Large black-hole scalar charges induced by cosmology in Horndeski theories

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arXiv:2504.07882 [gr-qc] → Phys. Rev. D

IAP, June 30th, 2025

Introduction

- No hair theorem in GR: BHs characterized by mass, spin, charge.

- Also true in standard scalar-tensor theories when $\dot{\varphi} = 0$, because of divergence at BH horizon $\Rightarrow$ $\varphi = \text{const}$.

- No longer true when $\dot{\varphi} \neq 0$ (imposed by cosmology), but small scalar charges in standard scalar-tensor theories.

- Large scalar charges in some Galileon/Horndeski theories. $\Rightarrow$ Are they consistent with observations?

Scalar-tensor theories

- Standard scalar kinetic term: $-(\partial_\mu\varphi)^2$
  $\Rightarrow$ Second order field equation: $\Box\varphi =$ source

- Decoupling limit of the **D**vali-**G**abadadze-**P**orrati brane model: cubic kinetic term $-(\partial_\mu\varphi)^2\Box\varphi$

  $$\begin{aligned} \Rightarrow \text{Field equation:} \quad 2\nabla_\mu\left(\partial^\mu\varphi\,\Box\varphi\right) - \Box\left[(\partial_\mu\varphi)^2\right] &= \text{source} \\ \Leftrightarrow \quad 2(\Box\varphi)^2 + 2\partial^\mu\varphi\partial_\mu\Box\varphi - 2\nabla^\nu\left(\partial^\mu\varphi\nabla_\nu\partial_\mu\varphi\right) &= \text{source} \\ \Leftrightarrow \quad 2(\Box\varphi)^2 + 2\partial^\mu\varphi\partial_\mu\Box\varphi - 2\left(\nabla_\mu\partial_\nu\varphi\right)^2 - 2\partial^\mu\varphi\Box\partial_\mu\varphi &= \text{source} \\ \Leftrightarrow \quad 2(\Box\varphi)^2 - 2\left(\nabla_\mu\partial_\nu\varphi\right)^2 - 2R^{\mu\nu}\partial_\mu\varphi\partial_\nu\varphi &= \text{source} \end{aligned}$$

  Nonlinear, but still second order! ($\Rightarrow$ no Ostrogradski ghost)

- Simplest explanation of this "miracle":
  $$-(\partial_\mu\varphi)^2\Box\varphi = \frac{1}{3}\varepsilon^{\mu\nu\rho\sigma}\,\varepsilon^{\alpha\beta}{}_{\rho\sigma}\,\partial_\mu\varphi\,\partial_\alpha\varphi\,\nabla_\nu\nabla_\beta\varphi + \text{tot. div.}$$

Scalar-tensor theories

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Galileons

Notation: $\varphi_\mu \equiv \partial_\mu \varphi, \qquad \varphi_{\mu\nu} \equiv \nabla_\mu \nabla_\nu \varphi$

Galileons (in 4 dimensions)

$$L_{(2,0)} \equiv \frac{1}{3!} \varepsilon^{\mu\nu\rho\sigma} \varepsilon^{\alpha}{}_{\nu\rho\sigma} \varphi_\mu \varphi_\alpha = -\varphi_\mu^2 \equiv X,$$

$$L_{(3,0)} \equiv \frac{1}{2!} \varepsilon^{\mu\nu\rho\sigma} \varepsilon^{\alpha\beta}{}_{\rho\sigma} \varphi_\mu \varphi_\alpha \varphi_{\nu\beta} \sim -\frac{3}{2} \varphi_\mu^2 \Box \varphi,$$

$$L_{(4,0)} \equiv \varepsilon^{\mu\nu\rho\sigma} \varepsilon^{\alpha\beta\gamma}{}_{\sigma} \varphi_\mu \varphi_\alpha \varphi_{\nu\beta} \varphi_{\rho\gamma},$$

$$L_{(5,0)} \equiv \varepsilon^{\mu\nu\rho\sigma} \varepsilon^{\alpha\beta\gamma\delta} \varphi_\mu \varphi_\alpha \varphi_{\nu\beta} \varphi_{\rho\gamma} \varphi_{\sigma\delta},$$

$$L_{(4,1)} \equiv \varepsilon^{\mu\nu\rho\sigma} \varepsilon^{\alpha\beta\gamma}{}_{\sigma} \varphi_\mu \varphi_\alpha R_{\nu\rho\beta\gamma} = 4\, G^{\mu\nu} \varphi_\mu \varphi_\nu,$$

$$L_{(5,1)} \equiv \varepsilon^{\mu\nu\rho\sigma} \varepsilon^{\alpha\beta\gamma\delta} \varphi_\mu \varphi_\alpha \varphi_{\nu\beta} R_{\rho\sigma\gamma\delta}.$$

[Cf. also Λ_{cosmo}, Einstein-Hilbert R, and Gauss-Bonnet $R^2_{\mu\nu\rho\sigma} - 4R^2_{\mu\nu} + R^2$]

Horndeski theories

Notation: $\varphi_\mu \equiv \partial_\mu \varphi, \qquad \varphi_{\mu\nu} \equiv \nabla_\mu \nabla_\nu \varphi, \qquad X \equiv -\varphi_\mu^2$

Horndeski theories

$$L_{(2,0)} \equiv \frac{1}{3!} f_2(\varphi, X) \varepsilon^{\mu\nu\rho\sigma} \varepsilon^{\alpha}{}_{\nu\rho\sigma} \varphi_\mu \varphi_\alpha,$$

$$L_{(3,0)} \equiv \frac{1}{2!} f_3(\varphi, X) \varepsilon^{\mu\nu\rho\sigma} \varepsilon^{\alpha\beta}{}_{\rho\sigma} \varphi_\mu \varphi_\alpha \varphi_{\nu\beta},$$

$$L_{(4,0)} \equiv f_4(\varphi, X) \varepsilon^{\mu\nu\rho\sigma} \varepsilon^{\alpha\beta\gamma}{}_{\sigma} \varphi_\mu \varphi_\alpha \varphi_{\nu\beta} \varphi_{\rho\gamma},$$

$$L_{(5,0)} \equiv f_5(\varphi, X) \varepsilon^{\mu\nu\rho\sigma} \varepsilon^{\alpha\beta\gamma\delta} \varphi_\mu \varphi_\alpha \varphi_{\nu\beta} \varphi_{\rho\gamma} \varphi_{\sigma\delta},$$

$$L_{(4,1)} \equiv s_4(\varphi, X) \varepsilon^{\mu\nu\rho\sigma} \varepsilon^{\alpha\beta\gamma}{}_{\sigma} \varphi_\mu \varphi_\alpha R_{\nu\rho\beta\gamma},$$

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$s_{4,5}$ related to $f_{3,4,5}$ in Horndeski theories, otherwise “beyond-Horndeski”

Horndeski theories

Notation: $\varphi_\mu \equiv \partial_\mu \varphi, \qquad \varphi_{\mu\nu} \equiv \nabla_\mu \nabla_\nu \varphi, \qquad X \equiv -\varphi_\mu^2$

Shift-symmetric Horndeski theories

$$L_{(2,0)} \equiv \frac{1}{3!} f_2(X)\, \varepsilon^{\mu\nu\rho\sigma} \varepsilon^{\alpha}{}_{\nu\rho\sigma}\, \varphi_\mu \varphi_\alpha,$$

$$L_{(3,0)} \equiv \frac{1}{2!} f_3(X)\, \varepsilon^{\mu\nu\rho\sigma} \varepsilon^{\alpha\beta}{}_{\rho\sigma}\, \varphi_\mu \varphi_\alpha\, \varphi_{\nu\beta},$$

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$s_{4,5}$ related to $f_{4,5}$ in Horndeski theories, otherwise "beyond-Horndeski"

