Inflationary Trispectrum of Gauge Fields from Scalar and Tensor Exchanges

(arXiv:2509.11143)

INDIAN INSTITUTE OF SCIENCE
भारतीय विज्ञान संस्थान

Haridev S R

Department of Physics

IISc, Bengaluru, India

Collaborators : P. Jishnu Sai and Prof. Rajeev Kumar Jain

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Outline

Motivation and introduction: Cosmological Correlators

Our Model : U(1) gauge field during inflation

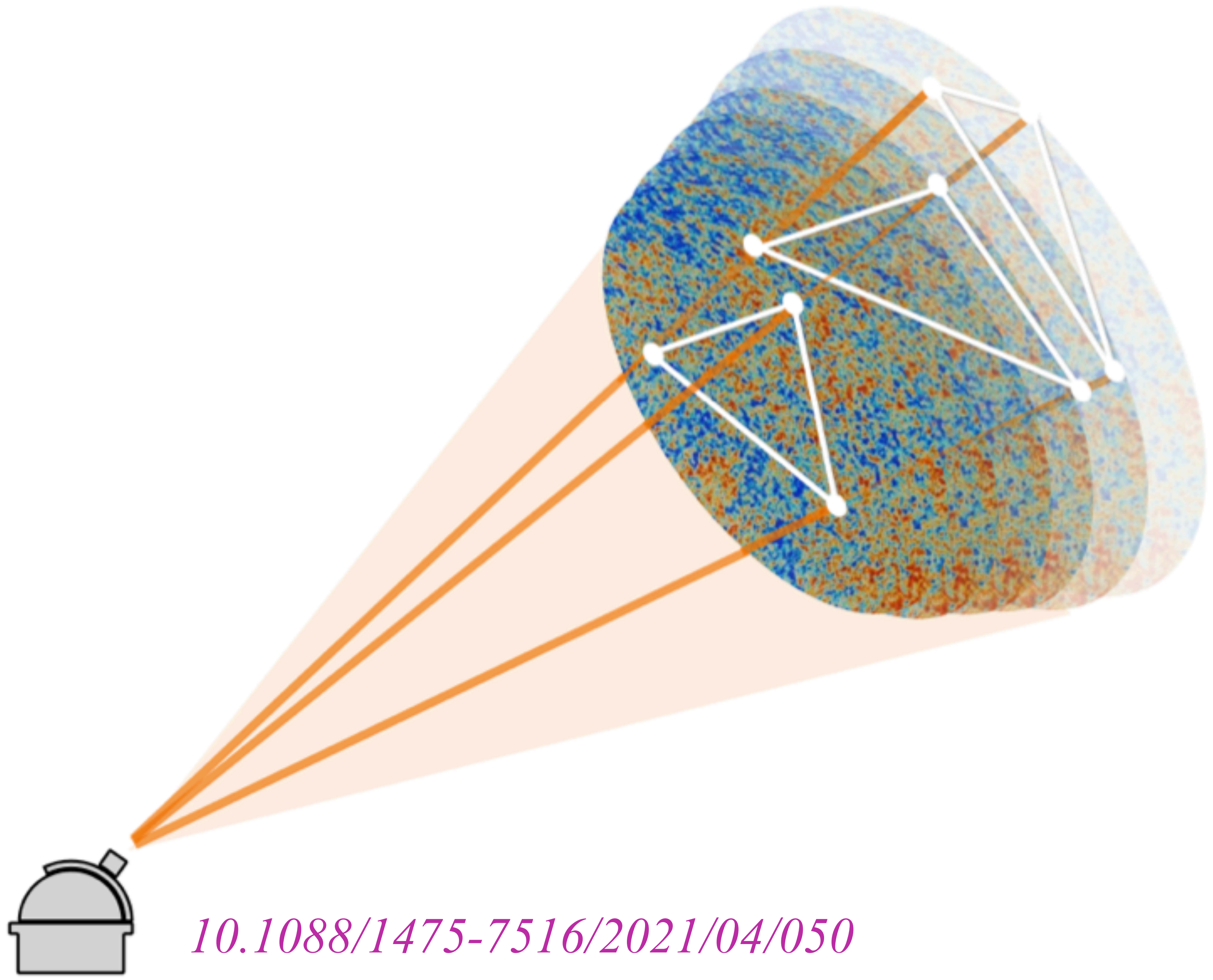
Method : In-in formalism & Diagrammatic rules

Trispectrum of gauge field (Exchange diagram)

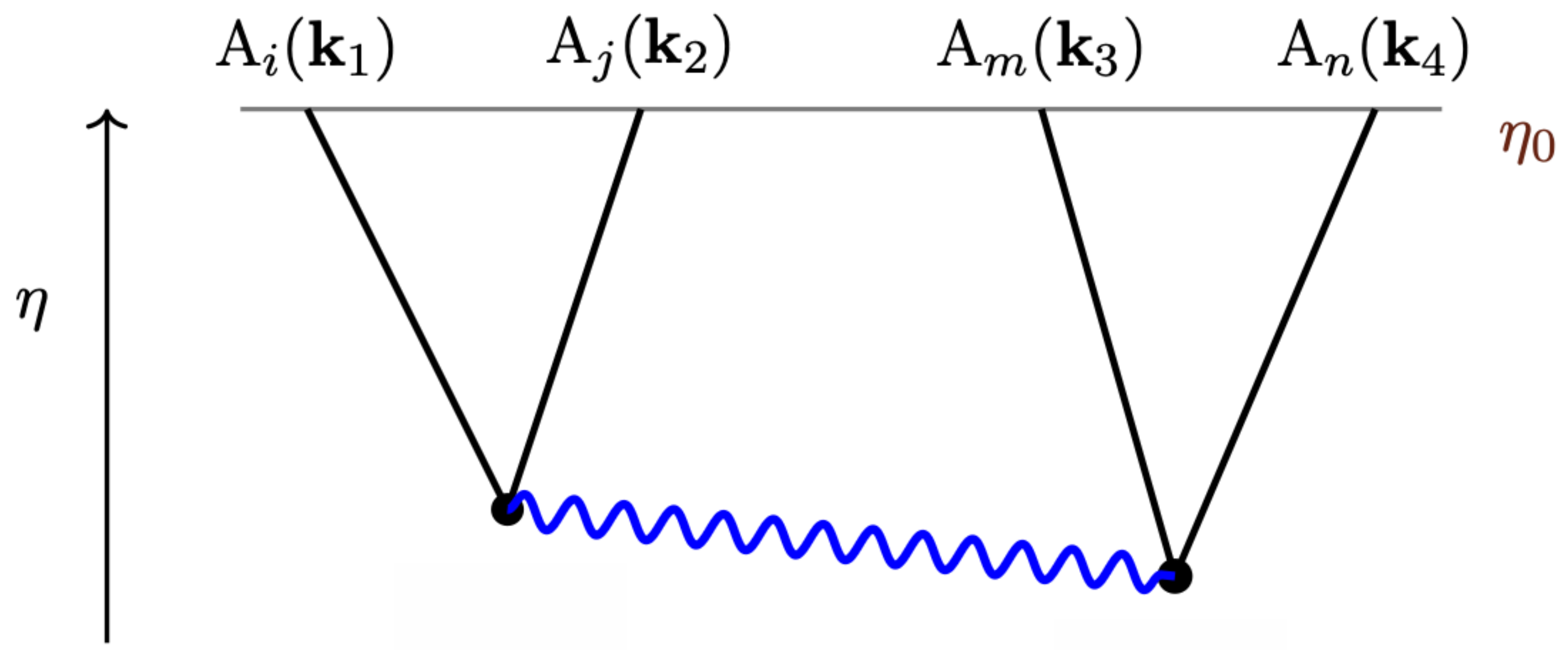
B and ***E***, trispectrum

Momentum shapes

Results and Discussions

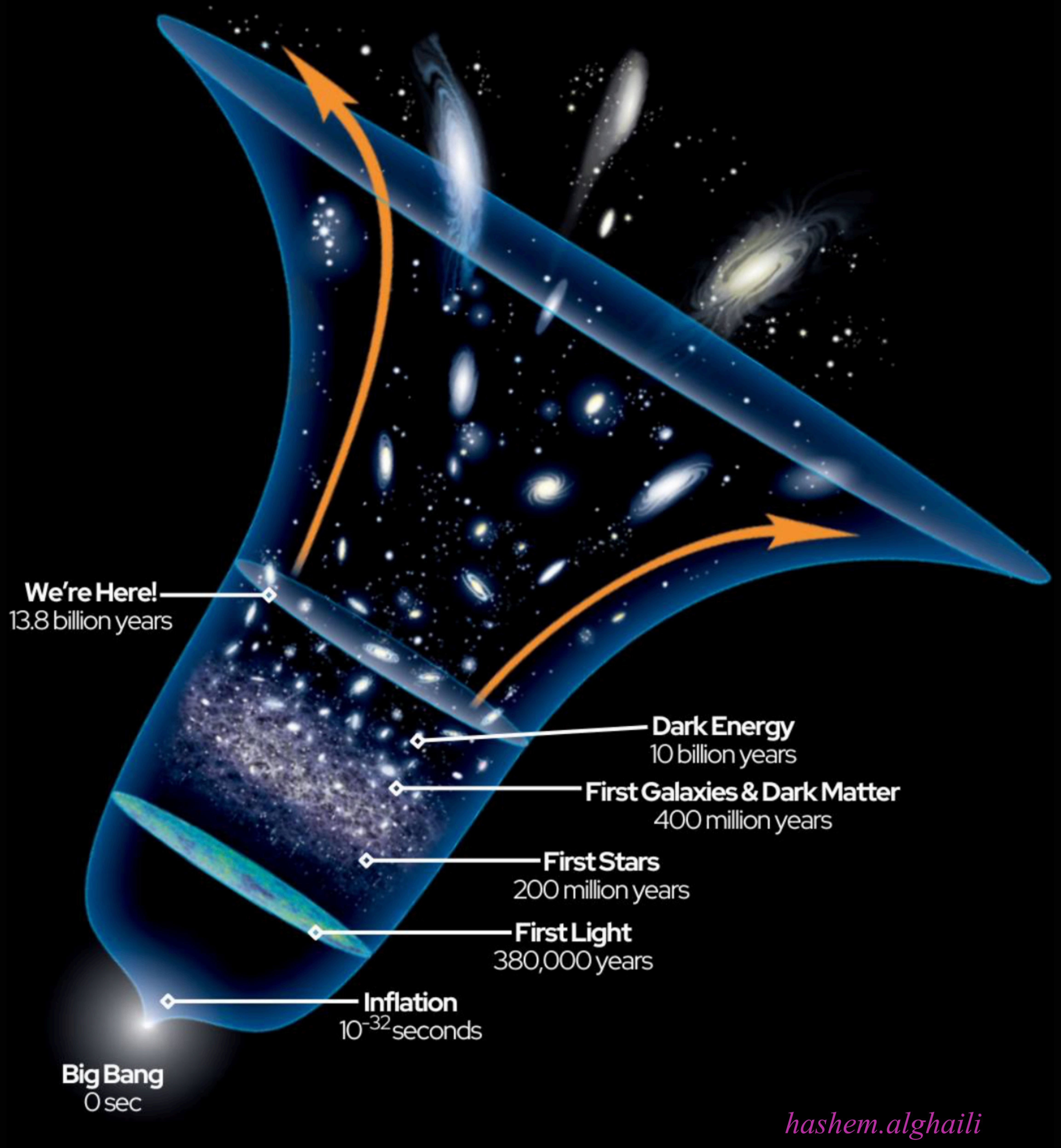


10.1088/1475-7516/2021/04/050



Inflation and Higher correlations

- Dynamics of Inflation $\leftrightarrow$ Statistics of perturbations
- Near Gaussian : Central Limit Theorem
- Gaussian statistics : Observationally well constrained
  - Plethora of theoretical models
  - Knows little about the properties of inflation
- Non- Gaussian: Anything other than Gaussian
  - Point towards specific features like number of fields, form of the potential and kinetic part, energy scale etc..



Gauge fields during Inflation

A. Kandus, K. E. Kunze, and C. G. Tsagas, Phys. Rept. 505(2011) 1–58,

- Weak coherent magnetic fields in the inter galactic medium ($\overrightarrow{B} \sim 10^{-16} G$) : May have primordial origin

- Amplification of gauge field $\Longrightarrow$ break conformal symmetry
  - Maxwell's theory is conformal and inflationary back ground is also conformal

$$A_k(\eta) \sim \frac{1}{\sqrt{2k}} e^{-ik\eta}$$

- Same as the flat space mode
- Red shifted away

- Conformal invariance is restored at the end of inflation

- String inspired effective field theory : Dilatonic couplings which breaks conformal symmetry

- Broad category : Does the gauge field has VEV or not
  - In our model we assume $< A > = 0$

- Interactions between gauge fields and gravitational modes : Cross correlations, exchanges etc.

