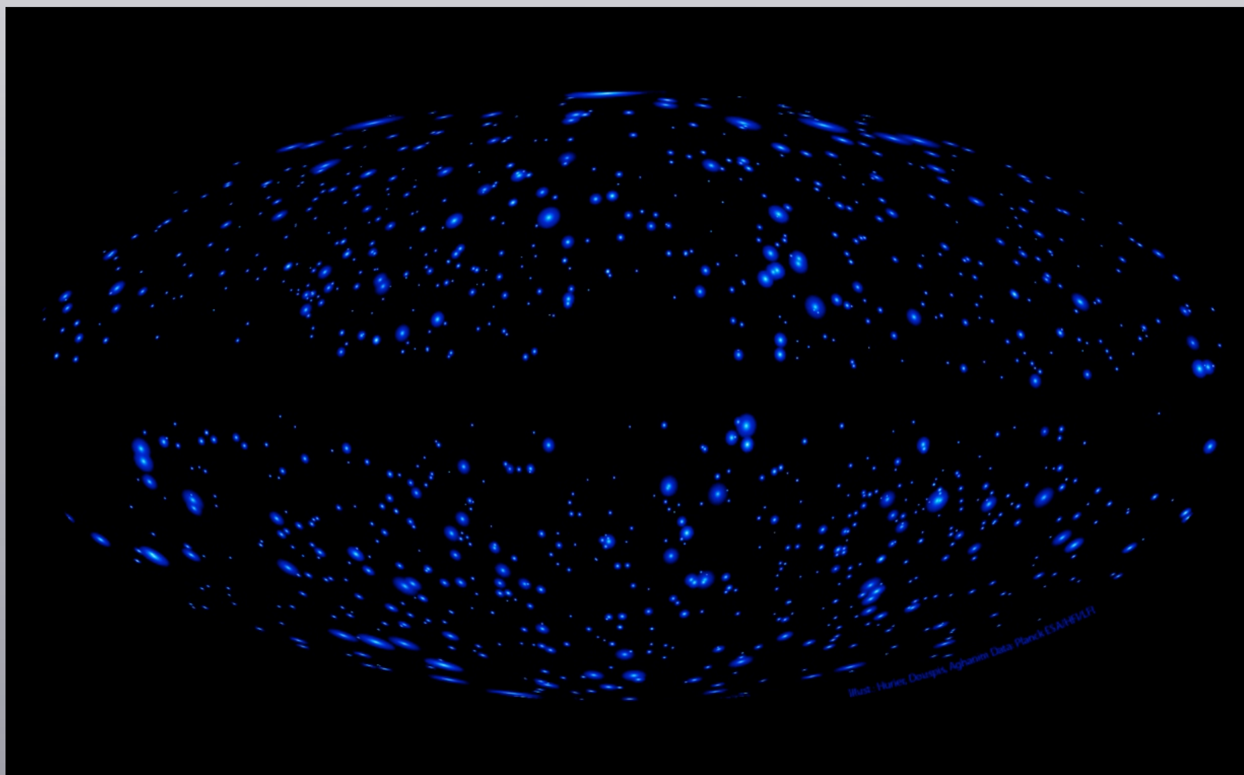
Combining probes : cluster counts and galaxy power spectrum



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THE DARK ENERGY SURVEY

ICTP SAIFR

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Outline

I. Covariance of galaxy spectrum and cluster counts

II. Results and Fisher analysis

III. Joint likelihood

arxiv:1407.1247

I. Combining probes

$$\mathbf{X} = \left(\begin{array}{c} N_{\rm cl} \\ C_\ell^{\rm gal} \end{array} \right)$$

$N_{cluster}(i_M, i_z)$

$C_l^{gal}(i_z) \leftrightarrow$ 2pcf

Motivations:

- increased constraints, can break degeneracies.
- mitigate super-sample covariance (SSC) on small scales.
- future : dark energy, modified gravity, f_{NL}.