Horndeski theories

Notation: $\varphi_\mu \equiv \partial_\mu \varphi$, $\varphi_{\mu\nu} \equiv \nabla_\mu \nabla_\nu \varphi$, $X \equiv -\varphi_\mu^2$

Subclass of shift-symmetric Horndeski theories

$$L_{(2,0)} \equiv \frac{1}{3!} f_2(X)\, \varepsilon^{\mu\nu\rho\sigma} \varepsilon^{\alpha}{}_{\nu\rho\sigma}\, \varphi_\mu \varphi_\alpha,$$

$$L_{(3,0)} \equiv \frac{1}{2!} f_3(X)\, \varepsilon^{\mu\nu\rho\sigma} \varepsilon^{\alpha\beta}{}_{\rho\sigma}\, \varphi_\mu \varphi_\alpha\, \varphi_{\nu\beta},$$

$$L_{(5,0)} \equiv f_5(X)\, \varepsilon^{\mu\nu\rho\sigma} \varepsilon^{\alpha\beta\gamma\delta}\, \varphi_\mu \varphi_\alpha\, \varphi_{\nu\beta} \varphi_{\rho\gamma} \varphi_{\sigma\delta},$$

$$L_{(5,1)} \equiv s_5(X)\, \varepsilon^{\mu\nu\rho\sigma} \varepsilon^{\alpha\beta\gamma\delta}\, \varphi_\mu \varphi_\alpha\, \varphi_{\nu\beta}\, R_{\rho\sigma\gamma\delta}.$$

s_5 related to f_5 in Horndeski theories, otherwise “beyond-Horndeski”

Horndeski theories

Notation: $\varphi_\mu \equiv \partial_\mu \varphi, \qquad \varphi_{\mu\nu} \equiv \nabla_\mu \nabla_\nu \varphi, \qquad X \equiv -\varphi_\mu^2$

Subclass of shift-symmetric Horndeski theories

$$L_{(2,0)} \equiv \frac{1}{3!} \; k_2 \; \varepsilon^{\mu\nu\rho\sigma} \, \varepsilon^{\alpha}{}_{\nu\rho\sigma} \, \varphi_\mu \, \varphi_\alpha = k_2 X,$$

$$L_{(3,0)} \equiv \frac{2}{3} \; \frac{k_3}{M^2} \; \varepsilon^{\mu\nu\rho\sigma} \, \varepsilon^{\alpha\beta}{}_{\rho\sigma} \, \varphi_\mu \, \varphi_\alpha \, \varphi_{\nu\beta} \sim \frac{k_3}{M^2} X \Box \varphi,$$

$$L_{(5,0)} \equiv 0 \times \varepsilon^{\mu\nu\rho\sigma} \, \varepsilon^{\alpha\beta\gamma\delta} \, \varphi_\mu \, \varphi_\alpha \, \varphi_{\nu\beta} \, \varphi_{\rho\gamma} \, \varphi_{\sigma\delta} = 0,$$

$$L_{(5,1)} \equiv -\frac{1}{6} \; \frac{k_5}{M^4} \; \varepsilon^{\mu\nu\rho\sigma} \, \varepsilon^{\alpha\beta\gamma\delta} \, \varphi_\mu \, \varphi_\alpha \, \varphi_{\nu\beta} \, R_{\rho\sigma\gamma\delta}.$$

Other writing of Horndeski theories

Equivalent to other notations used in the literature (still $X \equiv -\varphi_\mu^2$)

Shift-symmetric (beyond) Horndeski theories

$$
\begin{aligned}
L_{(2,0)} &= G_2(X), \\
L_{(3,0)} &= G_3(X)\Box\varphi + \text{tot. div.}, \\
L_{(4,0)} + L_{(4,1)} &= G_4(X)R + 2G_4'(X)\left[(\Box\varphi)^2 - \varphi_{\mu\nu}\varphi^{\mu\nu}\right] \\
&\quad + F_4(X)\varepsilon^{\mu\nu\rho\sigma}\,\varepsilon^{\alpha\beta\gamma}{}_{\sigma}\,\varphi_\mu\,\varphi_\alpha\,\varphi_{\nu\beta}\,\varphi_{\rho\gamma} + \text{tot. div.}, \\
L_{(5,0)} + L_{(5,1)} &= G_5(X)G^{\mu\nu}\varphi_{\mu\nu} \\
&\quad -\frac{1}{3}G_5'(X)\left[(\Box\varphi)^3 - 3\,\Box\varphi\,\varphi_{\mu\nu}\varphi^{\mu\nu} + 2\,\varphi_{\mu\nu}\varphi^{\nu\rho}\varphi_\rho{}^\mu\right] \\
&\quad +F_5(X)\varepsilon^{\mu\nu\rho\sigma}\,\varepsilon^{\alpha\beta\gamma\delta}\,\varphi_\mu\,\varphi_\alpha\,\varphi_{\nu\beta}\,\varphi_{\rho\gamma}\,\varphi_{\sigma\delta} + \text{tot. div.}
\end{aligned}
$$

In this work: $G_2(X) = k_2 X$, $\quad G_3(X) = \dfrac{k_3}{M^2}X$, $\quad G_5(X) = \dfrac{k_5}{M^4}X$.

Considered subclass of Horndeski theories

Notation: $\varphi_\mu \equiv \partial_\mu \varphi$, $\varphi_{\mu\nu} \equiv \nabla_\mu \nabla_\nu \varphi$, $X \equiv -\varphi_\mu^2$

Subclass of shift-symmetric Horndeski theories

$$S = M_{\rm Pl}^2 \int \sqrt{-g}\, d^4x \Big\{ \frac{R}{2} - \Lambda_{\rm bare} + k_2\, X + \frac{k_3}{M^2} X \Box\varphi + \frac{k_5}{M^4} X\, G^{\mu\nu} \varphi_{\mu\nu} - \frac{1}{3}\frac{k_5}{M^4}\Big[(\Box\varphi)^3 - 3\,\Box\varphi\,\varphi_{\mu\nu}\varphi^{\mu\nu} + 2\,\varphi_{\mu\nu}\varphi^{\nu\rho}\varphi_\rho{}^\mu\Big]\Big\} + S_{\rm matter}\big[\text{matter fields}, e^{2\alpha\varphi} g_{\mu\nu}\big]$$

Shift symmetry if $\alpha = 0 \Rightarrow \exists$ conserved current: $J^\mu \equiv -\frac{1}{\sqrt{-g}} \frac{\delta S}{\delta \partial_\mu \varphi}$

Scalar field equation: $\nabla_\mu J^\mu = 0 \quad [\text{or } -\alpha T_{\rm matter}]$

Considered subclass of Horndeski theories

FLRW metric $ds^2 = -d\tau^2 + a(\tau)^2 \left(d\rho^2 + \rho^2 d\Omega^2\right)$, $\quad H \equiv \dot{a}/a$

Cosmological field equations

$$3H^2 = \frac{\varepsilon}{M_{\text{Pl}}^2} + \Lambda_{\text{bare}} + k_2\dot{\varphi}^2 - 6H\,\frac{k_3}{M^2}\,\dot{\varphi}^3 + \mathcal{O}(k_5),$$

$$-\dot{H} = \frac{\varepsilon + p}{2M_{\text{Pl}}^2} + k_2\dot{\varphi}^2 - 3H\,\frac{k_3}{M^2}\,\dot{\varphi}^3 + \mathcal{O}(k_5),$$

$\alpha_T \equiv \left(\frac{c_{\text{grav}}}{c}\right)^2 - 1 \approx 2H\frac{k_5}{M^4}\dot{\varphi}^3$, $\quad$ GW detection: $|\alpha_T| < 10^{-15}$