Inflationary Dynamics

- Single field inflation in the slow roll model

$$S_\phi = -\frac{1}{2}\int dx^4\sqrt{-g}\left[g^{\mu\nu}\partial_\mu\phi\partial_\nu\phi + 2V(\phi) - R\right]$$

- Include the metric perturbation through ADM decomposition

$$ds^2 = -\underbrace{N^2}_{\text{Lapse}} dt^2 + h_{ij}\left(dx^i + \underbrace{N^i}_{\text{Shift}} dt\right)\left(dx^j + N^j dt\right) \quad \text{With} \quad h_{ij} = a^2 e^{2\zeta}[e^\gamma]_{ij}$$

Tensor fluctuations

Scalar fluctuations

- ζ and γ_{ij} satisfies the Mukhanov-Sasaki equation

$$\zeta'' + 2\frac{z'}{z}\zeta' - \nabla^2\zeta = 0 \xrightarrow{\text{B D normalisation}} \zeta_k(\eta) = \frac{1}{\sqrt{2\epsilon}}\frac{H}{\sqrt{2k^3}}(1+ik\eta)e^{-ik\eta}$$

$z = a\sqrt{2\epsilon}$, and ϵ is the first slow roll parameter

Inflationary Dynamics

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$\downarrow$

$z = a\sqrt{2\epsilon}$, and ϵ is the first slow roll parameter

- The dimensionless power spectrum

$$\mathscr{P}_\zeta \equiv \frac{k^3}{2\pi^2}|\zeta_k(\eta)|^2$$

- Nearly scale invariant in slow roll inflation
- Agrees with CMB observation

- Similarly for γ_{ij}

$$\gamma_k(\eta) = \frac{H}{\sqrt{k^3}}(1+ik\eta)e^{-ik\eta}$$

- Power spectrum is suppressed by tensor to scalar ratio

$$r = \frac{\mathscr{P}_\gamma}{\mathscr{P}_\zeta} < 0.036$$

BICEP, Keck collaboration, Phys.Rev. Lett. 127 (2021) 151301 [2110.00483]
P. Campeti and E. Komatsu, Astrophys. J. 941 (2022) 110 [2205.05617]

Our Model

- Dilatonic coupling: Explicitly break conformal symmetry *B. Ratra, Astrophys. J. Lett. 391 (1992) L1.*

$$S[A_\mu] = -\frac{1}{4}\int \mathrm{d}^4x\sqrt{-g}\;\lambda(\phi)\;F_{\mu\nu}F^{\mu\nu} \quad : \quad F_{\mu\nu} = \partial_\mu A_\nu - \partial_\nu A_\mu$$

$$\lambda \sim a^{2n} \sim \eta^{-2n} \quad : \quad ds^2 = a^2(\eta)\left[-d\eta^2 + d\vec{x}^2\right]$$

- $U(1)$ Symmetry : No self interactions

- We work in Coulomb gauge: $A_0 = 0$ and $\nabla . \overrightarrow{A} = 0$

- Equation of motion

$$A_k'' + \frac{\lambda'}{\lambda}A_k' + k^2 A_k = 0 \xrightarrow{\text{B D}} A_k(\eta) = \frac{1}{\sqrt{\lambda_e}}\frac{\sqrt{\pi}}{2}e^{i\pi(n+1)/2}\sqrt{-\eta}\left(\frac{\eta}{\eta_e}\right)^n H^{(1)}_{n+\frac{1}{2}}(-k\eta)$$

- A_k is an auxiliary field with $B_\mu =^* F_{\mu\nu}u^\nu$ and $E_\mu = F_{\mu\nu}u^\nu$ with $u^\mu = (1/a,0,0,0)$

$$\mathscr{P}_B \equiv \frac{d\rho_B}{d\ln k} = \frac{\lambda k^3}{4\pi^2}P_B\,, \qquad \mathscr{P}_E \equiv \frac{d\rho_E}{d\ln k} = \frac{\lambda k^3}{4\pi^2}P_E \quad \text{With} \quad P_B = 2\frac{k^2}{a^4}|A_k(\eta)|^2, \qquad P_E = \frac{2}{a^4}|A_k'(\eta)|^2$$

Our Model

- The dimensionless power spectrum is given as

V. Demozzi, V. Mukhanov, and H. Rubinstein, JCAP 0908 (2009) 025
R. J. Z. Ferreira, R. K. Jain, and M. S. Sloth, JCAP 1310 (2013) 004

$$\frac{d\rho_B}{d\log k} \approx \frac{\mathcal{F}(m)}{2\pi^2} H^4 \left(-k\eta\right)^{4+2m} \qquad m = \begin{cases} -n, n \geq -1/2 \\ 1+n, n \leq -1/2 \end{cases}$$

- Scale invariant for $n = 2$ and $n = -3$

$$\frac{d\rho_E}{d\log k} \approx \frac{\mathcal{G}(m)}{2\pi^2} H^4 \left(-k\eta\right)^{4+2m} \qquad m = \begin{cases} 1-n, n \geq 1/2 \\ n, n \leq 1/2 \end{cases}$$

- For $n = -3$, the electric spectrum diverges leads to back reaction problem (can't have $N > 13.8$)
- For n= 2 , $\lambda\left(\phi\right) \propto a^2$: $\lambda|_{\eta_0} \to 1 \implies \lambda|_{initial} = e^{-2N_{total}} = e^{-120}$: $e_{phy} = \lambda^{-1} = e^{120}$
  - This is the strong coupling problem, one can't do perturbative analysis
  - Can circumvent the strong coupling problem

R. J. Z. Ferreira, R. K. Jain, and M. S. Sloth, JCAP 1310 (2013) 004

Interaction Hamiltonian

- No self interaction $\Longrightarrow$ No three point auto correlation function
- We choose comoving gauge : Only possible interaction is through metric fluctuations
- Using the ADM decomposition to next order one can obtain with scalar fluctuations

$$H_{\zeta AA} = n \int d^3x \, \lambda \zeta \left(A_i'^2 - \frac{1}{2} F_{ij}^2 \right) + O(\epsilon)$$

R.K. Jain and M.S. Sloth, Phys. Rev. D 86 (2012) 123528
L. Motta and R.R. Caldwell, Phys. Rev. D 85 (2012) 103532

- With tensor fluctuations

$$H_{\gamma AA} = \frac{1}{2} \int d^3x \, \lambda \, \gamma_{ij} \left(A_i' A_j' - (\partial_i A_l \partial_j A_l + \partial_l A_i \partial_l A_j) + 2 \partial_i A_l \partial_l A_j \right) + O(\epsilon)$$

- We are avoiding a temporal boundary term

M. Braglia and L. Pinol, JHEP 08 (2024) 068 [2403.14558]

$$\mathscr{B}_{\zeta AA} = \frac{a\lambda}{2H} \zeta \left(\dot{A}_i^2 + \frac{1}{2a^2} F_{ij}^2 \right)$$

- Red shifted away or suppressed relative to bulk

In-in Formalism

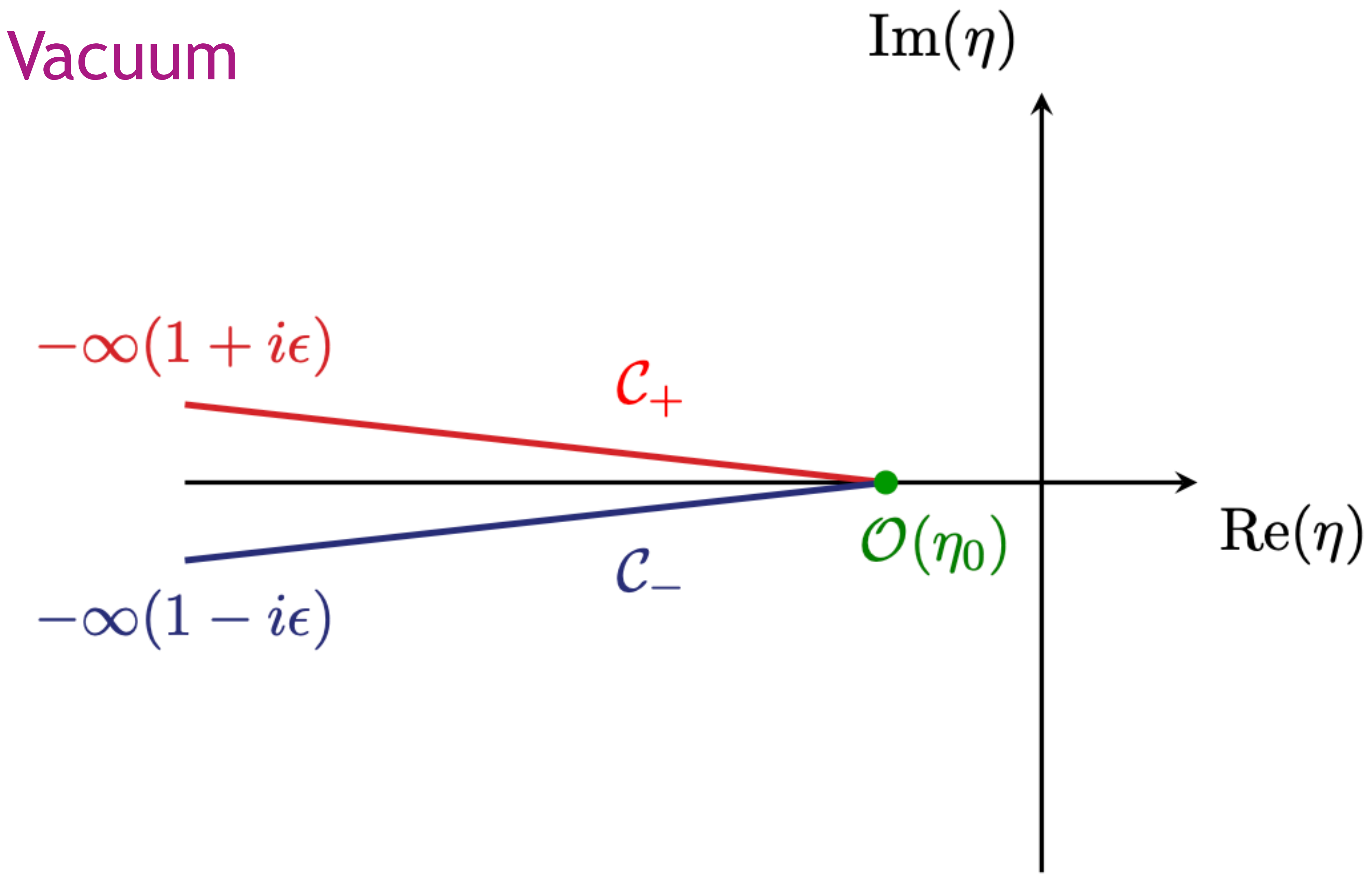
Master Formula

Interactive Vacuum — Free Vacuum

$$\langle \Omega | \mathcal{O}(\eta_0) | \Omega \rangle = \langle 0 | \, \bar{T}\left(e^{i\int_{-\infty}^{\eta_0} d\eta \, H_{\text{int}}}\right) \mathcal{O}(\eta_0) \, T\left(e^{-i\int_{-\infty}^{\eta_0} d\eta \, H_{\text{int}}}\right) | 0 \rangle$$

Anti- time ordering — Time ordering

- Contour choice: Bunch Davies Vacuum



In-in Formalism

Master Formula

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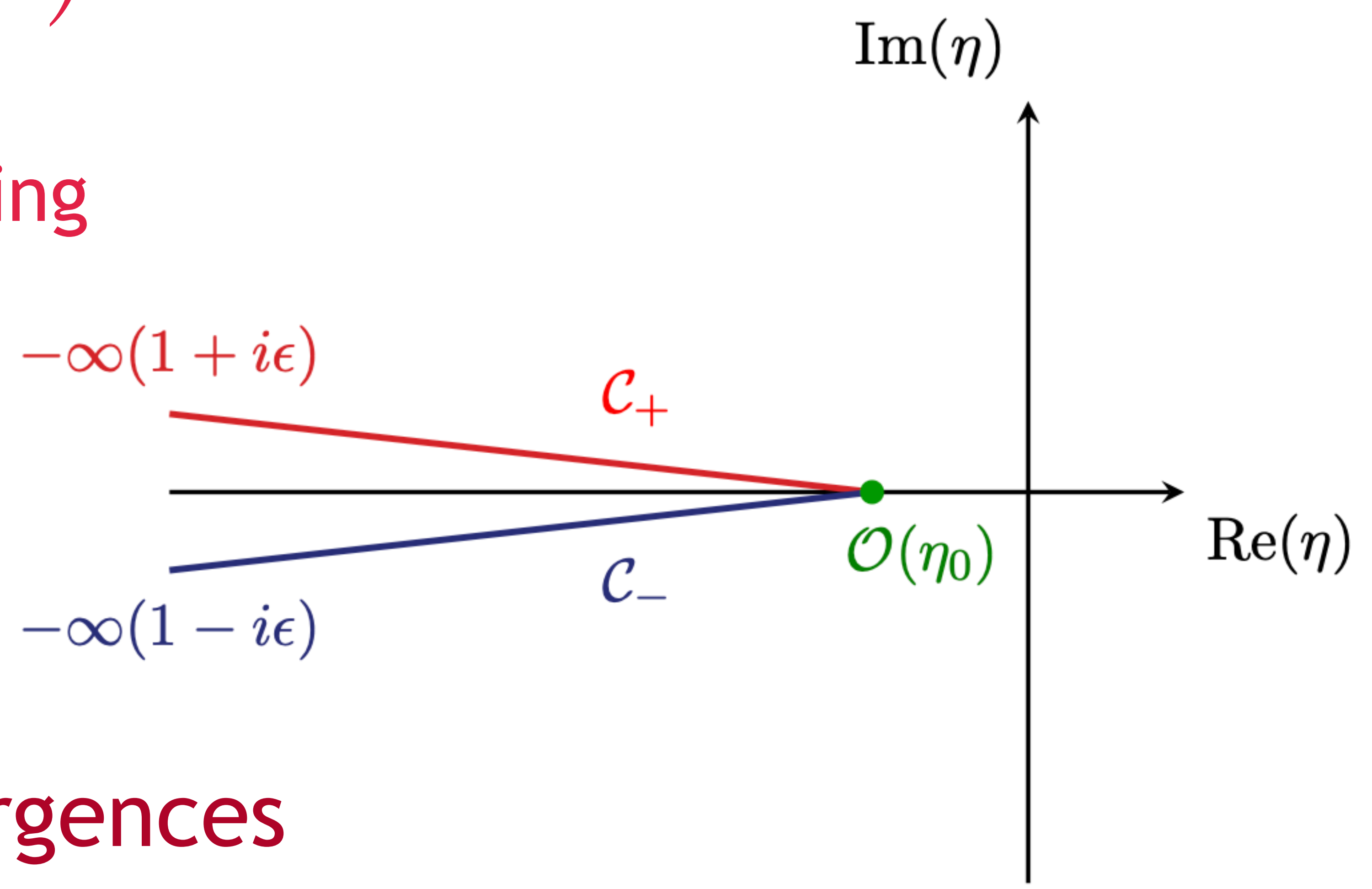
$$\langle \Omega | \mathcal{O}(\eta_0) | \Omega \rangle = \langle 0 | \, \bar{T}\left(e^{i\int_{-\infty}^{\eta_0} d\eta \, H_{\rm int}} \right) \mathcal{O}(\eta_0) \, T\left(e^{-i\int_{-\infty}^{\eta_0} d\eta \, H_{\rm int}} \right) | 0 \rangle$$

