Primordial black holes from inflation and their observational imprints

Rajeev Kumar Jain

Theoretical Cosmology group Department of Physics, IISc Bangalore

Based on: JCAP'20, PRD'21, JHEP'23, PRD'23, JHEP'24

Institut d'Astrophysique de Paris 10.06.2025



भारतीय विज्ञान संस्थान

Outline of the talk

- Why Primordial Black Holes (PBH) ?
- PBH generation mechanisms single field inflation inflection point models — PBH mass fraction
- Interesting observational imprints of PBHs
 - Induced secondary GWs from scalar perturbations
 - Ultralight PBHs and imprints from Hawking evaporation
 - Induced GWs and PTA results
 - Imprints of memory burden effect of PBHs
- Conclusions

Standard model of Cosmology

- The concordance model Radiation + Baryons + Dark matter + Dark energy
- Statistically homogeneous and isotropic on very large scales







Dark matter — observational evidence



Rajeev Kumar Jain

IAP Paris 2025

Why Primordial Black Holes (PBH) ?

- A novel and promising candidate for the cold dark matter
- Abundant production in the early universe e.g. after inflation
- Non-baryonic, non-relativistic and nearly collisionless
- No new physics (beyond inflation) required !
- Interesting observational signatures e.g. secondary GWs, Hawking evaporation etc.



PBH as DM — Current constraints



PBH as DM — Current constraints



Primordial black holes and their gravitational-wave signatures LISA Cosmology Working Group, Bagui, ... RKJ... et al., Living Reviews in Relativity, 28 (2025) 1, 1 [arXiv:2310.19857]

Rajeev Kumar Jain

PBH formation from inflation — in a nutshell



Fig. credit: G. Franciolini

PBH formation from inflation — in a nutshell



PBH formation from collapse of overdense fluctuations during radiation domination

Fig. credit: Front. Astron. Space Sci., 2021

PBH formation from inflation — in a nutshell



PBH from inflation — inflection point models



IAP Paris 2025

A single field inflection point scenario

$$V(x) = V_0 \frac{ax^2 + bx^4 + cx^6}{(1 + dx^2)^2}, \quad x = \phi/v$$

$$x \gg 1: V(x) \simeq \frac{V_0 c}{d^2} x^2$$
$$x \ll 1: V(x) \simeq V_0 a x^2$$

Quadratic for both large & small field values

$$r \sim 0.05$$



Rajeev Kumar Jain

IAP Paris 2025

Slow roll, ultra slow roll and all that...

Background evolution

$$H^{2} = \frac{V(\phi)}{M_{\text{Pl}}^{2}(3-\epsilon)},$$

$$\frac{d^{2}\phi}{dN^{2}} + (3-\epsilon)\frac{d\phi}{dN} + \frac{1}{H^{2}}V'(\phi) = 0,$$

$$SR: \quad \frac{d\phi}{dN} + \frac{1}{3H^2}V'(\phi) \simeq 0,$$
$$USR: \quad \frac{d^2\phi}{dN^2} + 3\frac{d\phi}{dN} \simeq 0,$$
$$\epsilon \sim \exp\left[-6(N - N_i)\right]$$
$$\eta \simeq \epsilon + (3 - \epsilon) \sim 3$$

 J_3

Curvature perturbations

$$\mathcal{R}_{k}^{\prime\prime} + 2\left(\frac{z^{\prime}}{z}\right)\mathcal{R}_{k}^{\prime} + k^{2}\mathcal{R}_{k} = 0.$$

$$P_{\mathcal{R}} = \frac{\kappa}{2\pi^{2}}|\mathcal{R}_{k}|^{2}$$

$$\frac{z^{\prime}}{z} = aH(1 + \epsilon - \eta)$$

$$\mathcal{R}_{k}(\tau) \simeq C_{1} + C_{2}\int \frac{d\tau}{z^{2}}$$

$$P_{\mathcal{R}} \sim \frac{H^{2}}{\epsilon}$$

Slow roll, ultra slow roll and all that...