Considered subclass of Horndeski theories

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$$-\dot{H} = \frac{\varepsilon + p}{2M_{\rm Pl}^2} + k_2\dot{\varphi}^2 - 3H\,\frac{k_3}{M^2}\,\dot{\varphi}^3 + \mathcal{O}(k_5),$$

$$\nabla_\mu J^\mu = 0 \quad \Rightarrow \quad \frac{\partial_\tau\left(a^3 J^0\right)}{a^3} = 0 \quad [\text{or } \alpha(\varepsilon - 3p)]$$

$$\text{with} \quad \frac{J^0}{M_{\rm Pl}^2} = -2k_2\dot{\varphi} + \frac{6H}{M^2}\left[k_3 - \left(\frac{H}{M}\right)^2 k_5\right]\dot{\varphi}^2.$$

Large a at late times $\Rightarrow J^0 \to 0$, and $\dot{\varphi} \neq 0$ if $k_2 < 0$

$\Rightarrow \dot{\varphi}_{\rm cosmo} = \frac{k_2 M^2}{3H} / \left[k_3 - \left(\frac{H}{M}\right)^2 k_5\right]$

Considered subclass of Horndeski theories

$$\text{Schwarzschild-de Sitter metric} \quad ds^2 = -f(r)dt^2 + \frac{dr^2}{f(r)} + r^2 d\Omega^2$$

$$\text{with} \quad f(r) = 1 - \frac{r_S}{r} - (Hr)^2 .$$

$$\text{Assume} \quad \varphi(t,r) = \dot{\varphi}_{\text{BH}}\, t + \phi(r).$$

Test scalar field equation near black hole

$$\nabla_\mu J^\mu = 0 \quad \Rightarrow \quad \partial_r \left(r^2 J^r\right) = 0 \quad \Rightarrow \quad J^r = \frac{\text{const}}{r^2}$$

$$\text{with} \quad \frac{J^r}{M_{\text{Pl}}^2} = A\varphi'^2 + B\varphi' + C,$$

$$A \equiv \frac{f}{M^2}\left[\left(\frac{4f}{r} + f'\right)k_3 + \frac{3f-1}{(Mr)^2} f' k_5\right],$$

$$B \equiv 2 f k_2,$$

$$C \equiv -\left[k_3 + \frac{f-1}{(Mr)^2}k_5\right]\frac{f' \dot{\varphi}_{\text{BH}}{}^2}{f M^2}.$$

Considered subclass of Horndeski theories

Test scalar field equation near black hole

$$
\begin{aligned}
A\varphi'^2 + B\varphi' + C &= \frac{\alpha_{\text{BH}} r_S}{r^2} \quad \text{(notation)},\\
\text{therefore} \quad \varphi' &= \frac{-B \pm \sqrt{\Delta}}{2A},\\
\text{with} \quad \Delta &\equiv B^2 - 4A\left(C - \frac{\alpha_{\text{BH}} r_S}{r^2}\right).
\end{aligned}
$$

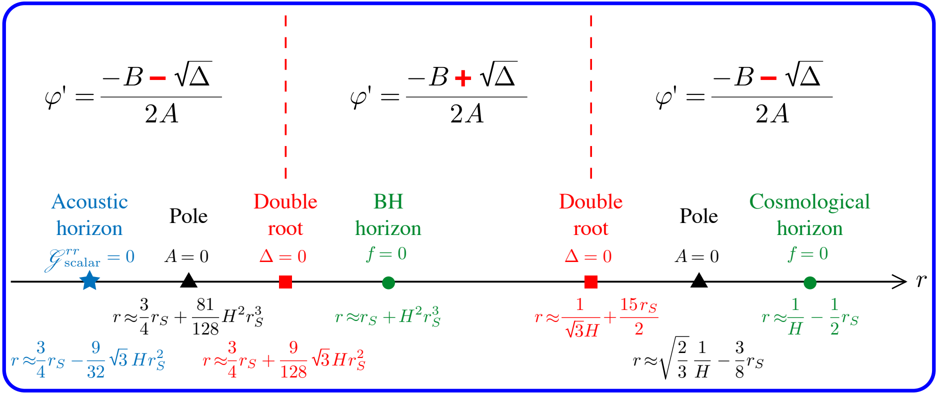
Three results to keep in mind:

- $\dot{\varphi} \neq 0$ imposed by cosmology
- $\dot{\varphi}^2$ is a *source* for the BH scalar hair
- $\Delta \geq 0$ necessary for real φ' solution
  $\Rightarrow$ any zero of Δ must be *double*

$G_2 + G_3$: Sign of $\pm\sqrt{\Delta}$

Simple model but allows self-acceleration ($\Lambda_{\text{effective}} \neq 0$ with $\Lambda_{\text{bare}} = 0$) or self-tuning ($\Lambda_{\text{effective}} \ll \Lambda_{\text{bare}} \sim M^2_{\text{Planck}}$)

- If no BH, homogenenous $\varphi_{\text{cosmo}} = \dot\varphi_c \tau \quad \Rightarrow \quad \varphi' = -\dot\varphi_c \dfrac{Hr}{1-(Hr)^2}$
- If $A \to 0$, then $\varphi' = \dfrac{|B| \pm |B|}{2A} \mp \dfrac{C}{|B|} + \mathcal{O}(A)$



$G_2 + G_3$: Solving for the double roots of Δ

Each double root of Δ imposes a relation between $\dot\varphi_{\rm BH}$ and $\alpha_{\rm BH}$
$\Rightarrow$ Two double roots fix both of them!

Regularity of φ' solution

$$\begin{aligned}
\dot\varphi_{\rm BH} &= \dot\varphi_{\rm cosmo} \times \left[1 + \frac{3}{2}\sqrt{3}\, H r_S + \mathcal{O}\left(H^2 {r_S}^2\right)\right],\\
\alpha_{\rm BH} &= 3k_3 \left(\frac{\dot\varphi_{\rm BH}}{M}\right)^2 \left[1 + \mathcal{O}\left(H r_S\right)\right]\\
&= \frac{1}{3k_3}\left(\frac{k_2 M}{H}\right)^2 \left[1 + \mathcal{O}\left(H r_S\right)\right].
\end{aligned}$$

- Consistent $\dot\varphi_{\rm BH} \approx \dot\varphi_{\rm cosmo}$
- $\alpha_{\rm BH} = \mathcal{O}(1) \Rightarrow$ a priori large deviations from general relativity!

$G_2 + G_3$: Gravitational-wave emission

GW energy flux

$$F_{\rm GR} \approx \frac{2}{5G}\left(\frac{r_S}{r_{AB}}\right)^5, \qquad [\text{with } r_{AB} = \text{interbody distance}]$$

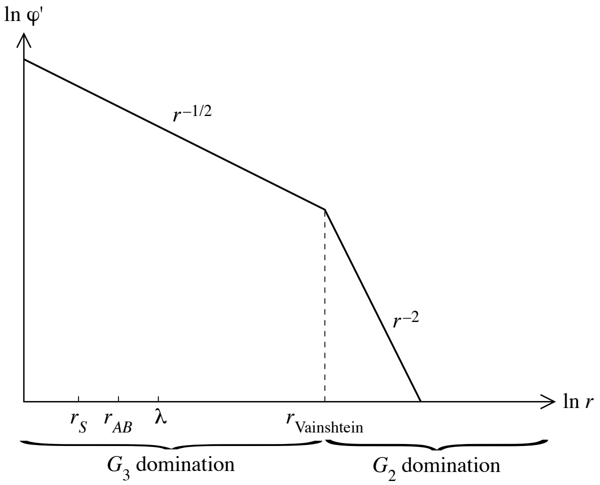
$$F_{\rm scalar} = \left(F^{\rm dipole}_{\rm scalar} + F^{\rm quadrupole}_{\rm scalar}\right) \times \big(\text{Vainshtein screening factor}\big),$$

$$F^{\rm dipole}_{\rm scalar} \approx \frac{1}{48G|k_2|}\left(\frac{r_S}{r_{AB}}\right)^4 (\alpha_A - \alpha_B)^2,$$

$$F^{\rm quadrupole}_{\rm scalar} \approx \frac{1}{15G|k_2|}\left(\frac{r_S}{r_{AB}}\right)^5 \alpha_A\alpha_B.$$

Here, $\alpha_{\rm BH} \approx \frac{1}{3k_3}\left(\frac{k_2 M}{H}\right)^2$ for all BHs $\Rightarrow$ no dipole

$G_2 + G_3$: Vainshtein screening



$$r^3_{\text{Vainshtein}} = \frac{|k_3 \alpha_{\text{BH}}| r_S}{k_2^2 M^2}, \quad \text{screening factor} \left(\frac{\lambda}{r_{\text{Vainshtein}}}\right)^{3/2}$$

$G_2 + G_3$: LIGO/Virgo/LISA?