Anti- time ordering — Time ordering



- Contour choice: Bunch Davies Vacuum
- Should keep track of time ordered and anti-time ordered part: Spurious divergences
- Perturbative analysis

P. Adshead, R. Easther and E.A. Lim, Phys. Rev. D 80 (2009) 083521

$$\bar{T}\left(1 + i\int_{-\infty}^{\eta_0} d\eta\, H_{\rm int} + i^2 \int_{-\infty}^{\eta_0} d\eta_1\, H_{\rm int} \int_{-\infty}^{\eta_0} d\eta_2\, H_{\rm int} + \dots\right)^* \mathcal{O}\left(\eta_0\right) \, T\left(1 - i\int_{-\infty}^{\eta_0} d\eta\, H_{\rm int} + (-i)^2 \int_{-\infty}^{\eta_0} d\eta_1\, H_{\rm int} \int_{-\infty}^{\eta_0} d\eta_2\, H_{\rm int} + \dots\right)$$

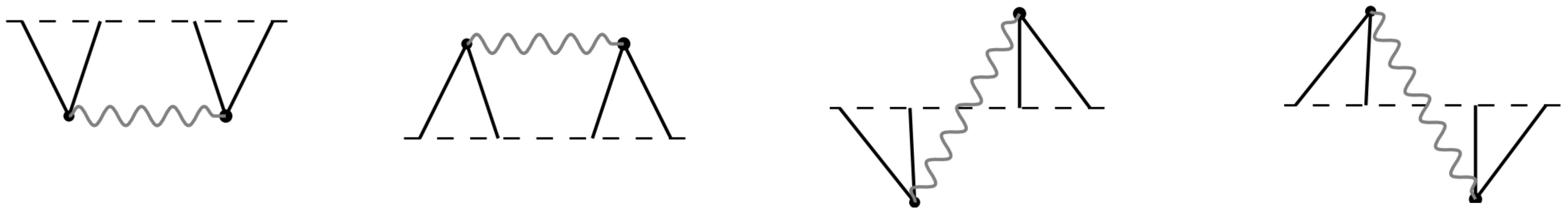
- One way to keep track : Cosmological diagrammatic rules *S.B. Giddings and M.S. Sloth, JCAP 07 (2010) 015*

Cosmological Diagrammatic Rules

S.B. Giddings and M.S. Sloth, JCAP 1007 (2010) 015[1005.3287]

- One can summarise the cosmological diagrammatic rules for scalar exchange ($H_{\zeta AA}$) as follows

1. Draw the final hyper surface, end of inflation $\eta = \eta_0$
2. Draw all possible diagrams for the process, allowing vertices to appear above and below the η_0 surface



etc.

3. Lines that cross or terminate are represented by Wightman function $W_k(\eta, \eta') = A_k(\eta) A_k^*(\eta')$

- Lines entirely below the final surface : $G_F\left(k; \eta, \eta'\right) = W_k\left(\eta, \eta'\right)\theta\left(\eta - \eta'\right) + W_k\left(\eta', \eta\right)\theta\left(\eta' - \eta\right)$

- Lines entirely above the final surface : $G_F^*\left(k; \eta, \eta'\right)$

4. Vertices located below : a factor of $-2in\lambda$ and integrate over the conformal time

- Vertices located above : a factor of $2in\lambda$ and integrate over the conformal time

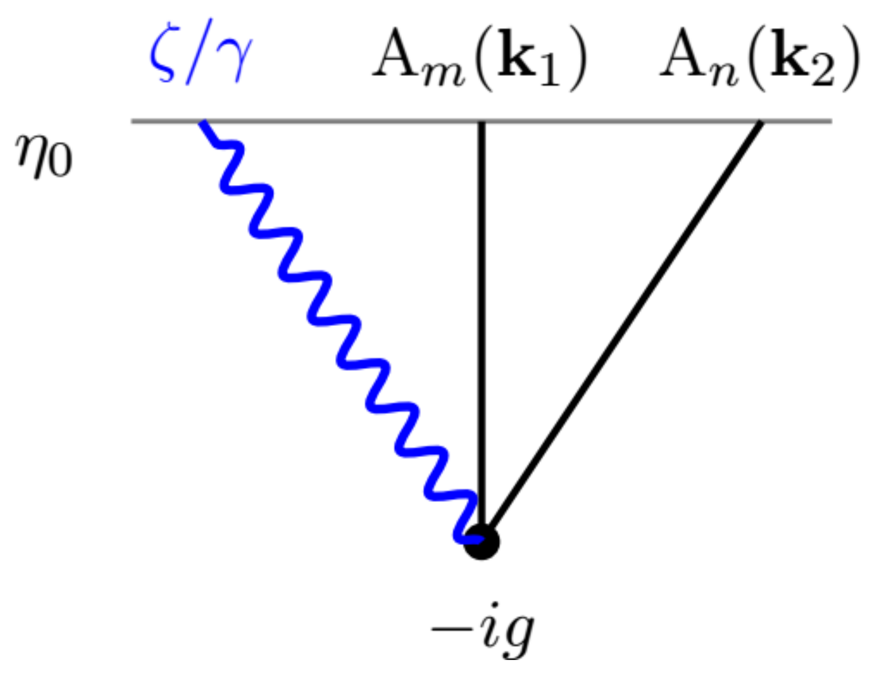
5. Divide by appropriate symmetry factor, diagram related by reflection across η_0 accounted by $2Re$

The Bispectrum

- Bispectrum (contact diagrams)

R.K. Jain, P.J. Sai and M.S. Sloth, JCAP 03 (2022) 054
R.K. Jain and M.S. Sloth,, JCAP 02 (2013) 003
R.R. Caldwell, L. Motta and M. Kamionkowski,,Phys. Rev. D 84 (2011) 123525

$\langle \zeta AA \rangle$ or $\langle \gamma AA \rangle$



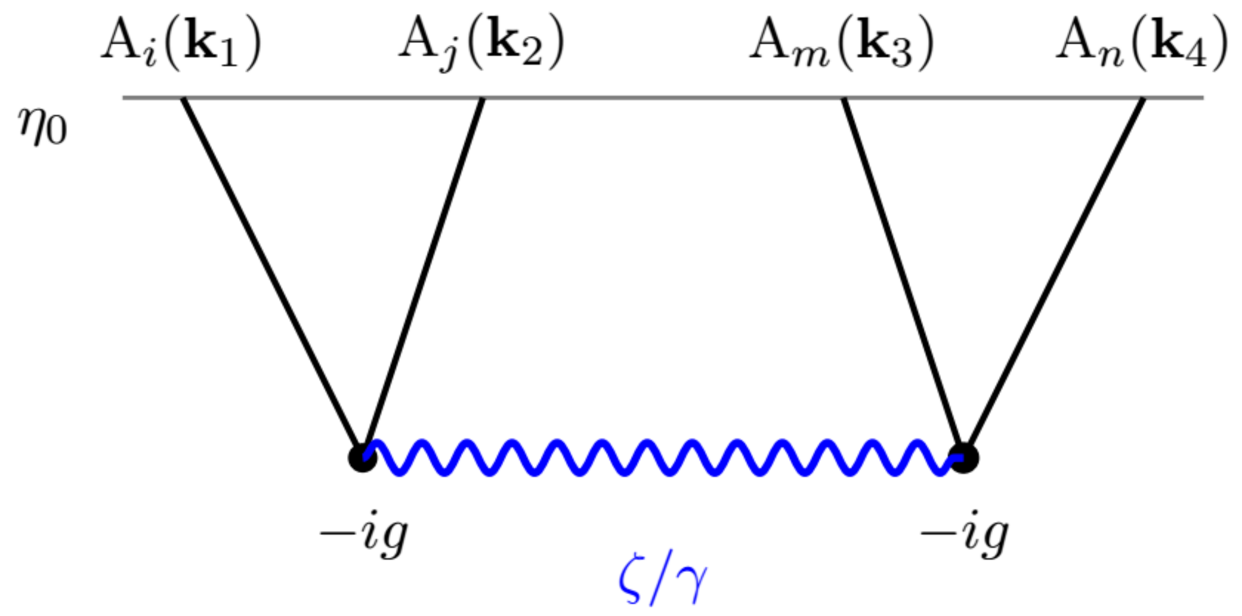
$$\langle\, \zeta(k_1)/\gamma(k_1)\, B(k_2)B(k_3)\rangle_{\eta_0} \sim \log(-k_t\eta_0)\, P_{\zeta/\gamma}(k_1)P_B(k_2)$$

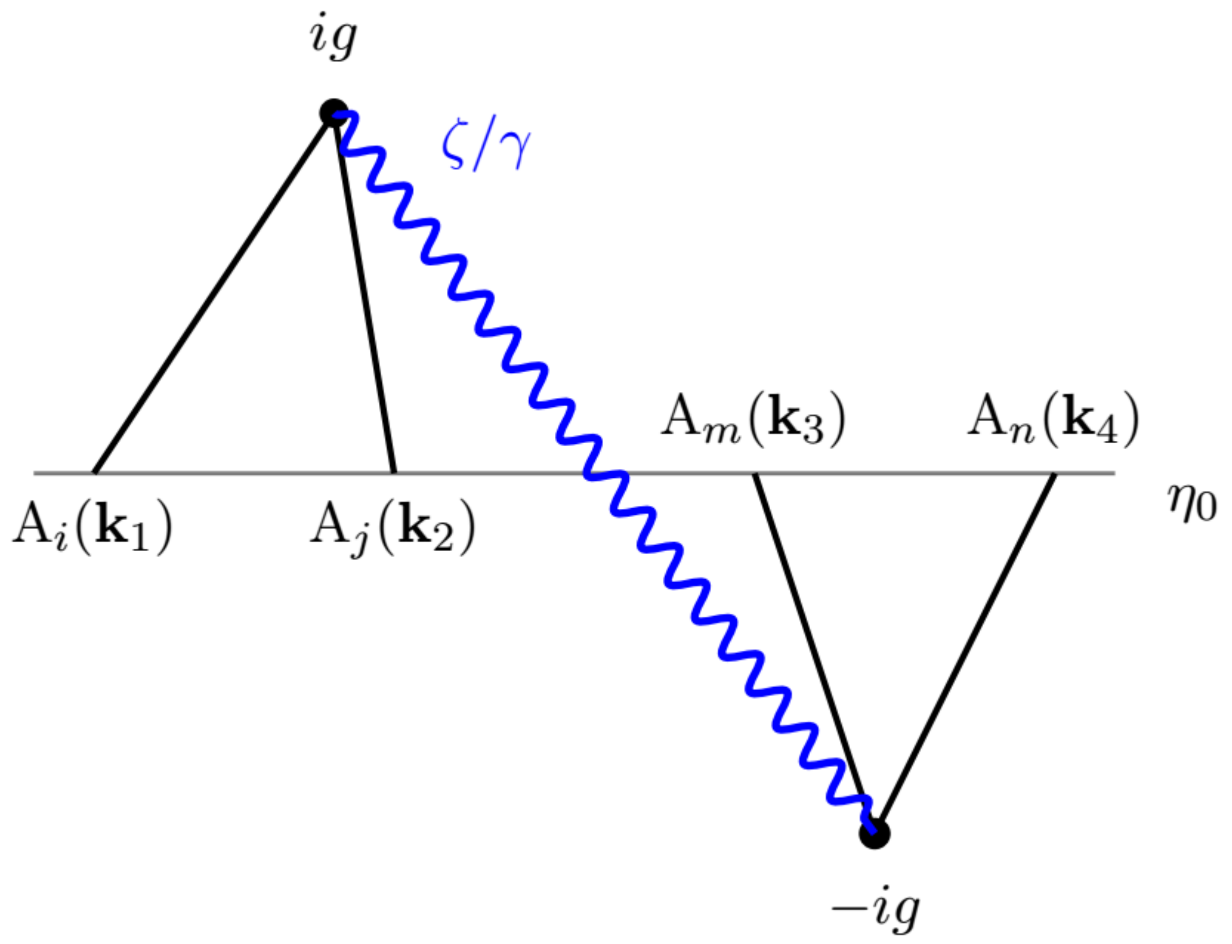
- The result is for $n = 2$
- The first check of non-Gaussian feature
- Logarithmically diverging contributions

- Trispectrum (Exchange diagrams)