Rajeev Kumar Jain

Primordial power spectra



Rajeev Kumar Jain

IAP Paris 2025

PBH mass fraction: Press-Schechter formalism

$$\delta(k,t) \simeq \frac{2(w+1)}{(3w+5)} \left(\frac{k}{aH}\right)^2 \mathcal{R}_k.$$

$$\sigma_{\delta}^2(R) \simeq \frac{16}{81} \int \frac{dk}{k} (kR)^4 P_{\mathcal{R}}(k) W^2(k,R).$$

$$\beta_f(M) \simeq \sqrt{\frac{1}{2\pi}} \frac{\sigma_\delta(M(R))}{\delta_c} \exp\left(-\frac{\delta_c^2}{2\sigma_\delta^2(M(R))}\right)$$

$$\beta(M) \equiv \frac{\rho_{\rm PBH}(M)}{\rho_{\rm tot}}.$$

$$\beta_{\rm eq}(M) \simeq \beta_f(M) \left(\frac{a_{\rm eq}}{a_f}\right)$$

$$f_{\rm PBH}(M) \equiv \frac{\Omega_{\rm PBH}(M)}{\Omega_{\rm DM}} = \frac{\beta(M)}{8 \times 10^{-16}} \left(\frac{\gamma}{0.2}\right)^{3/2} \left(\frac{g_*}{106.75}\right)^{-1/4} \left(\frac{M}{10^{18}\,\rm g}\right)^{-1/2}$$

Rajeev Kumar Jain



PBH mass fraction: Press-Schechter formalism



Non-gaussianities play very important role in PBH formation !

Rajeev Kumar Jain

PBH mass fraction



Effects of reheating — remapping of scales



Rajeev Kumar Jain

IAP Paris 2025

PBH observational imprints: Induced secondary GWs

GWs across the universe

THE GRAVITATIONAL WAVE SPECTRUM

The length of an observatory's 'baseline' affects its sensitivity to the gravitational wave spectrum. Ground-based observatories, such as LIGO, have a relatively short baseline and thus detect short wavelength events. Pulsar timing arrays have the longest 'baseline' and so are sensitive to longer wavelengths.



GWs are novel probes of different epochs of the universe

Rajeev Kumar Jain

Tensor modes sourced by scalar perturbations

$$h_{\mathbf{k}}''(\tau) + 2\mathcal{H}h_{\mathbf{k}}'(\tau) + k^2h_{\mathbf{k}}(\tau) = 4S_{\mathbf{k}}(\tau),$$

$$S_{\mathbf{k}} = \int \frac{\mathrm{d}^3 q}{(2\pi)^{3/2}} e_{ij}^{\lambda}(\mathbf{k}) q_i q_j \left[2\Phi_{\mathbf{q}} \Phi_{\mathbf{k}-\mathbf{q}} + \left(\mathcal{H}^{-1} \Phi_{\mathbf{q}}' + \Phi_{\mathbf{q}}\right) \left(\mathcal{H}^{-1} \Phi_{\mathbf{k}-\mathbf{q}}' + \Phi_{\mathbf{k}-\mathbf{q}}\right) \right].$$

$$\Phi_{\mathbf{k}}(\tau) = \frac{2}{3} \mathcal{T}(k\tau) \mathcal{R}(\mathbf{k}). \qquad \mathcal{T}(k\tau) = \frac{9}{(k\tau)^2} \left[\frac{\sqrt{3}}{k\tau} \sin\left(\frac{k\tau}{\sqrt{3}}\right) - \cos\left(\frac{k\tau}{\sqrt{3}}\right) \right].$$

$$\frac{k^3}{2\pi^2} \left\langle h_{\mathbf{k}}^{\lambda}(\tau) h_{\mathbf{k}'}^{\lambda'}(\tau) \right\rangle = \delta_{\lambda\lambda'} \delta^3(\mathbf{k} + \mathbf{k}') \mathcal{P}_h(\tau, k),$$

$$\Omega_{\rm GW}(\tau,k) \equiv \frac{1}{\rho_c} \frac{\mathrm{d}\,\rho_{\rm GW}}{\mathrm{d}\,\mathrm{ln}k} = \frac{\rho_{\rm GW}(\tau,k)}{\rho_{\rm tot}(\tau)} = \frac{1}{24} \left(\frac{k}{\mathcal{H}}\right)^2 \overline{\mathcal{P}_h(\tau,k)},$$