Largest effects when self-acceleration ($M^4 = 3^3 {k_3}^2 H^4 / |k_2|^3$)

GW energy flux

$$\frac{F^{\text{scalar}}}{F^{\text{GR}}} \approx \frac{2^5}{3\sqrt{3}} \frac{H}{\Omega_{\text{p}}} \frac{1}{(\Omega_{\text{p}} r_S)^{5/6}}$$

$$\Rightarrow \frac{F^{\text{scalar}}}{F^{\text{GR}}} \approx 6 \times 10^{-18} \text{ for LIGO/Virgo}$$

$$\frac{F^{\text{scalar}}}{F^{\text{GR}}} \approx 4 \times 10^{-11} \text{ for LISA}$$

In spite of $\mathcal{O}(1)$ scalar charge α_{BH}, all experimental tests are passed, thanks to Vainshtein screening.

$G_2 + G_5$

Not possible to have self-acceleration in this model:

$$|\alpha_T| = \left|\left(\frac{c_{\text{grav}}}{c}\right)^2 - 1\right| < 10^{-15} \Rightarrow \frac{M}{H} \lesssim 2 \times 10^{-2}$$

Regularity of φ' solution

$$\begin{aligned}
\dot\varphi_{\text{BH}} &\approx \dot\varphi_{\text{cosmo}},\\
\alpha_{\text{BH}} &= 2k_5 \left(\frac{2\,\dot\varphi_{\text{BH}}}{3M^2 r_S}\right)^2 \left[1 + \mathcal{O}\left(H^2 r_S^2\right)\right]\\
&= \frac{2}{k_5}\left(\frac{2k_2 M^2}{9H^3 r_S}\right)^2 \left[1 + \mathcal{O}\left(H^2 r_S^2\right)\right],\\
r_{\text{double root}} &= \frac{3}{2} r_S \left[1 + \mathcal{O}\left(H^2 r_S^2\right)\right].
\end{aligned}$$

$\Rightarrow$ Huge scalar charge $\alpha_{\text{BH}} \propto 1/(H r_S)^2$
in spite of negligible influence of φ for cosmological expansion

$G_2 + G_5$: Accretion

Stress-energy tensor of φ: $T^r{}_t = -J^r \dot\varphi_{\rm BH} = -M_{\rm Pl}^2 \frac{\alpha_{\rm BH} r_S}{r^2} \dot\varphi_{\rm BH}$

$\Rightarrow$ BH mass changes in characteristic time $\frac{1}{|\dot\varphi_{\rm BH}\, \alpha_{\rm BH}|} \propto \frac{1}{|\dot\varphi^3_{\rm BH}|}$

$\Rightarrow$ BH accretes local $\dot\varphi_{\rm BH}$ until characteristic time $>$ BH's age

(N.B.: This depends on theory parameter M and on BH's mass r_S)

After scalar accretion

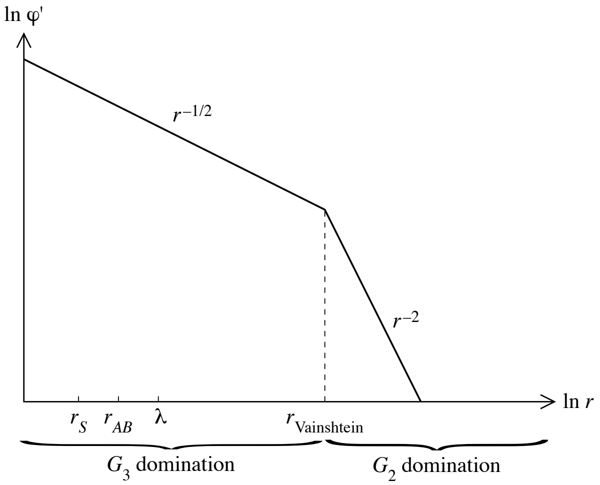
$$|\dot\varphi_{\rm BH}| \gtrsim \left(\frac{9 H M^4 {r_S}^2}{8|k_5|}\right)^{1/3},$$

$$|\alpha_{\rm BH}| \gtrsim 2\left(\frac{|k_5| H^2}{9 M^4 {r_S}^2}\right)^{1/3}.$$

$\Rightarrow$ Still large scalar charge $\alpha_{\rm BH} \propto (H r_S)^{-2/3}$

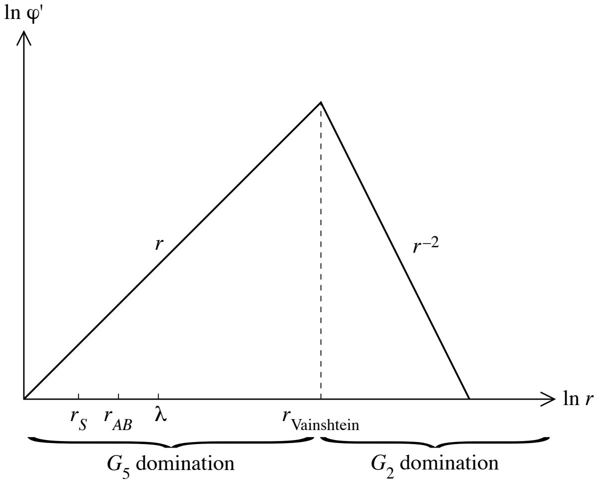
Depends on $r_S \Rightarrow$ dipolar radiation $\propto (\alpha_A - \alpha_B)^2$

$G_2 + G_3$: Vainshtein screening



$$r^3_{\text{Vainshtein}} = \frac{|k_3 \alpha_{\text{BH}}| r_S}{k_2^2 M^2}, \quad \text{screening factor} \left(\frac{\lambda}{r_{\text{Vainshtein}}}\right)^{3/2}$$

$G_2 + G_5$: Vainshtein screening



$$r^3_{\text{Vainshtein}} = \frac{\sqrt{|k_5 \alpha_{\text{BH}}|}\, r_S}{\sqrt{2}\,|k_2| M^2}, \qquad \text{screening factor } \left(\frac{\lambda}{r_{\text{Vainshtein}}}\right)^3$$

$G_2 + G_5$: LIGO/Virgo/LISA

Large scalar accretion if $\frac{M}{H} \gtrsim \left(\frac{3^5 {k_5}^2}{2^3 |k_2|^3} H^2 r_S^2\right)^{1/8}$

GW energy flux

$$\frac{F^{\text{scalar}}}{F^{\text{GR}}} \gtrsim \frac{5H}{72\Omega_{\text{p}}} \frac{1}{(\Omega_{\text{p}} r_S)^{8/3}}$$

$$\Rightarrow \frac{F^{\text{scalar}}}{F^{\text{GR}}} \gtrsim 4 \times 10^{-16} \text{ for LIGO/Virgo}$$

$$\frac{F^{\text{scalar}}}{F^{\text{GR}}} \gtrsim 10^{-6} \text{ for LISA} > \text{ expected bounds}$$

- LIGO/Virgo tests passed although large $\alpha_{\text{BH}} \propto (H r_S)^{-2/3}$.
- LISA should constrain $\frac{M}{H} < 5 \times 10^{-5} \Leftrightarrow |\alpha_T| < 3 \times 10^{-36}$, 10^{-21} tighter than GW speed bound!