  - Naturally next in order

$\langle AAAA \rangle$



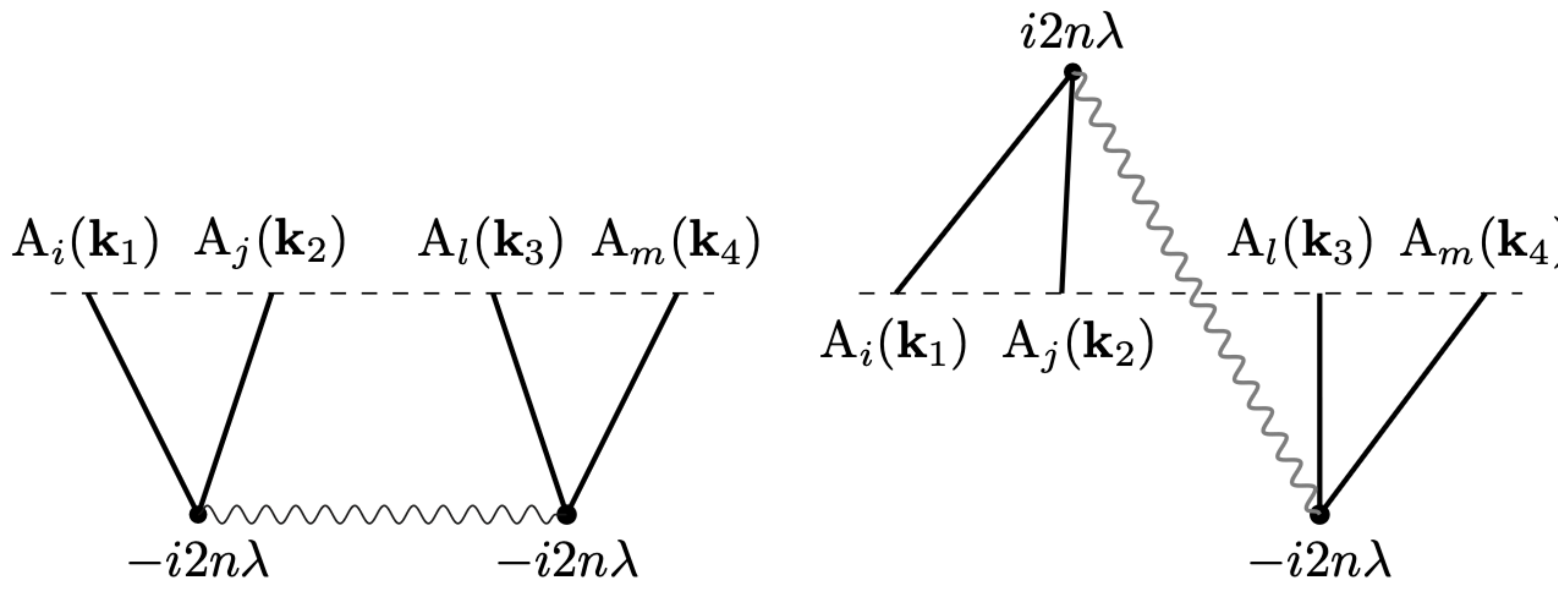


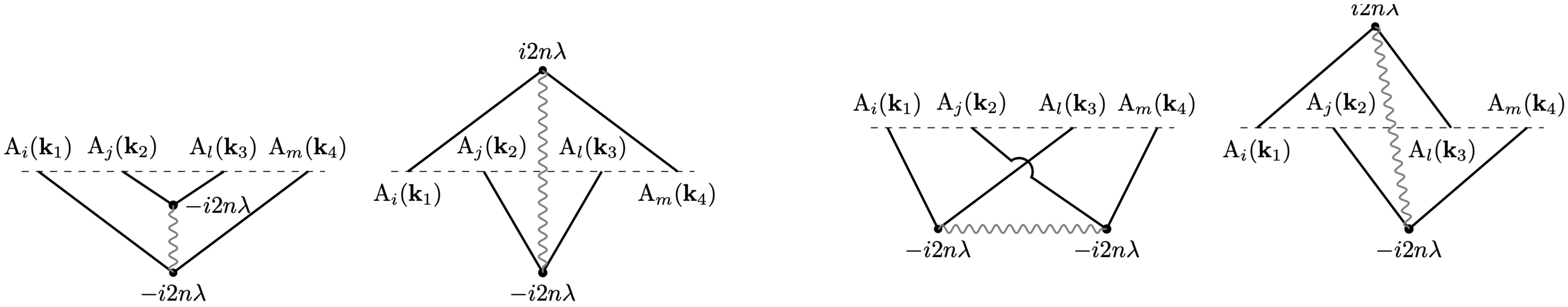
The Trispectrum

- $< A_i A_j A_m A_n >$ and $< A'_i A'_j A'_m A'_n >$ $\longrightarrow$ $< B_\mu B_\nu B_\rho B_\sigma >$ and $< E_\mu E_\nu E_\rho E_\sigma >$

  With $B_\mu =^* F_{\mu\nu} u^\nu$ and $E_\mu = F_{\mu\nu} u^\nu$ with $u^\mu = (1/a, 0, 0, 0)$

- All possible diagrams are

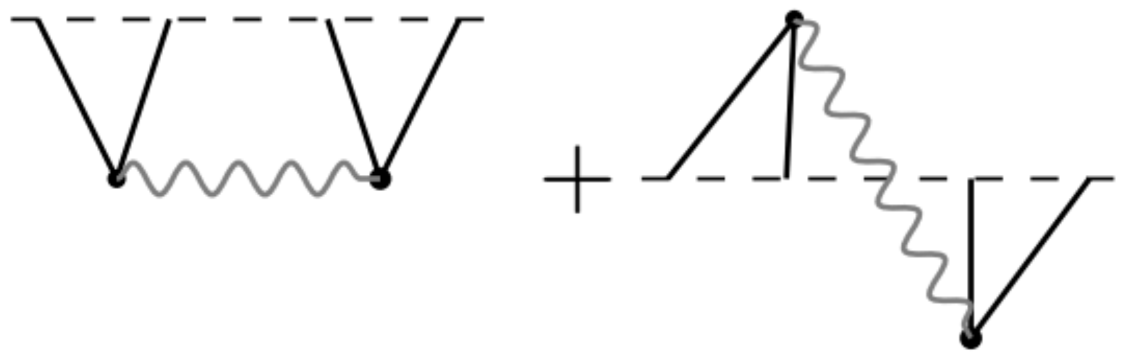




Computations

- Solution can be written in a schematic way as

$$\langle A_i(\mathbf{k}_1)\, A_j(\mathbf{k}_2)\, A_l(\mathbf{k}_3)\, A_m(\mathbf{k}_4)\rangle = 2\,\mathrm{Re}\left\{ \right.$$



$$\left. \right\} + \text{permutations}$$

- Trispectrum can be written as

Nested time Integral of mode functions

$$\langle A_i(\mathbf{k}_1)\, A_j(\mathbf{k}_2)\, A_l(\mathbf{k}_3)\, A_m(\mathbf{k}_4)\rangle = (2\pi)^3\,\delta\left(\sum_{i=1}^{4}\mathbf{k}_i\right)\left(\frac{\dot\lambda}{H\lambda}\right)^2 \sum_{X,Y\in A,A'} {}^{\zeta/\gamma}Q^{XY}_{ijlm}\, I^{XY} + (j,k_2)\leftrightarrow(n,k_3) + (j,k_2)\leftrightarrow(m,k_4)$$

Connected $\quad (2n)^2 \quad$ Polarisation $\quad$ permutations

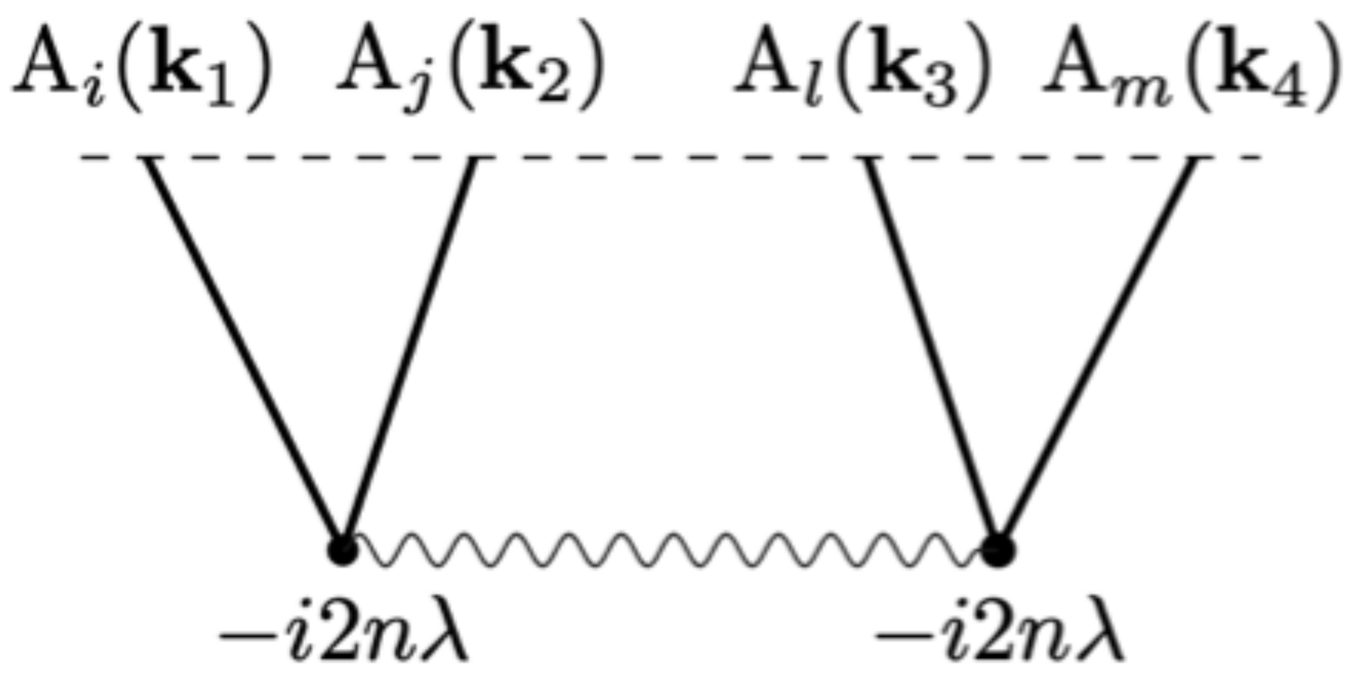
$$H_{\zeta AA} = n\int d^3x\,\lambda\zeta\left(A_i'^2 - \frac{1}{2}F_{ij}^2\right) + O(\epsilon)$$

- Results for arbitrary values of external momentum and for different 'n'
- Individual diagrams has spurious diverges as $\eta_0 \to 0$, which cancels each other.

Nested time integral

- The cubic order interaction Hamiltonian for the scalar exchange is

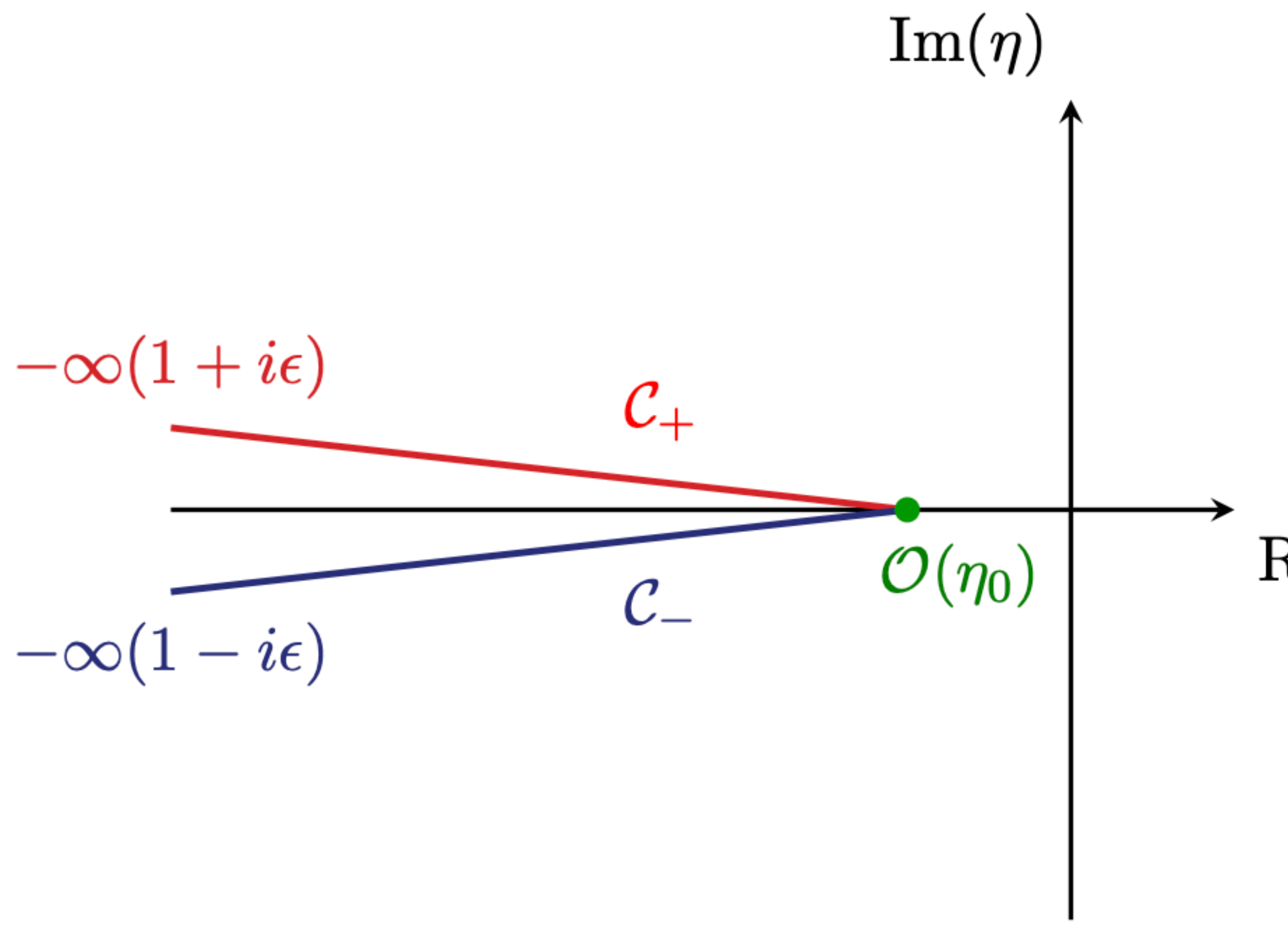
$$H_{\zeta AA} = n\int d^3x\, \lambda\zeta\left(A_i'^2 - \frac{1}{2}F_{ij}^2\right) + O(\epsilon)$$



- Using cosmological diagrammatic rules one can write

$$I_{++} = -\int_{-\infty}^{\eta_0} d\eta_1\, \lambda(\eta_1) \int_{-\infty}^{\eta_0} d\eta_2\, \lambda(\eta_2) W_{k_1}(\eta_0,\eta_1)\, W_{k_2}(\eta_0,\eta_1)\, G^{\zeta}_{k_I}(\eta_1,\eta_2)\, W_{k_3}(\eta_0,\eta_2)\, W_{k_4}(\eta_0,\eta_2)$$