$$\Omega_{\rm GW}(\tau_0,k) h^2 \simeq 2.4 \times 10^{-5} \left(\frac{\Omega_{r,0} h^2}{4.0 \times 10^{-5}} \right) \left(\frac{k}{\mathcal{H}(\tau_f)} \right)^2 \int_{-\frac{1}{\sqrt{3}}}^{\frac{1}{\sqrt{3}}} \mathrm{d}d \int_{\frac{1}{\sqrt{3}}}^{\infty} \mathrm{d}s \left[\frac{(d^2 - 1/3)(s^2 - 1/3)}{s^2 - d^2} \right]^2 \\ \times \mathcal{P}_{\mathcal{R}}\left(\frac{k\sqrt{3}}{2}(s+d) \right) \mathcal{P}_{\mathcal{R}}\left(\frac{k\sqrt{3}}{2}(s-d) \right) \left[\mathcal{I}_c^2(d,s) + \mathcal{I}_s^2(d,s) \right].$$

$$\mathcal{I}_{c}(d,s) = -36\pi \frac{(s^{2} + d^{2} - 2)^{2}}{(s^{2} - d^{2})^{3}} \theta(s-1),$$

$$\mathcal{I}_{s}(d,s) = -36\frac{(s^{2} + d^{2} - 2)}{(s^{2} - d^{2})^{2}} \left[\frac{(s^{2} + d^{2} - 2)}{(s^{2} - d^{2})} \log \frac{(1 - d^{2})}{|s^{2} - 1|} + 2 \right]$$

$$d = (u - v)/\sqrt{3}$$
$$v = p/k, u = |\mathbf{k} - \mathbf{p}|/k$$
$$s = (u + v)/\sqrt{3},$$



N. Bhaumik & **RKJ**, Phys. Rev. D 104, 023531 (2021)



- A universal IR scaling of GW spectrum, Cai, Pi & Sasaki, 2019
- IR scaling with log corrections, Yuan, Chen & Huang, 2019

Rajeev Kumar Jain

IAP Paris 2025

The 'three' peaks

An interesting and useful relation between the 'three' peaks



• Advanced LIGO: f ~ 30 Hz $\longrightarrow M_{\rm PBH} \sim 10^{13} \,\mathrm{g} \sim 10^{-20} M_{\odot}$

Induced secondary GWs – effects of reheating



N. Bhaumik & **RKJ**, Phys. Rev. D 104, 023531 (2021)

(future) constraints on small scales



Observational imprints of ultralight PBHs: early domination and evaporation

How to constrain imprints of ultralight PBHs ?



PBH with $M \le 10^9 - 10^{10} g$ completely evaporate by BBN hence remain unconstrained.

Ultralight PBHs — Hawking evaporation

• Non-spinning PBH

$$T_{\rm PBH}^{\rm S} = \frac{M_{\rm Pl}^2}{M_{\rm PBH}} \simeq 1.053 \,\text{GeV}\left(\frac{10^{13} \,\text{g}}{M_{\rm PBH}}\right) \qquad \Delta t_{\rm PBH}^{\rm S} \approx \frac{160 M_{\rm PBH}^3}{\pi \,\mathcal{G} \,\overline{g_{*,H}} \,M_{Pl}^4}$$

• Spinning PBH

$$T_{\rm PBH} = \frac{M_{\rm Pl}^2}{M_{\rm PBH}} \left(\frac{2\sqrt{1-a_*^2}}{1+\sqrt{1-a_*^2}} \right) \qquad a_* = JM_{\rm Pl}^2/M_{\rm PBH}^2$$
$$\frac{dM(t)}{dt} = -\varepsilon(M(t), a(t))\frac{M_{\rm Pl}^4}{M(t)^2}, \qquad \Delta t_{\rm PBH} = \Delta t_{\rm PBH}^{\rm S} \mathcal{F}(a_*, M_{\rm PBH})$$
$$\frac{da(t)}{dt} = -a(t) \Big[\gamma(M(t), a(t)) - 2\varepsilon(M(t), a(t)) \Big] \frac{M_{\rm Pl}^4}{M(t)^3} \qquad \textbf{BlackHawk}$$