$G_2 + G_3 +$ small G_5

Assume k_2 and k_3 of $\mathcal{O}(1)$ but k_5 small:

$$(Hr_S)^2 \ll \left|\frac{k_5}{k_3}\right| \left(\frac{H}{M}\right)^2 \ll Hr_S$$

- k_2 and k_3 dominate at large distances $\Rightarrow$ self-acceleration
- k_5 dominates close to BH $\Rightarrow$ large deviations from GR

Regularity of φ' solution for small accretion

$$\begin{aligned}
\dot\varphi_{\text{BH}} &\approx \dot\varphi_{\text{cosmo}},\\
\alpha_{\text{BH}} &\approx 2k_5\left(\frac{2\,\dot\varphi_{\text{BH}}}{3M^2 r_S}\right)^2 \approx 8k_5\left(\frac{k_2}{9k_3 H r_S}\right)^2,\\
r_{\text{double root}} &\approx \frac{3}{2} r_S.
\end{aligned}$$

$\Rightarrow$ Large scalar charge $\alpha_{\text{BH}} \propto k_5/(Hr_S)^2$

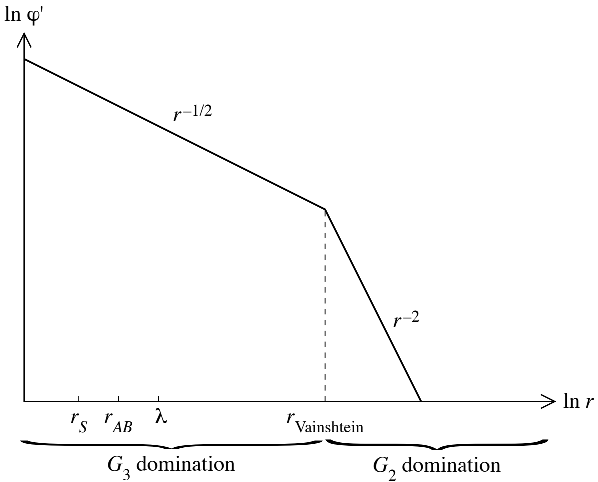
$G_2 + G_3 +$ small G_5: Accretion

When accretion is large:

- BHs decouple from cosmological background produced by $G_2 + G_3$
- Their final state is generated by G_5, which dominates locally

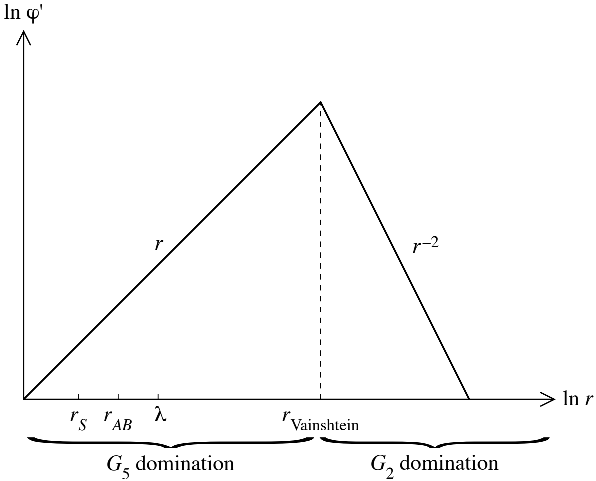
$\Rightarrow$ After accretion, same results as $G_2 + G_5$ model

$G_2 + G_3$: Vainshtein screening



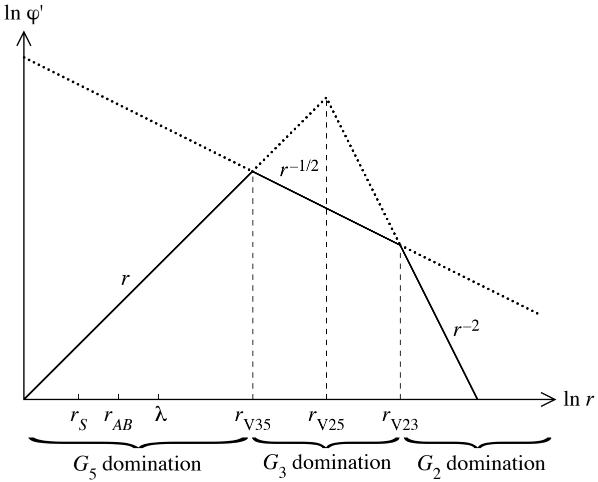
$$r^3_{\text{Vainshtein}} = \frac{|k_3 \alpha_{\text{BH}}| r_S}{k_2^2 M^2}, \quad \text{screening factor} \left(\frac{\lambda}{r_{\text{Vainshtein}}}\right)^{3/2}$$

$G_2 + G_5$: Vainshtein screening



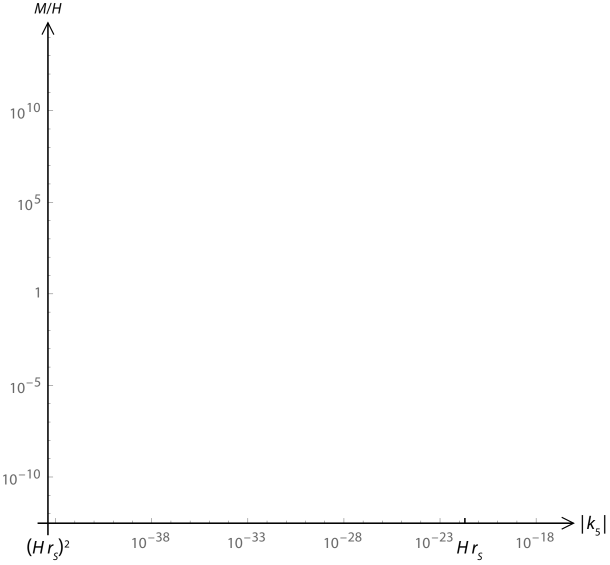
$$r^3_{\text{Vainshtein}} = \frac{\sqrt{|k_5 \alpha_{\text{BH}}|}\, r_S}{\sqrt{2}\, |k_2| M^2}, \qquad \text{screening factor} \left(\frac{\lambda}{r_{\text{Vainshtein}}}\right)^3$$

$G_2 + G_3 +$ small G_5: Vainshtein screening

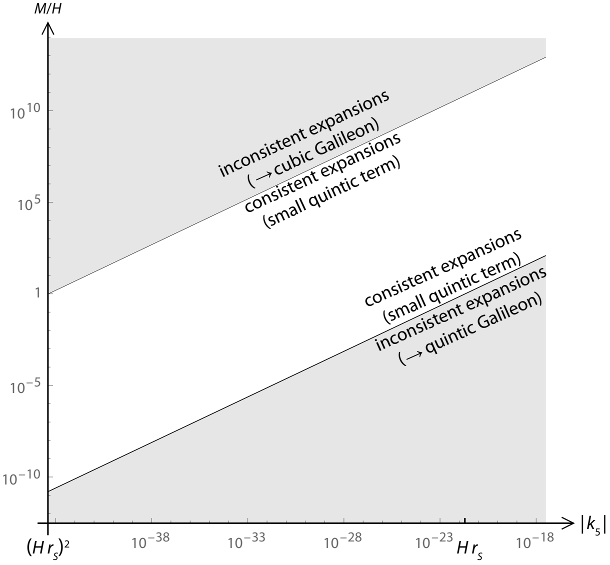


$$r_{V35}^3 = \frac{|k_5|\, r_S}{2|k_3|M^2}, \quad r_{V25}^3 = \frac{\sqrt{|k_5 \alpha_{\rm BH}|}\, r_S}{\sqrt{2}\,|k_2|M^2}, \quad r_{V23}^3 = \frac{|k_3 \alpha_{\rm BH}|\, r_S}{k_2^2 M^2}$$