On the positive contour



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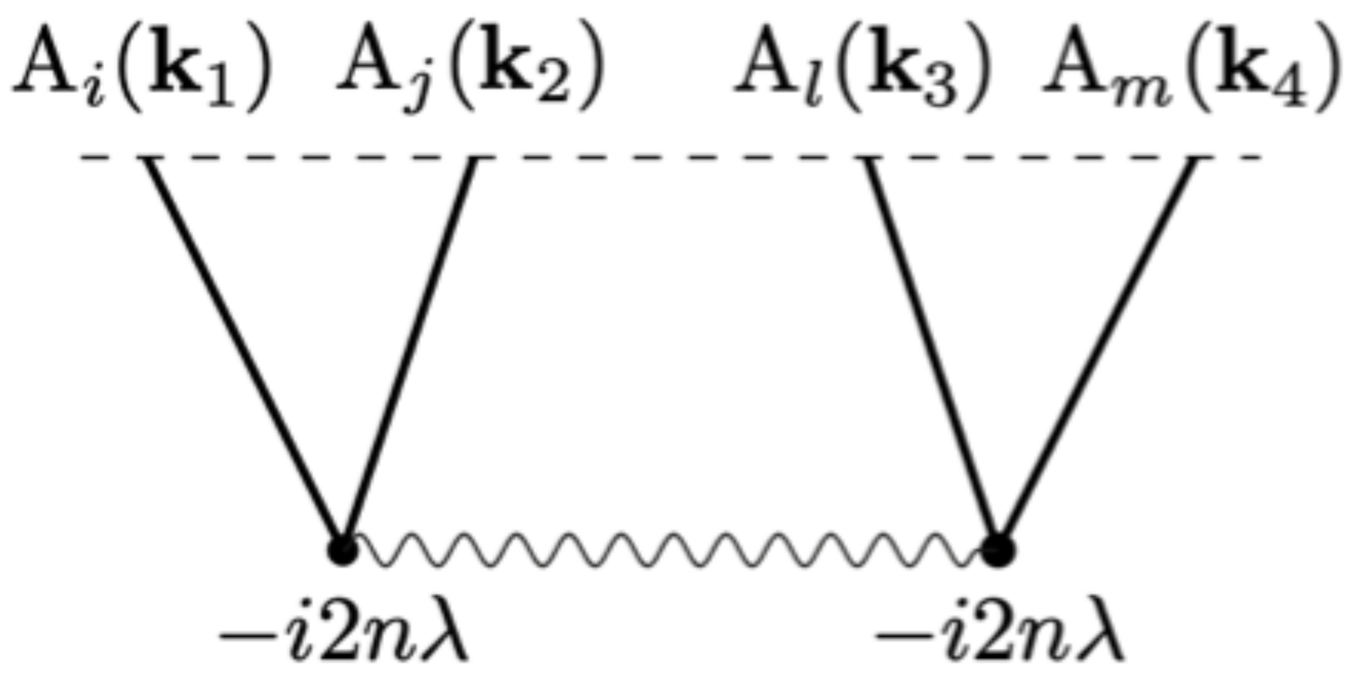


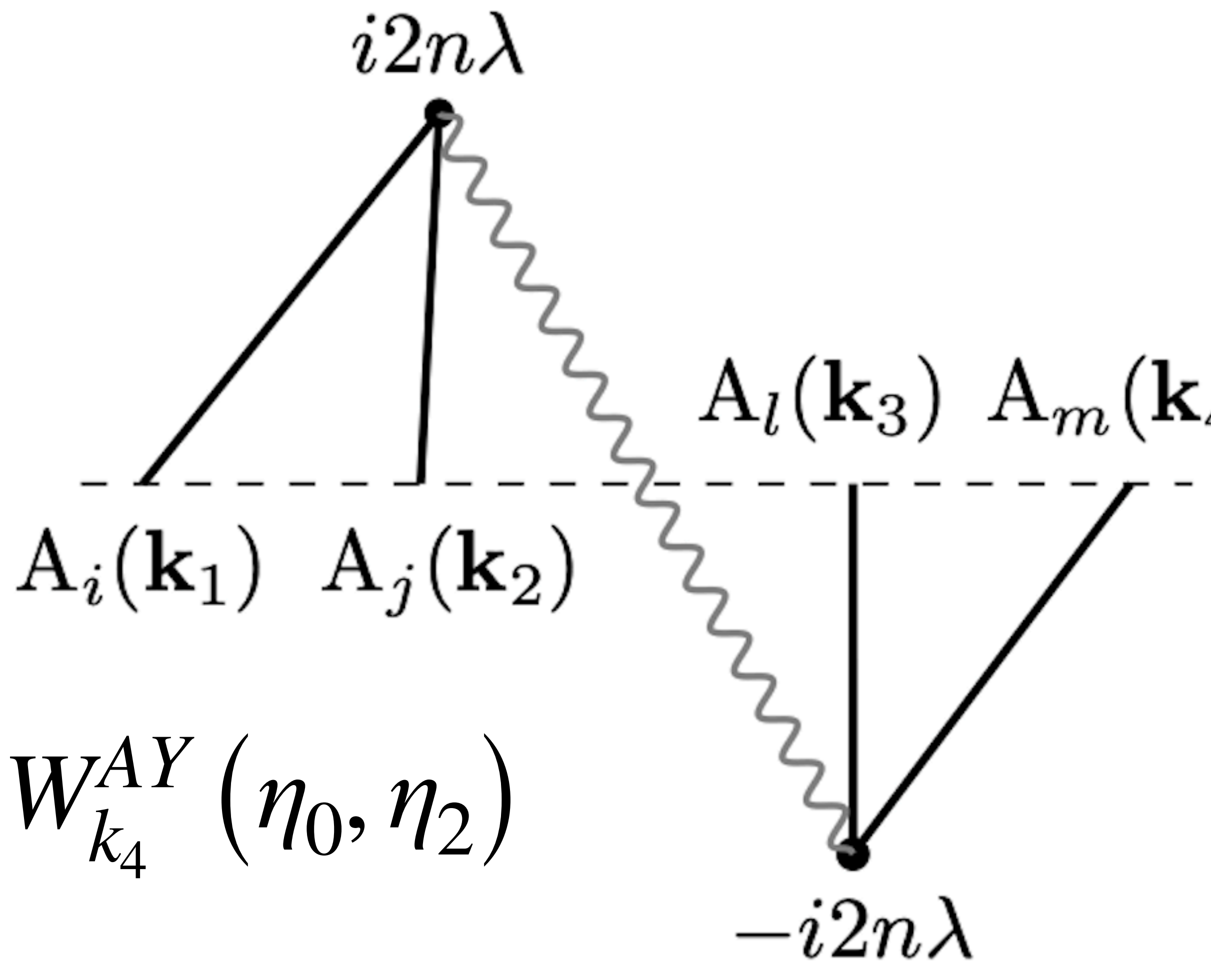
$$H_{\zeta AA} = n\int d^3x\,\lambda\zeta\left(A_i'^2 - \frac{1}{2}F_{ij}^2\right) + O(\epsilon)$$

- Using cosmological diagrammatic rules one can write

$$I^{XY}_{++} = -\int_{-\infty}^{\eta_0} d\eta_1\,\lambda\left(\eta_1\right)\int_{-\infty}^{\eta_0} d\eta_2\,\lambda\left(\eta_2\right)\, W^{AX}_{k_1}\left(\eta_0,\eta_1\right) W^{AX}_{k_2}\left(\eta_0,\eta_1\right) G^{\zeta}_{k_I}\left(\eta_1,\eta_2\right) W^{AY}_{k_3}\left(\eta_0,\eta_2\right) W^{AY}_{k_4}\left(\eta_0,\eta_2\right)$$

- "XY" comes due to the form of interaction Hamiltonian

- The second diagram



$$I^{XY}_{-+} = \int_{-\infty}^{\eta_0} d\eta_1\,\lambda\left(\eta_1\right)\int_{-\infty}^{\eta_0} d\eta_2\,\lambda\left(\eta_2\right)\, W^{AX}_{k_1}\left(\eta_1,\eta_0\right) W^{AX}_{k_2}\left(\eta_1,\eta_0\right) W^{\zeta}_{k_I}\left(\eta_1,\eta_2\right) W^{AY}_{k_3}\left(\eta_0,\eta_2\right) W^{AY}_{k_4}\left(\eta_0,\eta_2\right)$$

- The quantities we need to compute $\quad I^{XY} = 2\mathrm{Re}\left[I^{XY}_{++} + I^{XY}_{-+}\right]$

Polarisation Coefficient

- The polarisation coefficients are

$$Q^{\zeta AA}_{ijlm} = \Pi_{ia}\left(\hat{\boldsymbol{k}}_1\right)\mathbb{P}_{ab}(\boldsymbol{k}_2,\boldsymbol{k}_1)\,\Pi_{bj}\left(\hat{\boldsymbol{k}}_2\right)\ \Pi_{lc}\left(\hat{\boldsymbol{k}}_3\right)\mathbb{P}_{cd}(\boldsymbol{k}_4,\boldsymbol{k}_3)\,\Pi_{dm}\left(\hat{\boldsymbol{k}}_4\right)$$

$$Q^{\zeta AA'}_{ijlm} = \Pi_{ia}\left(\hat{\boldsymbol{k}}_1\right)\mathbb{P}_{ab}(\boldsymbol{k}_2,\boldsymbol{k}_1)\,\Pi_{bj}\left(\hat{\boldsymbol{k}}_2\right)\ \Pi_{lc}\left(\hat{\boldsymbol{k}}_3\right)\Pi_{cm}\left(\hat{\boldsymbol{k}}_4\right)$$

$$Q^{\zeta A'A}_{ijlm} = \Pi_{ia}\left(\hat{\boldsymbol{k}}_1\right)\Pi_{aj}\left(\hat{\boldsymbol{k}}_2\right)\ \Pi_{lc}\left(\hat{\boldsymbol{k}}_3\right)\mathbb{P}_{cd}(\boldsymbol{k}_4,\boldsymbol{k}_3)\,\Pi_{dm}\left(\hat{\boldsymbol{k}}_4\right)$$

$$Q^{\zeta A'A'}_{ijlm} = \Pi_{ia}\left(\hat{\boldsymbol{k}}_1\right)\Pi_{aj}\left(\hat{\boldsymbol{k}}_2\right)\ \Pi_{lc}\left(\hat{\boldsymbol{k}}_3\right)\Pi_{cm}\left(\hat{\boldsymbol{k}}_4\right)$$

With

Projection operator

$$\Pi_{ij}\left(\hat{\boldsymbol{k}}\right) = \delta_{ij} - \hat{k}_i\hat{k}_j$$

$$\mathbb{P}_{ab}\left(\boldsymbol{k}_1,\boldsymbol{k}_2\right) = \left(\boldsymbol{k}_1\cdot\boldsymbol{k}_2\right)\delta_{ab} - k_{1a}k_{2b}$$

Generalisation of projection operator

- Enforces the Coulomb gauge constrain : $\boldsymbol{k}\,.\,\Pi(\boldsymbol{k}) = 0$
- Only contains scalar products of external momentum
- Properties of projection tensor gives index free form for all $Q's$

Scalar Exchange: Magnetic Trispectra

- We focus on the index free trispectra

Different channels

$$\langle \boldsymbol{B}(\boldsymbol{k}_1)\cdot\boldsymbol{B}(\boldsymbol{k}_2)\,\boldsymbol{B}(\boldsymbol{k}_3)\cdot\boldsymbol{B}(\boldsymbol{k}_4)\rangle = (2\pi)^3\,\delta^{(3)}\left(\sum_{\alpha=1}^{4}\mathbf{k}_\alpha\right)\mathscr{T}_{\zeta B}(\boldsymbol{k}_1,\boldsymbol{k}_2,\boldsymbol{k}_3,\boldsymbol{k}_4) \quad \text{With} \quad \mathscr{T}_{\zeta B} = \frac{1}{a^8}\left(\frac{\dot\lambda}{H\lambda}\right)^2 \sum_{S\in 1,2,3}\ \sum_{X,Y\in A,A'} Q^{\zeta}_{SXY} I^{SXY}$$

- Polarisation function is

$$Q^{\zeta}_{1AA} = k_1^2k_2^2k_3^2k_4^2\left(1+\cos^2\theta_{12}\right)\left(1+\cos^2\theta_{34}\right)$$

