of oscillation), where the absolute value $|X|$ is not differentiable for $X \to 0$.

Subtleties of the reduction

- In the spherically symmetric case this can be circumvented by finding a Hamiltonian system $(\Phi(r), p_\Phi(r))$ whose equations of motion in configuration space reduce to

$$\nabla \cdot (\mathcal{M}(|\nabla\Phi|)\Phi) + \mu^2\Phi = 4\pi G\rho$$

  but whose equations of motion are solved entirely in momentum space p_Φ where, it turns out, there are no singularities.
- The desired solution $\Phi(r)$ is then obtained from p_Φ.
- A simpler resolution is not to reduce at all but stay with the two-component case

$$\nabla^2\Phi - \nabla^2\chi + \mu^2\Phi = 4\pi G\rho_b \quad (6)$$

$$\nabla\cdot(\beta(|\nabla\chi|/a_0)\nabla\chi) = \nabla\cdot(\nabla\Phi) \quad (7)$$

  integrating the second equation

$$\beta(|\nabla\chi|/a_0)\nabla\chi = \nabla\Phi.$$

AeST vacuum solutions: the field away from a source

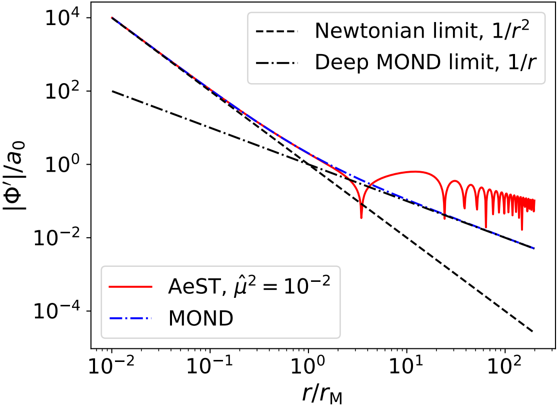


Figure: The magnitude of the force away from a point source in AeST (red). Newtonian regime $1/r^2$, MOND regime $1/r$ and oscillatory regime (with power-law envelope). Distance in units of $r_{\rm M} = \sqrt{GM/a_0}$ and $\hat{\mu} \equiv \mu r_{\rm M}$.

AeST vacuum solutions: the field away from a source

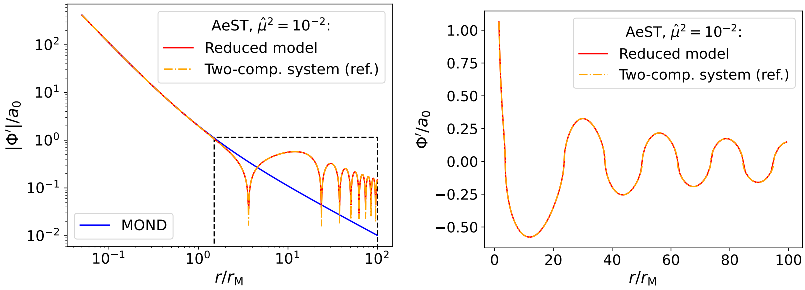


Figure: Oscillations in force. Left: Log-log plot of abs. value of force (AeST red-orange, MOND blue).Right: Linear plot of force.

AeST vacuum solutions: the field away from a source

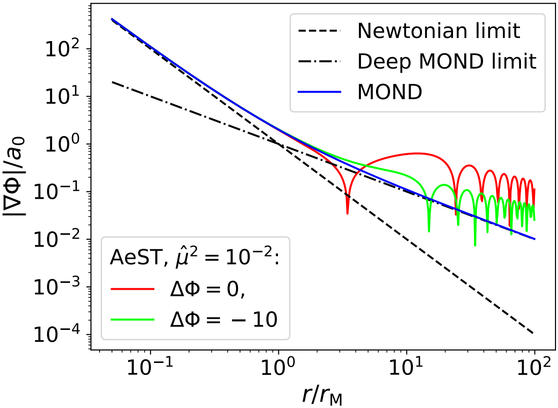


Figure: Onset of oscillatory regime depends on the boundary value of potential. Can be delayed by a lowered potential (green line).