$$\mathrm{Cov}\left(\mathbf{X},\mathbf{X}\right) = \left(\begin{array}{cc} \mathrm{Cov}\left(N_{\rm cl}, N_{\rm cl}\right) & \mathrm{Cov}\left(N_{\rm cl}, C_\ell^{\rm gal}\right) \\ \mathrm{Cov}\left(C_\ell^{\rm gal}, N_{\rm cl}\right) & \mathrm{Cov}\left(C_\ell^{\rm gal}, C_\ell^{\rm gal}\right) \end{array} \right)$$

I. Covariance of the galaxy spectrum and cluster counts

Cluster count is the monopole of the halo density field

$$\hat{N}_{\rm cl}(i_M, i_z) = \overline{N}_{\rm cl}(i_M, i_z) + \frac{1}{\Omega_S} \int dM\, d^2\hat{n}\, dz\, r^2 \frac{dr}{dz} \frac{d^2 n_h}{dM\, dV} \delta_{\rm cl}(\mathbf{x}, z|M, z)$$

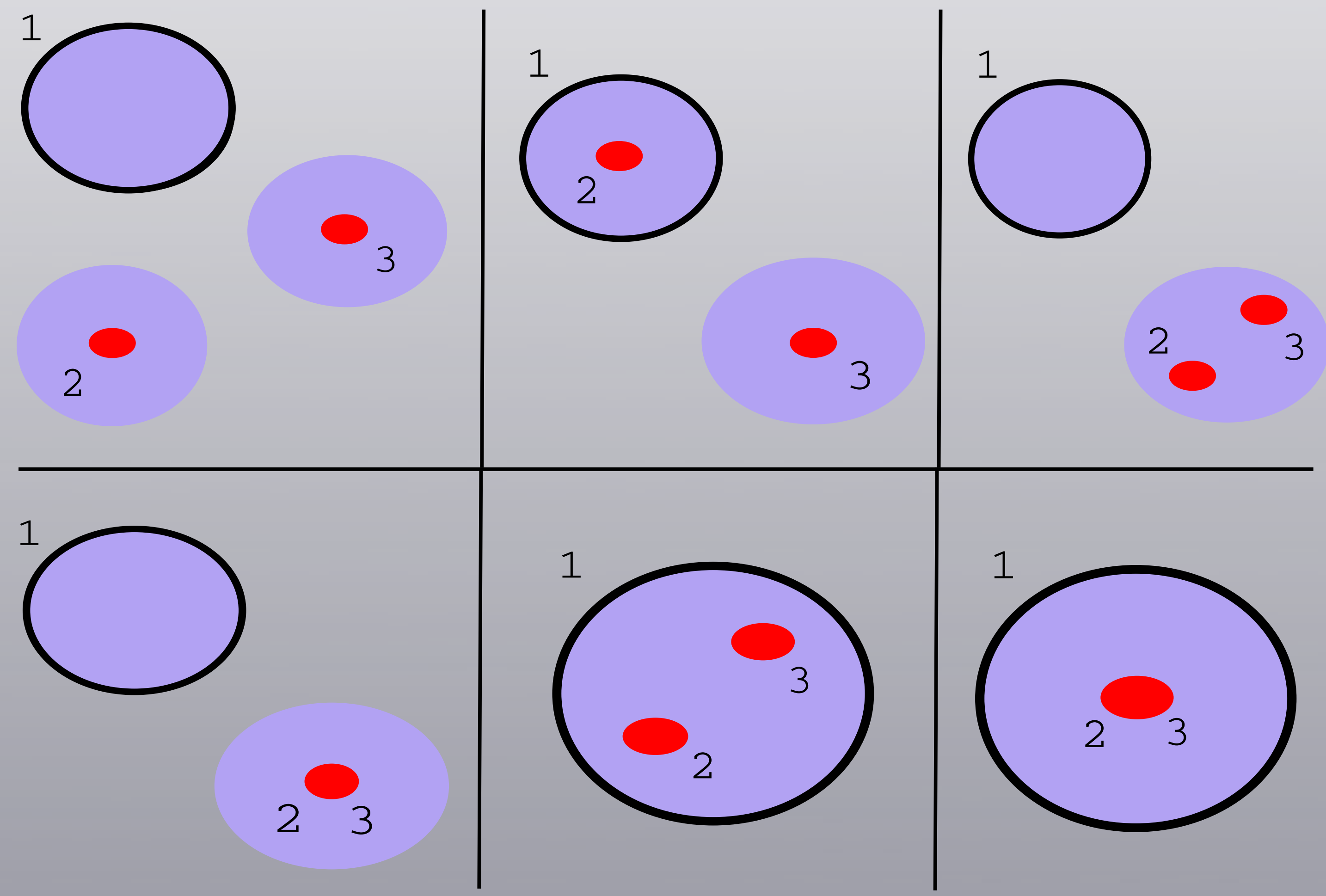
$$\mathrm{Cov}\left(\hat{N}_{\rm cl}(i_M, i_z), \hat{C}_\ell^{\rm gal}(j_z, k_z)\right) = \int \frac{dM_1\, dz_{123}}{4\pi} \frac{dV}{dz_1} \left.\frac{d^2 n_h}{dM\, dV}\right|_{M_1, z_1} b_{0\ell\ell}^{\rm hgg}(M_1, z_{123})$$