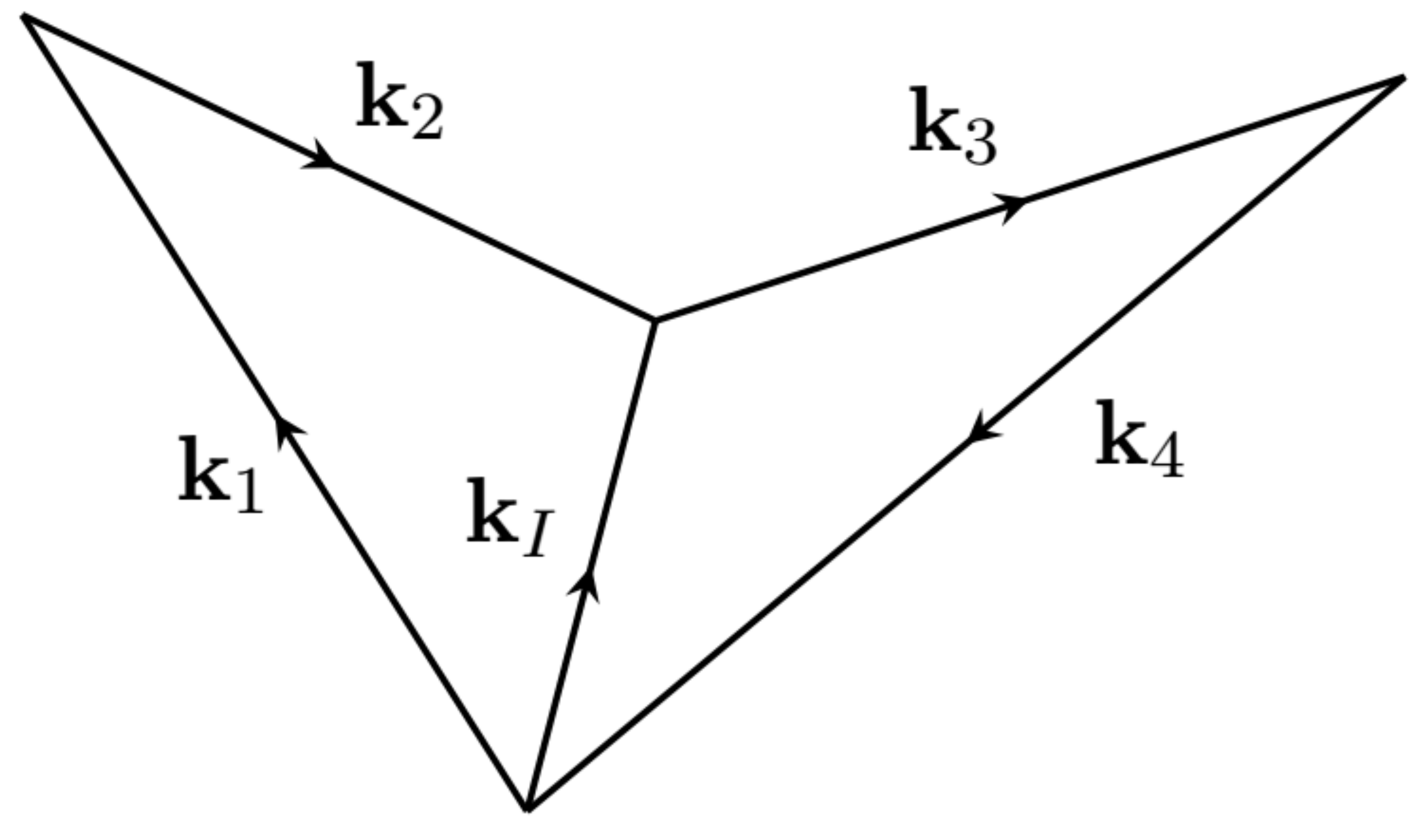
$$Q^{\zeta}_{1AA'} = 2k_1^2k_2^2k_3k_4\left(1+\cos^2\theta_{12}\right)\cos\theta_{34}$$

$$Q^{\zeta}_{1A'A} = 2k_1k_2k_3^2k_4^2\cos\theta_{12}\left(1+\cos^2\theta_{34}\right)$$

$$Q^{\zeta}_{1A'A'} = 4k_1k_2k_3k_4\cos\theta_{12}\cos\theta_{34}$$

Momentum Quadrilateral

- Momentum conservation $\Longrightarrow$ closed shapes



- Shape is not planar

Scalar Exchange: Electric fields Trispectra

- The electric field trispectrum

$$\langle \boldsymbol{E}(\boldsymbol{k}_1)\cdot\boldsymbol{E}(\boldsymbol{k}_2)\,\boldsymbol{E}(\boldsymbol{k}_3)\cdot\boldsymbol{E}(\boldsymbol{k}_4)\rangle = (2\pi)^3\,\delta^{(3)}\left(\sum_{\alpha=1}^{4}\boldsymbol{k}_\alpha\right)\left(\frac{\dot{\lambda}}{H\lambda}\right)^2\frac{1}{a^8}\,|\zeta^{(0)}_{k_I}|^2\,|A'^{(0)}_{k_1}|\,|A'^{(0)}_{k_2}|\,|A'^{(0)}_{k_3}|\,|A'^{(0)}_{k_4}|$$

$$\times\, k_1k_2k_3k_4\left(\frac{\pi^2}{8}\sum_S\sum_{X,Y}\tilde{Q}^{\zeta}_{SXY}\,\tilde{I}^{SXY}\right)$$

With

$$\tilde{Q}^{\zeta}_{1AA} = 4\cos\theta_{12}\cos\theta_{34}$$

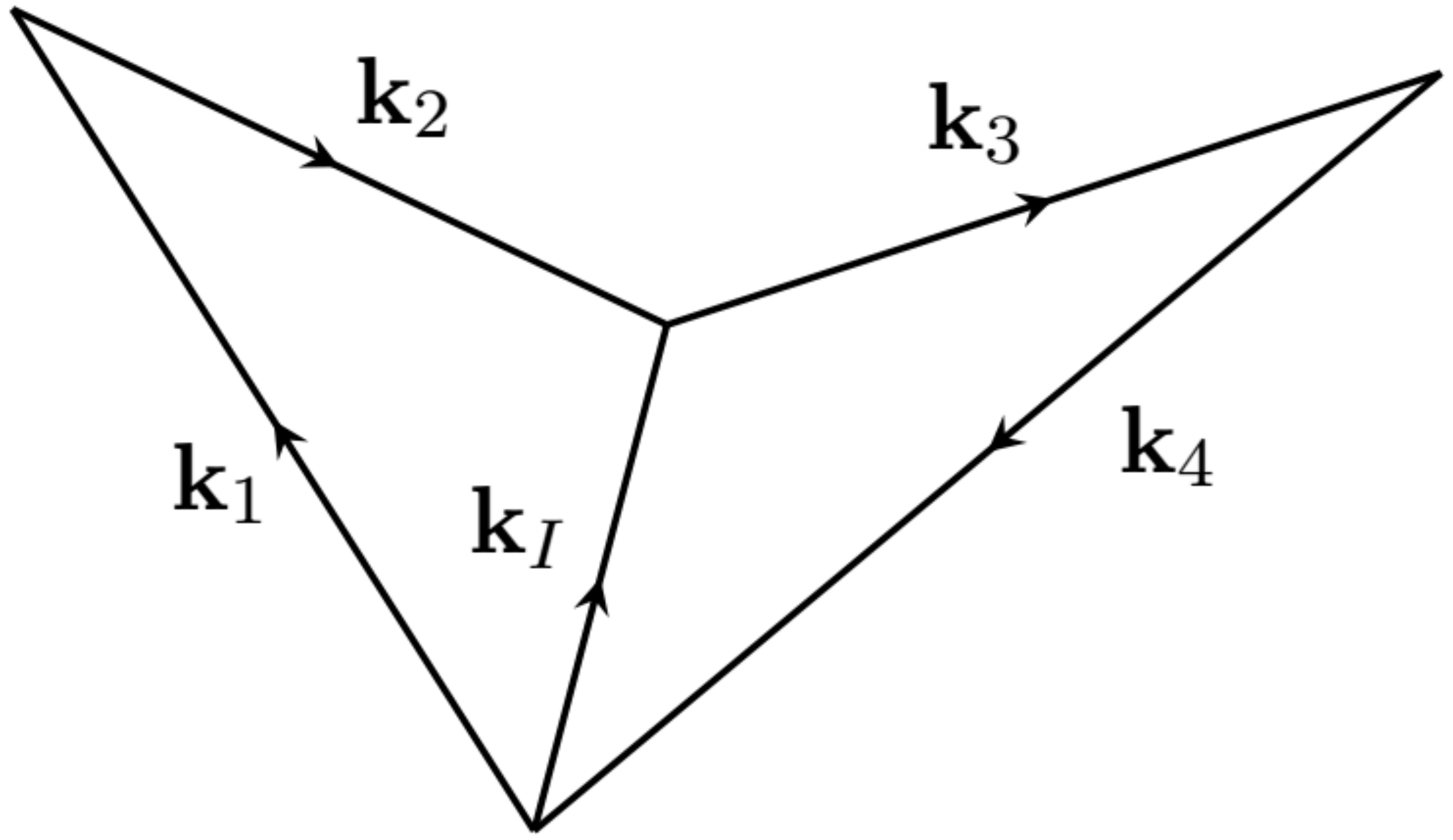
$$\tilde{Q}^{\zeta}_{1AA'} = 2\,\cos\theta_{12}\left(1+\cos^2\theta_{34}\right)$$

$$\tilde{Q}^{\zeta}_{1A'A} = 2\,\left(1+\cos^2\theta_{12}\right)\,\cos\theta_{34}$$

$$\tilde{Q}^{\zeta}_{1A'A'} = \left(1+\cos^2\theta_{12}\right)\left(1+\cos^2\theta_{34}\right)$$

- For $n > 0$ the electric trispectrum is much suppressed compare to the magnetic trispectrum

Equisided Configurartions



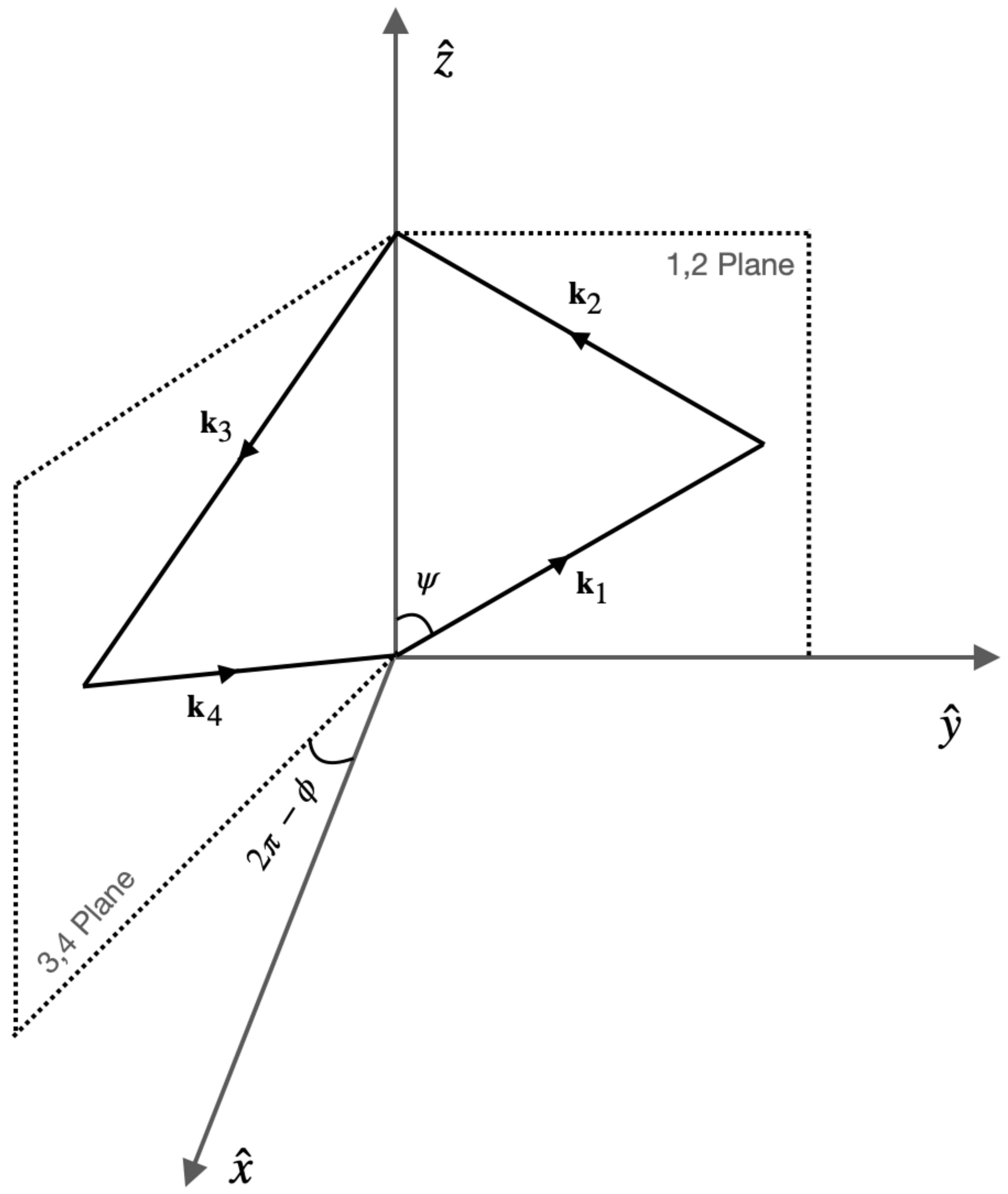
- Equisided configuration: $|\mathbf{k}_1| = |\mathbf{k}_2| = |\mathbf{k}_3| = |\mathbf{k}_4|$
- Maximises the signal for interactions with higher derivative operators

  - Each derivative contributes powers of momenta
- Removes the directional hierarchies
- Shapes can be analysed using two characteristic scale

  - One can count the DOF as : 12 - 3- 6-1= 2 or 12- 3- 4 - 3
- Choose the coordinate as follows:

Equisided Configurartions

- Choose the coordinate as follows:

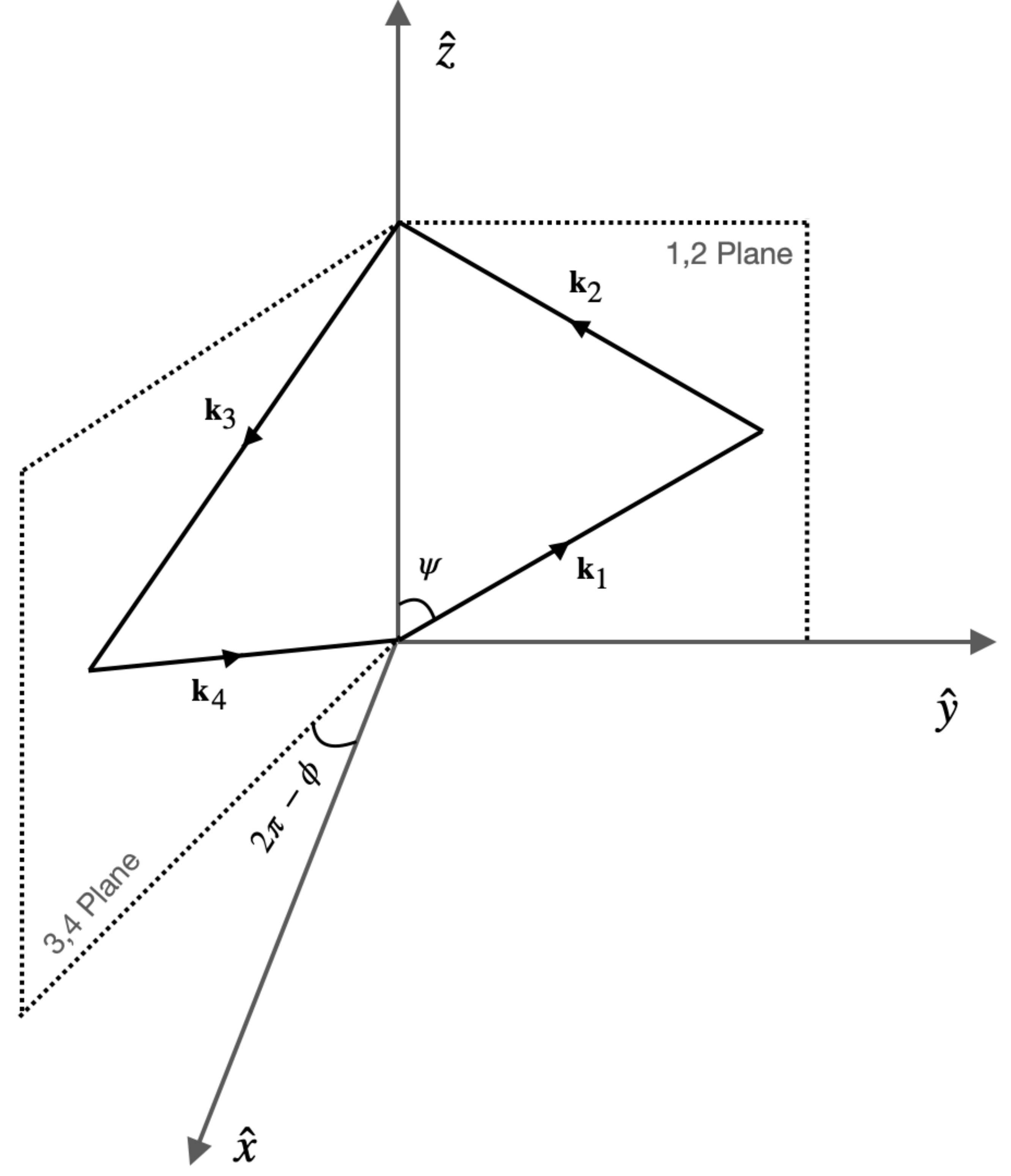


Equisided Configurartions

- The relative angles $C_{ij} = \cos\theta_{ij}$

$$C_{12} = C_{34} = \cos 2\psi$$
$$C_{13} = C_{24} = \sin^2\psi \sin\phi - \cos^2\psi$$
$$C_{14} = C_{23} = -\sin^2\psi \sin\phi - \cos^2\psi$$

With $$\cos\psi = \frac{k_I}{2k}$$



- One can study the configuration specifying k_I, k and ϕ

- The exchange momentum in different channels is

$$k_{\mathrm{I}} = k\sqrt{2\left(1 + C_{12}\right)}, \quad k_{\mathrm{II}} = k\sqrt{2\left(1 + C_{13}\right)}, \quad k_{\mathrm{III}} = k\sqrt{2\left(1 + C_{14}\right)}$$

- Value of ϕ different configurations: $\phi = \pi/2$ is counter collinear limit in third channel

Equisided Configurartions

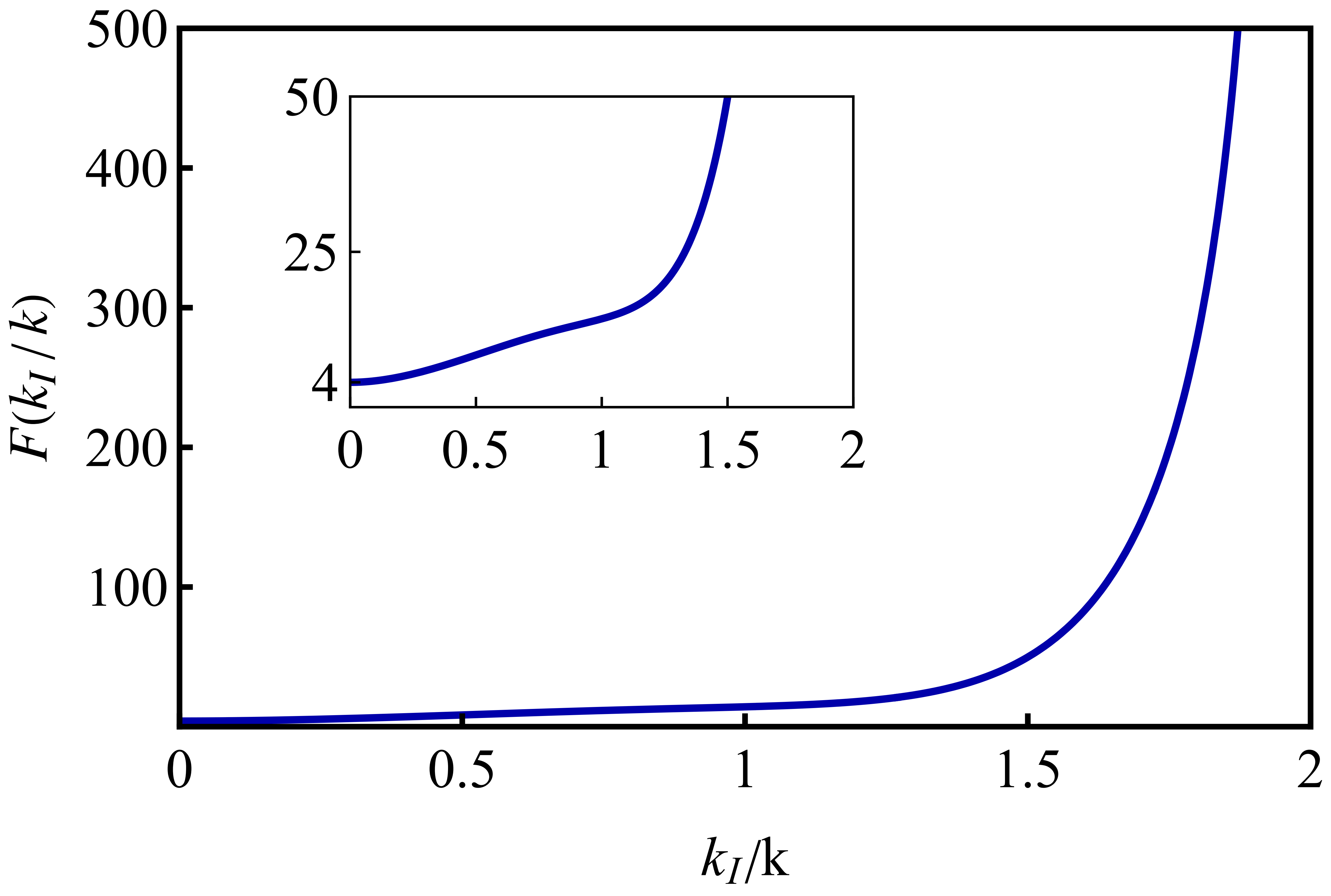
- The magnetic trispectrum is

$$\langle \boldsymbol{B}(\boldsymbol{k}_1)\cdot\boldsymbol{B}(\boldsymbol{k}_2)\boldsymbol{B}(\boldsymbol{k}_3)\cdot\boldsymbol{B}(\boldsymbol{k}_4)\rangle = (2\pi)^3\,\delta^{(3)}\left(\sum_{\alpha=1}^{4}\mathbf{k}_\alpha\right)\left(\frac{\dot\lambda}{H\lambda}\right)^2 P_\zeta(k_I)P_B^2(k)\;F\left(\frac{k_I}{k},\phi\right)$$

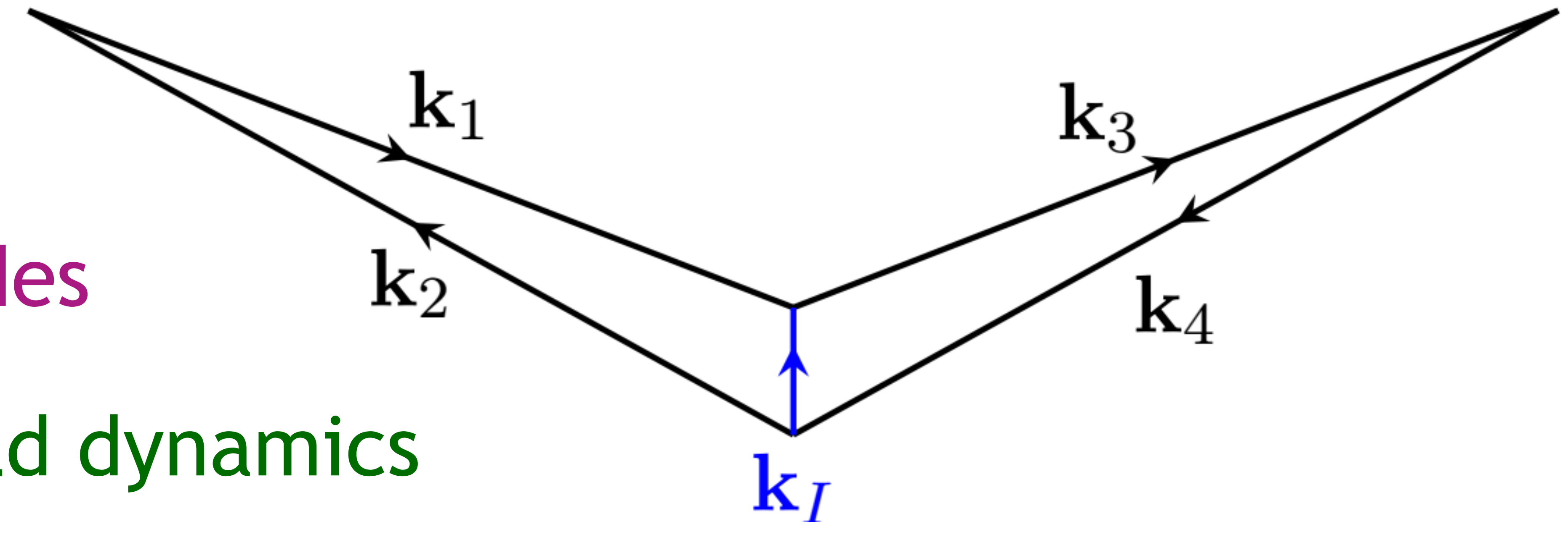
Contains all the informations of three channels and the nested time integrals

- For $\phi = 0$

  - Monotonically increasing function of k_I/k
  - Correlation strength $\propto k_I$
  - $F(0) \neq 0$, is the counter collinear limit



Counter Collinear Limit



- The counter collinear limit is defined as $k_I \to 0$ or $\boldsymbol{k}_1 \approx -\boldsymbol{k}_2$
- Captures the long-wavelength modulation of short-wavelength modes
- Amplifies the exchange contributions and sensitive to the multi field dynamics
- Primary objects in the construction of soft theorem or the consistency relations
- The magnetic field trispectrum in the limit is

$$\lim_{k_I \to 0} \langle \boldsymbol{B}(\boldsymbol{k}_1)\cdot\boldsymbol{B}(\boldsymbol{k}_2)\boldsymbol{B}(\boldsymbol{k}_3)\cdot\boldsymbol{B}(\boldsymbol{k}_4)\rangle = (2\pi)^3\,\delta^{(3)}\left(\sum_{\alpha=1}^{4}\boldsymbol{k}_\alpha\right)\beta_{NL}\,P_\zeta(k_I)P_B(k_1)P_B(k_3)$$

Non-linearity parameter

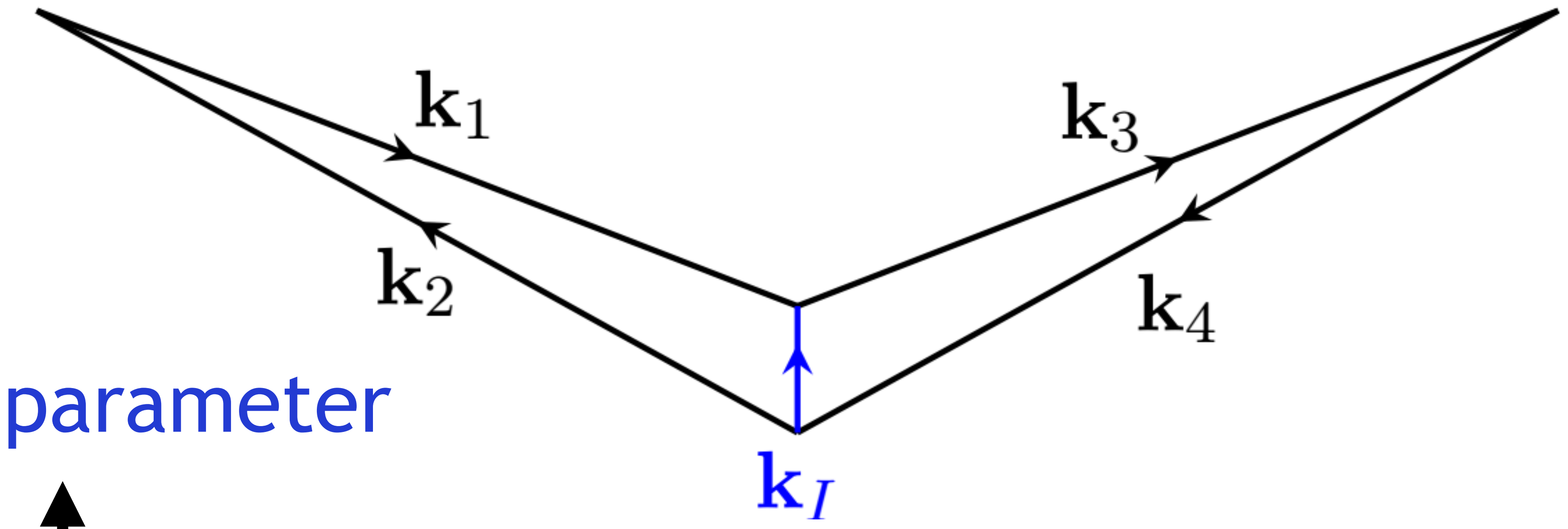
- From

Equation of Motion

$$\lim_{k_I \to 0} \langle \boldsymbol{B}(\boldsymbol{k}_1)\cdot\boldsymbol{B}(\boldsymbol{k}_2)\,\boldsymbol{B}(\boldsymbol{k}_3)\cdot\boldsymbol{B}(\boldsymbol{k}_4)\rangle = (2\pi)^3\,\delta^{(3)}\left(\sum_{\alpha=1}^{4}\boldsymbol{k}_\alpha\right)\left(\frac{\dot{\lambda}}{H\lambda}\right)^2 P_\zeta(k_I)P_B(k)P_B(p)$$