N. Bhaumik, A. Ghoshal, **RKJ** & M. Lewicki, JHEP 05, 169 (2023)

Rajeev Kumar Jain

IAP Paris 2025

Ultralight PBHs — early matter dominated epoch



Ultralight PBHs — early matter dominated epoch



IAP Paris 2025

Ultralight PBHs — Contributions to perturbations

- Poisson Distribution of PBHs
- Cutoff for scales below PBH mean distance

$$\mathcal{P}_{\text{PBH}}(k,\tau_r) = \frac{2}{3\pi} \left(\frac{k}{k_{\text{UV}}}\right)^3 \left(5 + \frac{4}{9} \frac{k^2}{k_m^2}\right)^{-2}$$
$$k_{\text{UV}} = \gamma^{-1/3} \beta^{1/3} k_f$$

$$\mathcal{P}_{\mathrm{infl}}(k,\tau_r) = A_s \left(\frac{k}{k_p}\right)^{n_s-1} \theta_H(k_m-k)$$

$$\Phi(k, au_r) = \Phi_{infl}(k, au_r) + \Phi_{PBH}(k, au_r)$$

 $\mathcal{P}(k, au_r) = \mathcal{P}_{infl}(k, au_r) + \mathcal{P}_{PBH}(k, au_r)$

Papanikolaou et. al. (2021), Domenech et. al. (2021)



N. Bhaumik, A. Ghoshal, **RKJ** & M. Lewicki, JHEP 05, 169 (2023)

IAP Paris 2025

Extra relativistic dof from PBH evaporation



N. Bhaumik, A. Ghoshal, RKJ & M. Lewicki, JHEP 05, 169 (2023)

IAP Paris 2025

Heavy dark matter from PBH evaporation



N. Bhaumik, A. Ghoshal, **RKJ** & M. Lewicki, JHEP 05, 169 (2023)

Future GWs and CMB complementarity



N. Bhaumik, A. Ghoshal, **RKJ** & M. Lewicki, JHEP 05, 169 (2023)

Rajeev Kumar Jain

IAP Paris 2025

PBH, stochastic GWs and Pulsar Timing Array (PTA) detection

Stochastic GWs – PTA detection

THE ASTROPHYSICAL JOURNAL LETTERS, 951:L11 (56pp), 2023 July 1

© 2023. The Author(s). Published by the American Astronomical Society.

OPEN ACCESS

https://doi.org/10.3847/2041-8213/acdc91



The NANOGrav 15 yr Data Set: Search for Signals from New Physics



Stochastic GWs – PTA detection



Rajeev Kumar Jain

Stochastic GWs from PBHs – PTA signal



Rajeev Kumar Jain

IAP Paris 2025

Stochastic GWs from PBHs — PTA signal



N. Bhaumik, RKJ and M. Lewicki, Phys. Rev. D 108,123532 (2023)

Rajeev Kumar Jain

IAP Paris 2025

Stochastic GWs from PBHs — PTA signal

Bayesian analysis using PTArcade

 $\mathrm{BF}_{Y,X} \equiv \mathcal{P}(\mathcal{D}|Y) / \mathcal{P}(\mathcal{D}|X),$

Model	Parameters	Posterior mean	
		NG15	IPTA2
Ultra-low mass PBH model	$\log_{10}(rac{M_{\mathrm{PBH}}}{\mathrm{1g}})$	$7.99\substack{+0.13\\-0.15}$	$8.11\substack{+0.35 \\ -0.12}$
	$\log_{10}(eta_f)$	$-9.57\substack{+0.15\\-0.11}$	$-9.69^{+0.22}_{-0.28}$
	n_0	$1.503\substack{+0.025\\-0.042}$	$1.507\substack{+0.047\\-0.040}$

Model X	Model V	$\mathrm{BF}_{Y,X}$	
	Widdel 1	NG15	IPTA2
SMBHB	Ultra-low mass PBHs	18.00 ± 1.75	3.31 ± 0.09



N. Bhaumik, **RKJ** and M. Lewicki, Phys. Rev. D 108,123532 (2023)

IAP Paris 2025

Stochastic GWs from PBHs — PTA signal Other observational constraints

- **PBH evaporation bound**: Depending on initial PBH abundance and mass range, PBH can either dominate the universe briefly or evaporate before they can dominate. We set $\tau_{rat} \gg 1$ to avoid the parameter region where PBHs evaporate before they can dominate.
- $\Delta N_{\rm eff}$ bound: GWs generated before BBN act as an extra relativistic component, which both BBN and CMB observations severely constrain in terms of the effective number of neutrinos, $\Delta N_{\rm eff}$ [94, 95], thereby restricting our parameter space,

$$\Omega_{\rm GW} h^2|_{\rm peak} \lesssim 6.9 \times 10^{-6} \,. \tag{14}$$

• Bound from CMB scales: We limit our parameter search to the region where the scalar spectrum is unaffected at CMB scales. We ensure that at $k = 1 \text{ Mpc}^{-1}$,

$$A_s \left(\frac{k}{k_{\rm p}}\right)^{n_s - 1} \ge A_0 \left(\frac{k}{k_{\rm pk}}\right)^{n_0 - 1} . \tag{15}$$