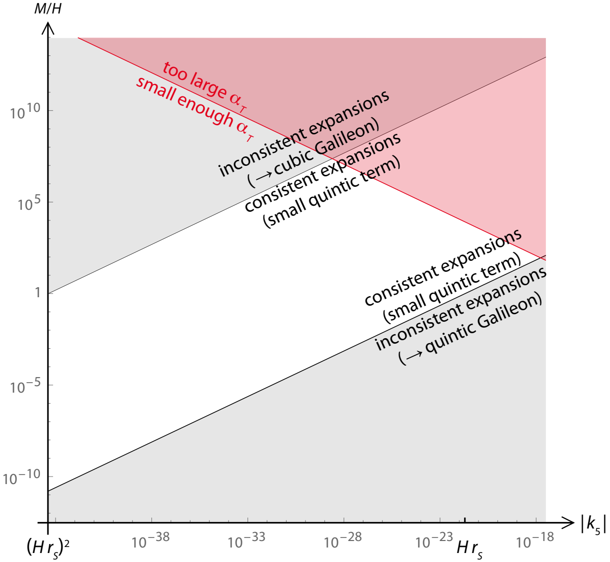
$G_2 + G_3+$ small G_5: LIGO/Virgo constraints



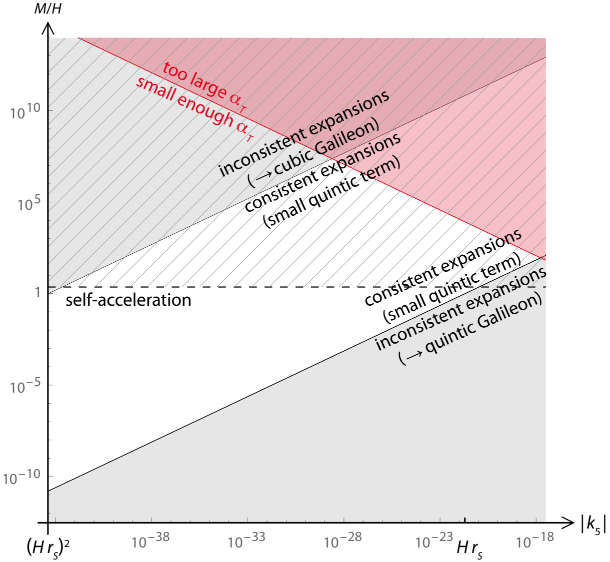
$G_2 + G_3+$ small G_5: LIGO/Virgo constraints



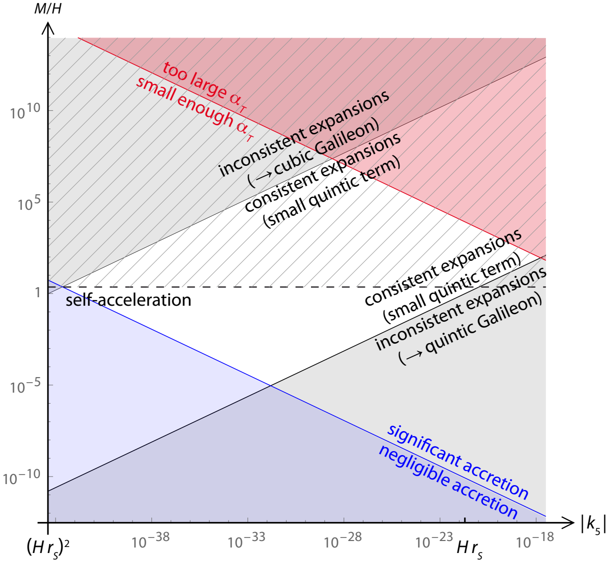
$G_2 + G_3 +$ small G_5: LIGO/Virgo constraints



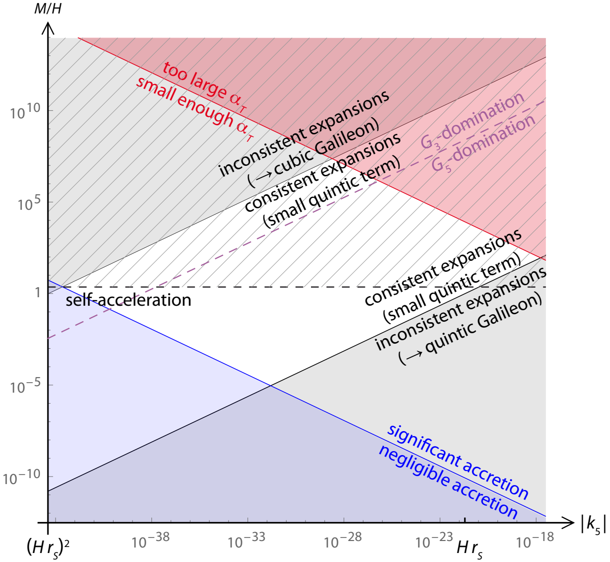
$G_2 + G_3 +$ small G_5: LIGO/Virgo constraints



$G_2 + G_3+$ small G_5: LIGO/Virgo constraints



$G_2 + G_3 +$ small G_5: LIGO/Virgo constraints



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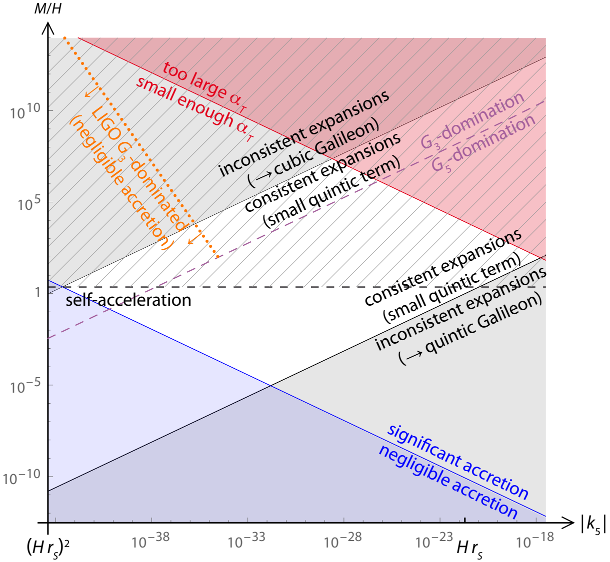
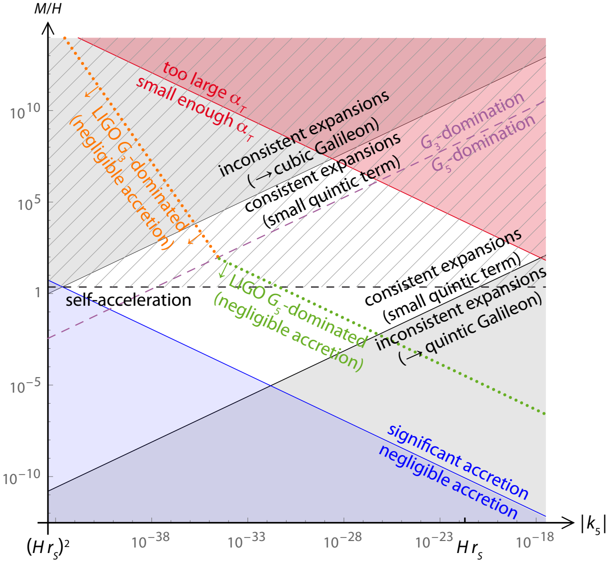
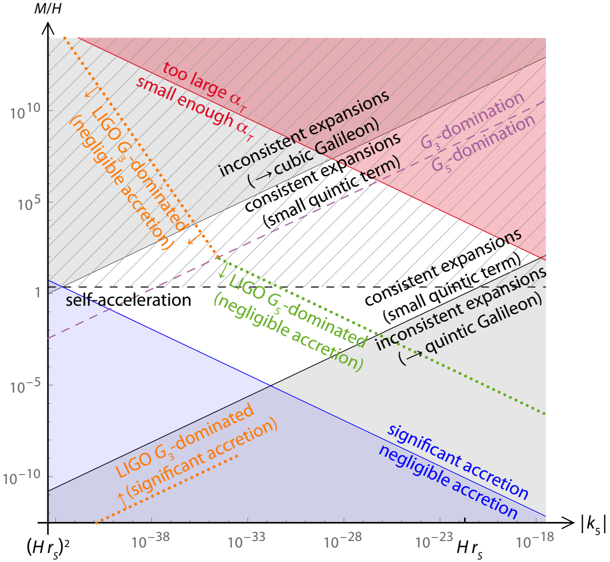