- For generic coupling (n) and for any channel

Counter Collinear Limit



$$\Longrightarrow \quad \beta_{NL}^{\zeta} = \left(\frac{\dot{\lambda}}{H\lambda}\right)^2 = \left(b_{NL}^{\zeta}\right)^2$$

Non-linearity parameter

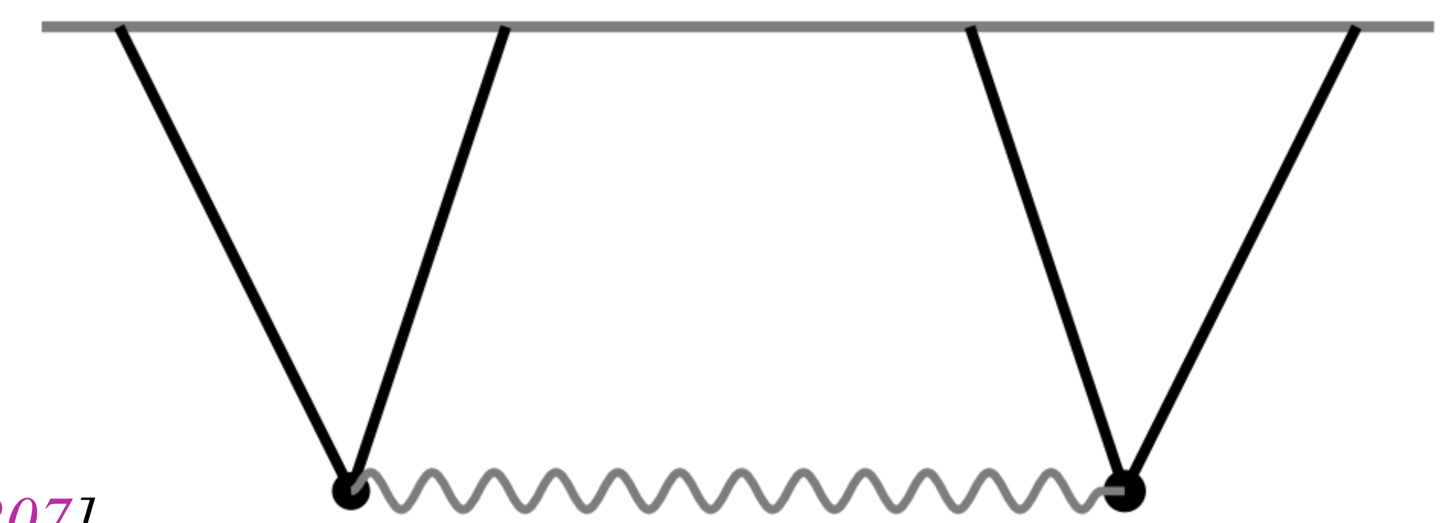
Where
$$\lim_{\boldsymbol{k}_1 \to 0} \langle \zeta(\boldsymbol{k}_1)\boldsymbol{B}(\boldsymbol{k}_2)\cdot\boldsymbol{B}(\boldsymbol{k}_3)\rangle = (2\pi)^3\,\delta^{(3)}\left(\sum_{\alpha=1}^{3}\boldsymbol{k}_\alpha\right) b_{NL}^{\zeta} P_\zeta(k_1) P_B(k)$$

T. Suyama and M. Yamaguchi,Phys.Rev. D 77 (2008) 023505 [0709.2545]

- Novel consistency relation similar to Suyama-Yamaguchi relation
- The bispectrum : single trilinear vertex, the trispectrum : two such vertices.
- The equality is a lower bound, in general $\beta_{NL}^{\zeta} \geq \left(b_{NL}^{\zeta}\right)^2$

  - The presence of γ exchange validate this inequality

V. Assassi, D. Baumann and D. JCAP 11 (2012) 047 [1204.4207]



Tensor Exchanges

- The exchange mode is a spin 2 graviton γ_{ij} : helicity
- The mode expansion is

$$\gamma_{ij}(\mathbf{x}, \eta) = \int \frac{d^3\mathbf{k}}{(2\pi)^3} \sum_{s=\pm 2} \left[\gamma_k(\eta)\, e^{i\mathbf{k}\cdot\mathbf{x}}\, \underbrace{\epsilon^s_{ij}(\hat{\mathbf{k}})}_{\text{polarisation tensor}}\, b^s_{\mathbf{k}} + h.c. \right]$$

- Scalar Exchange $\xrightarrow{W^\gamma_k(\eta_1, \eta_2) = r\, W^\zeta_k(\eta_1, \eta_2)}$ Tensor Exchange

- So the tensor exchange trispectrum is suppressed by tensor to scalar ratio
- The trispectrum in terms of gauge field is

$$< A_i(\boldsymbol{k}_1)\, A_j(\boldsymbol{k}_2)\, A_l(\boldsymbol{k}_3)\, A_m(\boldsymbol{k}_4) > = r\, (2\pi)^3\, \delta^{(3)}\left(\sum_{\alpha=1}^{4} \boldsymbol{k}_\alpha\right) \sum_{X,Y \in A,A'} Q^{\gamma XY}_{ijlm}\, I^{XY}$$

Different channels

$$+(j, \boldsymbol{k}_2) \leftrightarrow (l, \boldsymbol{k}_3) + (j, \boldsymbol{k}_2) \leftrightarrow (m, \boldsymbol{k}_4)$$

Tensor Exchanges: Angular dependence

- The polarisation coefficients are

$$Q^{\gamma AA}_{ijlm} = \alpha_{abcd}(\boldsymbol{k}_1, \boldsymbol{k}_2)\, \Pi_{ic}\left(\hat{\boldsymbol{k}}_1\right)\, \Pi_{jd}\left(\hat{\boldsymbol{k}}_2\right)\, \omega_{aba'b'}\left(\hat{\boldsymbol{k}}_{\mathrm{I}}\right) \Pi_{lc'}\left(\hat{\boldsymbol{k}}_3\right)\, \Pi_{md'}\left(\hat{\boldsymbol{k}}_4\right)\, \alpha_{a'b'c'd'}(\boldsymbol{k}_3, \boldsymbol{k}_4)$$

$$Q^{\gamma AA'}_{ijlm} = \alpha_{abcd}(\boldsymbol{k}_1, \boldsymbol{k}_2)\, \Pi_{ic}\left(\hat{\boldsymbol{k}}_1\right)\, \Pi_{jd}\left(\hat{\boldsymbol{k}}_2\right)\, \omega_{aba'b'}\left(\hat{\boldsymbol{k}}_{\mathrm{I}}\right) \Pi_{la'}\left(\hat{\boldsymbol{k}}_3\right)\, \Pi_{mb'}\left(\hat{\boldsymbol{k}}_4\right)$$

$$Q^{\gamma A'A}_{ijlm} = \Pi_{ia}\left(\hat{\boldsymbol{k}}_1\right)\, \Pi_{jb}\left(\hat{\boldsymbol{k}}_2\right)\, \omega_{aba'b'}\left(\hat{\boldsymbol{k}}_{\mathrm{I}}\right) \Pi_{lc'}\left(\hat{\boldsymbol{k}}_3\right)\, \Pi_{md'}\left(\hat{\boldsymbol{k}}_4\right)\, \alpha_{a'b'c'd'}(\boldsymbol{k}_3, \boldsymbol{k}_4)$$

$$Q^{\gamma A'A'}_{ijlm} = \Pi_{ia}\left(\hat{\boldsymbol{k}}_1\right)\, \Pi_{jb}\left(\hat{\boldsymbol{k}}_2\right)\, \omega_{aba'b'}\left(\hat{\boldsymbol{k}}_{\mathrm{I}}\right) \Pi_{la'}\left(\hat{\boldsymbol{k}}_3\right)\, \Pi_{mb'}\left(\hat{\boldsymbol{k}}_4\right)$$

With

$$\omega_{abcd}\left(\hat{\boldsymbol{q}}\right) = -\Pi_{ab}\left(\hat{\boldsymbol{q}}\right)\Pi_{cd}\left(\hat{\boldsymbol{q}}\right) + \Pi_{ac}\left(\hat{\boldsymbol{q}}\right)\Pi_{bd}\left(\hat{\boldsymbol{q}}\right) + \Pi_{ad}\left(\hat{\boldsymbol{q}}\right)\Pi_{bc}\left(\hat{\boldsymbol{q}}\right)$$

$$\alpha_{abcd}\left(\boldsymbol{k}, \boldsymbol{p}\right) = \delta_{cd} k_a p_b + \left(\boldsymbol{k}\cdot\boldsymbol{p}\right)\delta_{ac}\delta_{bd} - 2\delta_{bd} k_a p_c$$

- $\omega \implies$ contraction among exchange momentum and external momentum
- Different exchange momentums, $\boldsymbol{k}_I, \boldsymbol{k}_{II}$ and $\boldsymbol{k}_{III}$, in different channels

Tensor Exchanges: Angular Dependence

$$\omega_{abcd}(\hat{\boldsymbol{q}}) = -\Pi_{ab}(\hat{\boldsymbol{q}})\,\Pi_{cd}(\hat{\boldsymbol{q}}) + \Pi_{ac}(\hat{\boldsymbol{q}})\,\Pi_{bd}(\hat{\boldsymbol{q}}) + \Pi_{ad}(\hat{\boldsymbol{q}})\,\Pi_{bc}(\hat{\boldsymbol{q}}) \qquad \alpha_{abcd}(\boldsymbol{k},\boldsymbol{p}) = \delta_{cd}k_a p_b + (\boldsymbol{k}\cdot\boldsymbol{p})\,\delta_{ac}\delta_{bd} - 2\delta_{bd}k_a p_c$$

- ω : Correlating the azimuthal orientation of $\boldsymbol{k}_I$ with $\boldsymbol{k}$
- α : mixed contractions with free indices $\Longrightarrow$ higher order angular functions
- In counter collinear limit : enhanced correlation along $\boldsymbol{k}_I$
- Equisided configuration : oscillatory angular dependence (spin -2 nature)

Magnetic trispectrum

- The richer angular dependence is preserved
- In the counter collinear limit

$$Q^{\gamma}_{1AA} \to k_1^4 k_3^4\, \hat{k}_{1a}\hat{k}_{1b}\,\omega_{aba'b'}(\hat{\boldsymbol{k}}_{\rm I})\,\hat{k}_{3a'}\hat{k}_{3b'}$$

Spin-2 projection from ω is survived

Super Hubble Growth

L. Motta and R.R. Caldwell Phys. Rev. D85 (2012) 103532 [1203.1033]
R.K. Jain and M.S. Sloth, Phys. Rev. D 86 (2012) 123528 [1207.4187]
M. Shiraishi, S. Saga and S. Yokoyama, JCAP 1211 (2012) 046[1209.3384]
R.K. Jain, P.J. Sai and M.S. Sloth, JCAP 03 (2022) 054 [2108.10887]

- For $n=2$, the bispectrum : $\langle\, \zeta(k_1)/\gamma(k_1)\, B(k_2)B(k_3)\rangle_{\eta_0} \sim \log(-k_t\eta_0)\, P_{\zeta/\gamma}(k_1)P_B(k_2)$
- No such logarithmic time dependence for $n=2$ trispectrum
- However for $n=3$

$$\langle AAAA > \sim \log(-k_t\eta_0)$$

- For $n>3$, the η_0 dependence is inverse polynomial
- A diverging behaviour results in large observational signal

Magnetic Energy Density

- The trspectrum also encodes the following:
- The magnetic energy density

$$\rho_B(\boldsymbol{k}) = \int \frac{d^3p}{(2\pi)^3} B_i(\boldsymbol{p})\, B_i(\boldsymbol{k}-\boldsymbol{p}) \implies \left\langle \rho_B(\boldsymbol{k})\, \rho_B(\boldsymbol{k}') \right\rangle = \int \frac{d^3p}{(2\pi)^3} \int \frac{d^3p'}{(2\pi)^3} \left\langle B_i(\boldsymbol{p})\, B_i(\boldsymbol{k}-\boldsymbol{p})\, B_j(\boldsymbol{p}')\, B_j(\boldsymbol{k}'-\boldsymbol{p}') \right\rangle$$

$$= (2\pi)^3 \delta^{(3)}(\boldsymbol{k}+\boldsymbol{k}') \left[\underbrace{P^{\rm disconn}_{\rho_B}(k)}_{\downarrow} + P^{\rm conn}_{\rho_B}(k) \right]$$

Gaussian: from P_B

And

$$P^{\rm conn}_{\rho_B}(k) = \int \frac{d^3p}{(2\pi)^3} \int \frac{d^3p'}{(2\pi)^3}\, \mathscr{T}_B\left(\boldsymbol{p}, \boldsymbol{k}-\boldsymbol{p}, \boldsymbol{p}', -\boldsymbol{k}-\boldsymbol{p}'\right)$$

$$\downarrow$$

$$\mathscr{T}_B = \mathscr{T}_{\zeta B} + \mathscr{T}_{\gamma B}$$

Summary and conclusion

- We did the systematic computation of four point autocorrelation function of $U(1)$ gauge field with full in-in formalism upto cubic order interation with both tensor and scalar perturbation.
- From the auxiliary gauge field we also obtain the trispectrum for $\boldsymbol{B}$ and $\boldsymbol{E}$, in the contracted form which contributes to energy density.
- In the equisided configurations the non-Gaussian signal is monotonically increasing as the shape approaches the flattened configurations.
- In the counter collinear limit, we showed that the magnetic field trispectrum follows a novel consistency relation.

$$\beta_{NL}^{\zeta} = \left(b_{NL}^{\zeta}\right)^2$$

- Magnetic trispectrum sourced by tensor exchange, inherits a richer angular dependence leads to distinctive signatures. But suppressed by tensor to scale ratio

Thank You

(haridevsr@iisc.ac.in)