PBH evaporation and the memory burden effect

- Semiclassical description of Hawking evaporation can not hold over the entire black hole lifetime
- Quantum backreaction must be taken into account through memory burden effect
- Semiclassical formalism of Hawking evaporation breaks down at the latest by the time a black hole loses half of its mass due to Hawking evaporation
- The gapless microscopic energy states at the start of Hawking evaporation, allowing a maximal storage capacity of information, can no longer stay gapless after the mass of the black hole reduces significantly from its initial value.



Dvali et. al. (2020), Dvali et. al. (2024), Alexandre et. al. (2024)

Rajeev Kumar Jain

IAP Paris 2025

- What are the imprints of memory burden effect on the induced GW spectrum?
- Can non-trivial reheating mimic these signatures?



Rajeev Kumar Jain

$$\frac{\mathrm{d}M}{\mathrm{d}t} = -\frac{\pi \ \mathcal{G} \ g_{*,H} M_{\mathrm{Pl}}^4}{480 M^2} \begin{cases} 1 & \text{for } M > q M_{\mathrm{PBH}} \\ S^{-n}(M) & \text{for } M < q M_{\mathrm{PBH}} \end{cases}$$

 $S(M) = M^2/2M_{\rm Pl}^2$: mass dependent entropy of PBH

Memory burden effects can slow down the evaporation of a black hole, thus prolonging its lifetime

$$PS1 \equiv \{M_{\text{PBH1}}, \beta_{f1}, w_1 = 1/3, q_1, n_1\}$$
$$PS2 \equiv \{M_{\text{PBH2}}, \beta_{f2}, w_2, q_2 = 0, n_2 = 0\}$$

Is there a degeneracy between the two effects ?

N. Bhaumik, M. R.Haque, **RKJ** & M. Lewicki, JHEP 10, 142 (2024)



N. Bhaumik, M. R.Haque, **RKJ** & M. Lewicki, JHEP 10, 142 (2024)

IAP Paris 2025



N. Bhaumik, M. R.Haque, **RKJ** & M. Lewicki, JHEP 10, 142 (2024)

Rajeev Kumar Jain

IAP Paris 2025



N. Bhaumik, M. R.Haque, RKJ & M. Lewicki, JHEP 10, 142 (2024)

Rajeev Kumar Jain

IAP Paris 2025

A recent review on PBH and GW

Living Reviews in Relativity (2025)28:1 https://doi.org/10.1007/s41114-024-00053-w

REVIEW ARTICLE



Primordial black holes and their gravitational-wave signatures

Eleni Bagui¹ · Sébastien Clesse¹ · Valerio De Luca² · Jose María Ezquiaga³ · Gabriele Franciolini⁴ · Juan García-Bellido⁵ · Cristian Joana^{6,7} · Rajeev Kumar Jain⁸ · Sachiko Kuroyanagi^{5,9} · Ilia Musco^{10,11} · Theodoros Papanikolaou^{12,13,14} · Alvise Raccanelli^{4,15,16,17} · Sébastien Renaux-Petel¹⁸ · Antonio Riotto¹⁹ · Ester Ruiz Morales^{5,20} · Marco Scalisi²¹ · Olga Sergijenko^{22,23,24,25} · Caner Ünal^{26,27,28} · Vincent Vennin²⁹ · David Wands³⁰ · For the LISA Cosmology Working Group

Received: 22 December 2023 / Accepted: 5 November 2024 © The Author(s) 2024

A very comprehensive up-to-date review (~200 pages)



- PBHs are novel candidates for the cold DM in the universe an important probe of small scale dynamics during inflation
- Inflation can produce significant abundance of PBHs single field inflection point models are useful — model dependent results
- Interesting observational implications induced GWs on scales probed by LISA, DECIGO or BBO
- Hawking evaporation imprints dark radiation and dark matter (nonthermal mechanism) — complementarity between GW and CMB
- Induced GWs from ultralight PBHs can explain the PTA results
- Interesting imprints of memory burden effect probing quantum effects through classical GWs

Funding acknowledgments





Thank you for your attention!

Rajeev Kumar Jain

IAP Paris 2025