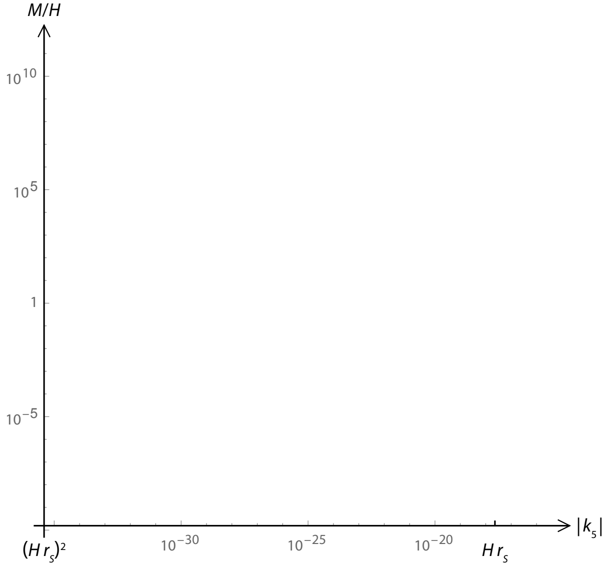
$G_2 + G_3 +$ small G_5: LIGO/Virgo constraints



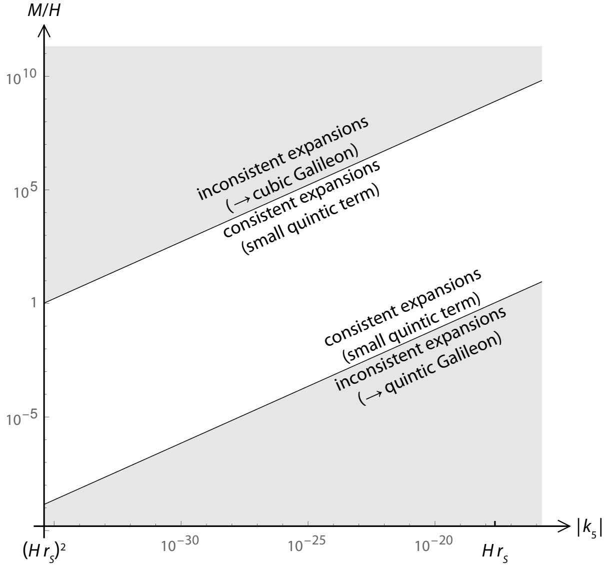
$G_2 + G_3 +$ small G_5: LIGO/Virgo constraints



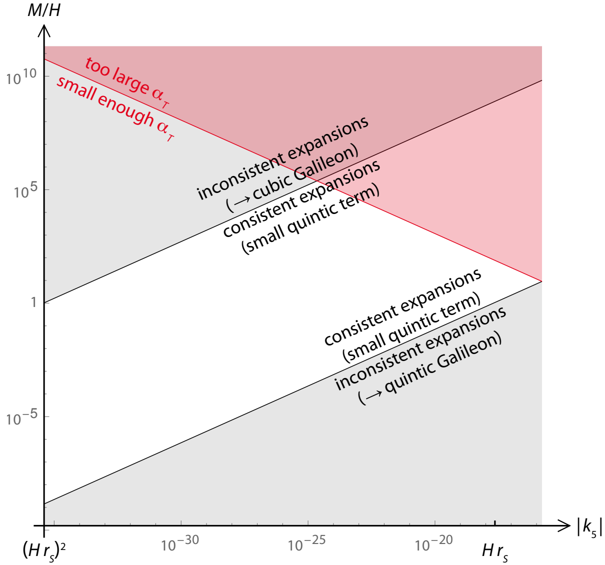
$G_2 + G_3+$ small G_5: LISA constraints



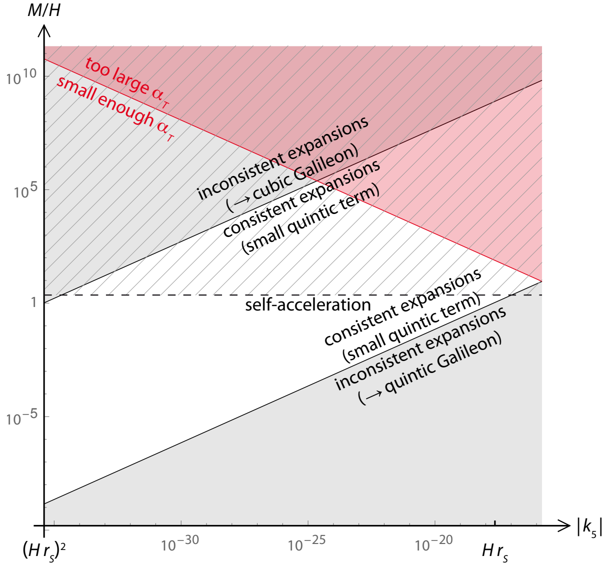
$G_2 + G_3 +$ small G_5: LISA constraints



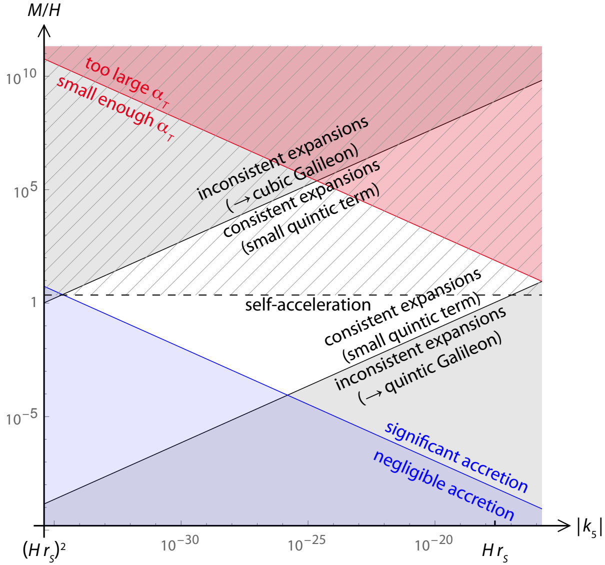
$G_2 + G_3+$ small G_5: LISA constraints



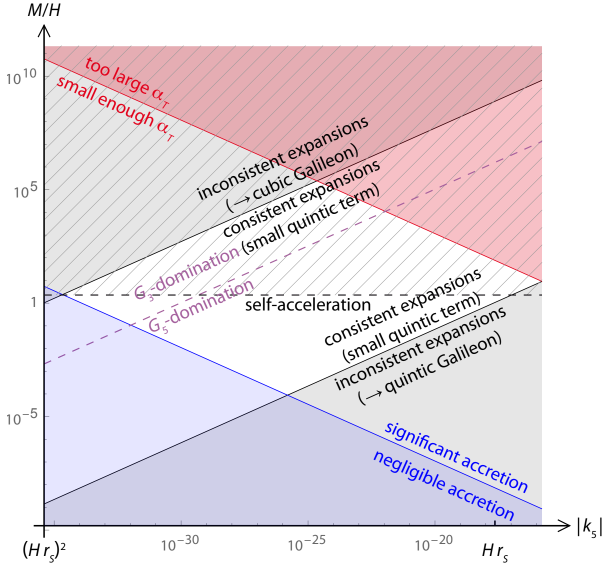
$G_2 + G_3 +$ small G_5: LISA constraints