Static spherically symmetric weak-field solutions of AeST

Hydrostatic isothermal gas, a simplified galaxy cluster model

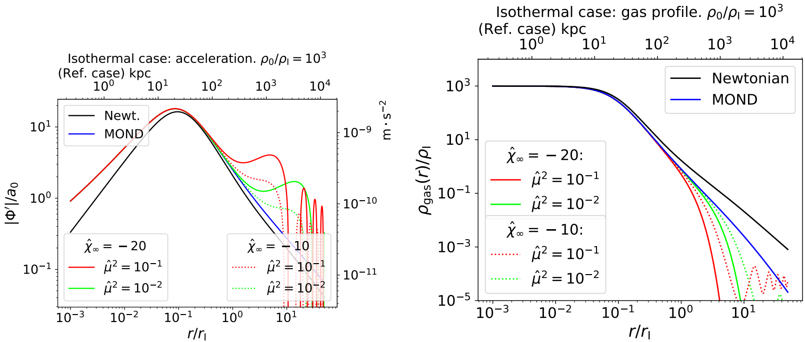


Figure: Isothermal gas in hydrostatic equilibrium: $\rho_b = \exp(-\Phi)$. Left: The force in AeST (red line) is stronger than in MOND (blue line). Oscillations persist. Right: The gas is seen to be more compressed in AeST than in either MOND (blue line) or Newtonian gravity (black line).

The Radial Acceleration Relation of AeST for the isothermal sphere

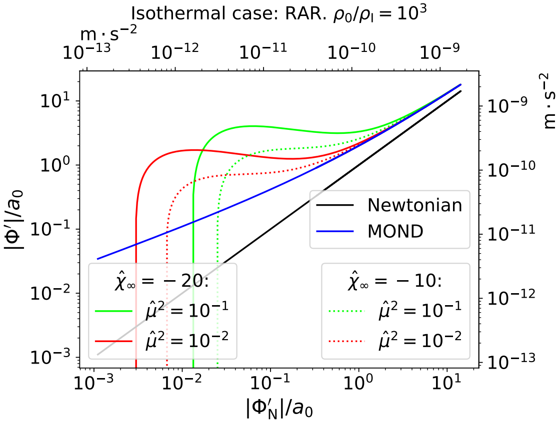


Figure: The RAR for the isothermal sphere in MOND (blue) and AeST for a higher (green) and a lower (red) value of $\hat{\mu} \propto \mu\sqrt{GM/a_0}$ and a smaller (dotted) and a greater (full line) shift of the asymptotic potential. The AeST displays a peak in a limited acceleration range.

A challenge to AeST

- The RAR extended by weak lensing is a potential challenge to AeST. As AeST introduces a new length scale $1/L = \mu$ the MOND behaviour should stop around a scale depending on μ and $r_{\rm M} \sim \sqrt{GM/a_0}$.
- The stacking used may wash out the oscillations. Nodes of the oscillations are affected by the boundary conditions of the potential.
- Shifts in the potential may also delay the onset of oscillatory behaviour.

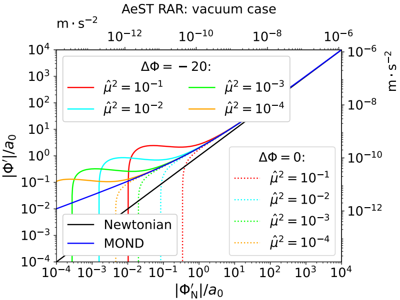


Figure: The AeST Radial Acceleration Relation for the vacuum case. Deviation from MOND is inevitable in AeST.

Conclusions

- There is observational support for MOND in galactic systems.
- Galaxy clusters remain a challenge for MOND.
- A recently proposed relativistic embedding of MOND, Aether-Scalar-Tensor theory (AeST) has a ΛCDM limit and a MOND regime.
- MOND only appears in a limited range.
- The weak-field effects explicitly depend on the potential which may potentially distinguish galaxies from galaxy clusters.
- The AeST Radial Acceleration Relation (RAR) for the isothermal sphere shares similar behaviour to the observed RAR for galaxy clusters, displaying a peak w.r.t. the MOND RAR in a limited acceleration range.
- However, a quantitative study with a realistic galaxy cluster model is needed to fully address whether AeST can account for the missing matter in galaxy clusters where (it is accepted that) MOND fails.