$G_2 + G_3 +$ small G_5: LISA constraints



$G_2 + G_3 +$ small G_5: LISA constraints



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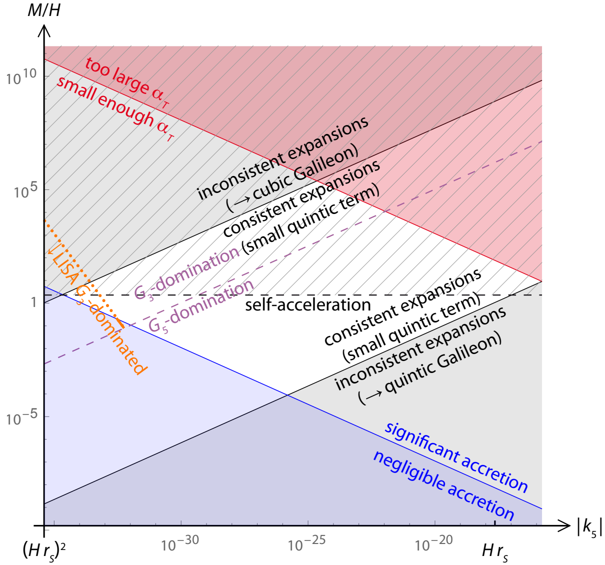
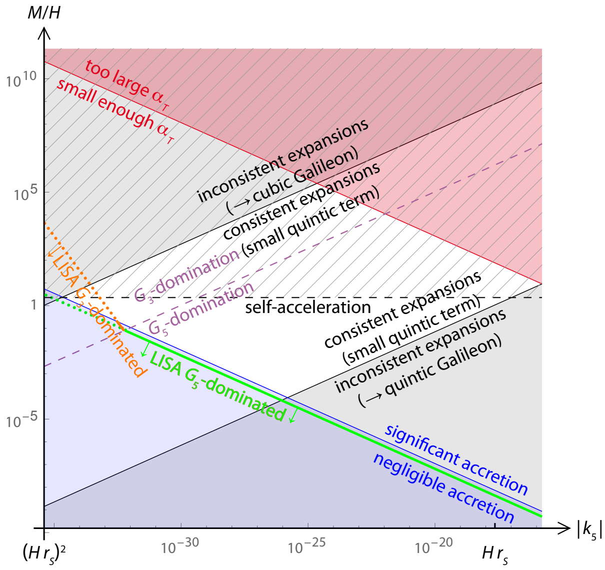
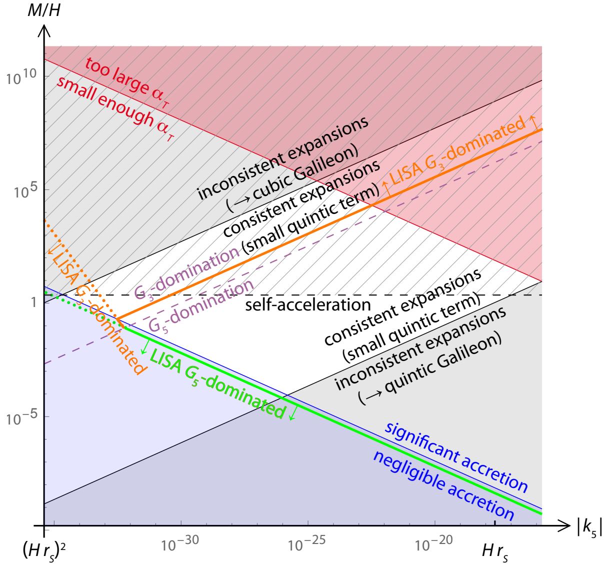


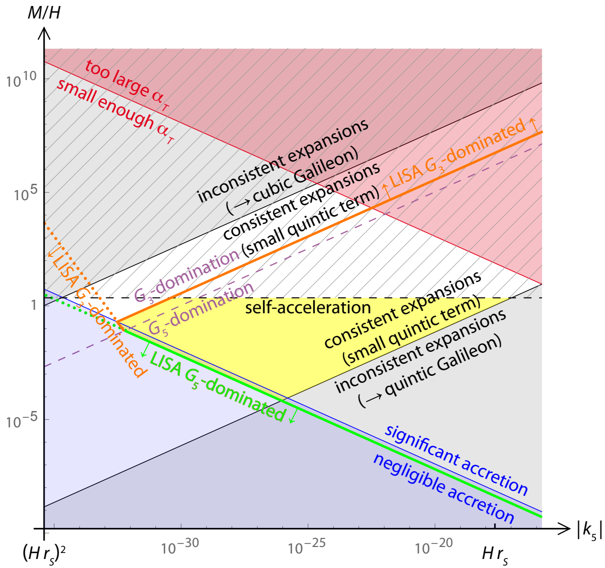
$G_2 + G_3 +$ small G_5: LISA constraints



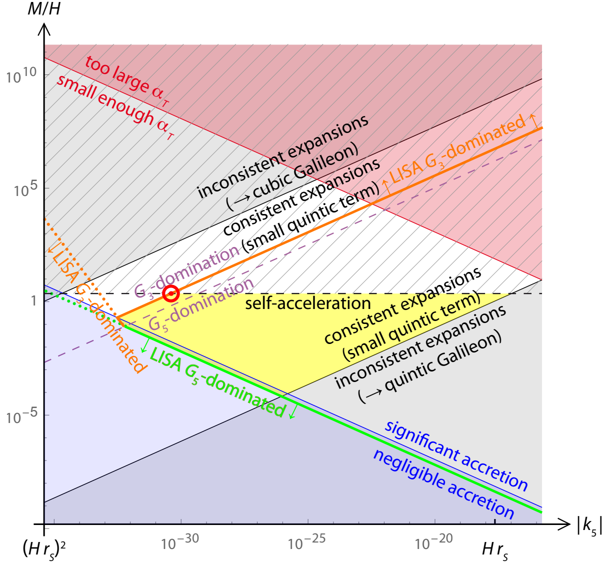
$G_2 + G_3 +$ small G_5: LISA constraints



$G_2 + G_3 +$ small G_5: LISA constraints



$G_2 + G_3 +$ small G_5: LISA constraints



Conclusions

- Very interesting predictions of Horndeski theories:
  Generic self-tuning ($\Lambda_{\text{effective}} \ll \Lambda_{\text{bare}} \sim M^2_{\text{Planck}}$)
  or self-acceleration ($\Lambda_{\text{effective}} \neq 0$ with $\Lambda_{\text{bare}} = 0$),
  Vainshtein screening in the solar system, no ghosts.
- Accelerated expansion of Universe $\Rightarrow$ large BH scalar charges in models containing G_3 and/or G_5 ($\neq$ general relativity!)
- Cubic Galileon model $G_2 + G_3$ predicts $\mathcal{O}(1)$ scalar charges, but consistent with GW data thanks to Vainshtein screening.
- Quintic model $G_2 + G_5$: huge scalar charges $\propto 1/(Hr_S)^2 \Rightarrow$ strong scalar accretion. After this, scalar charge $\propto (Hr_S)^{-2/3}$.
  LISA should improve GW speed constraint (on α_T) by 10^{-21}.
- Full model $G_2 + G_3 +$ small G_5: BH physics dominated by $G_2 + G_5$, cosmology by $G_2 + G_3$ (self-acceleration possible).
  LISA should improve GW speed constraint by 10^{-16}.
- **To be further studied**: stability, radial vs orthoradial sound velocities, precise scalar radiation when $\dot{\varphi} \neq 0$, time evolution of accretion?