Galaxy angular power spectrum between two redshift bins (in the following $j_z = k_z$)

Cluster count in a bin of mass (i_M) and redshift (i_z)

Halo-galaxy-galaxy angular bispectrum

I. Diagrammatic method for the hgg bispectrum

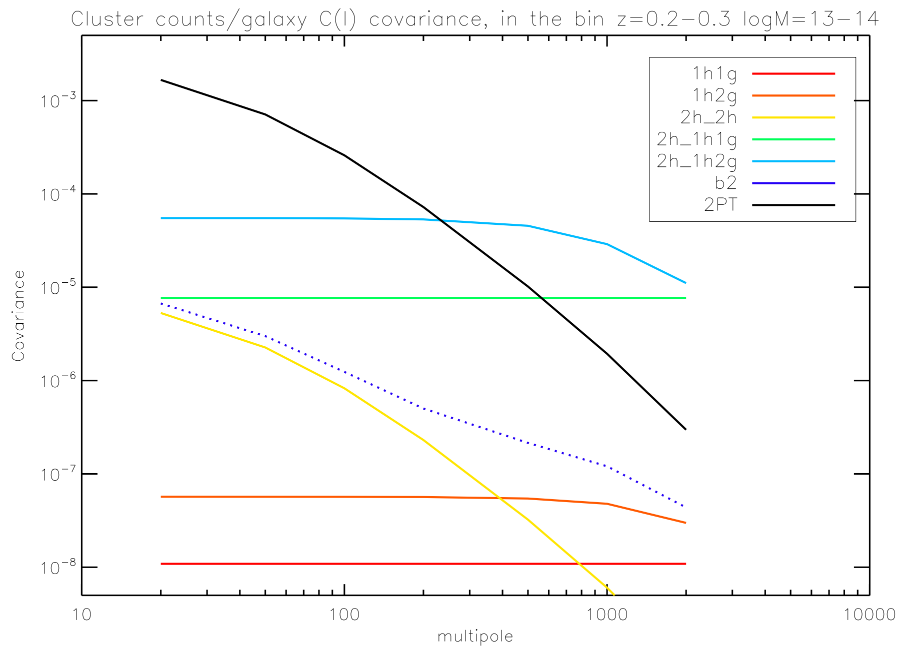


Ex: $$B_{\rm hgg}^{2h-1h2g}(k_{123}|M_1, z_{123}) = \frac{\delta_{z_2,z_3}}{\bar{n}_{\rm gal}^2(z_2)} \int dM \frac{d^2 n_h}{dM\, dV} \langle N_{\rm gal}(N_{\rm gal}-1)(M)\rangle\, u(k_2|M)\, u(k_3|M)\, P_{\rm halo}(k_1|M_1, M, z_1, z_2)$$

Ingredients : cosmology, halo model, Halo Occupation Distribution (HOD)

II. Ideal results I : scale dependence of the covariance

1



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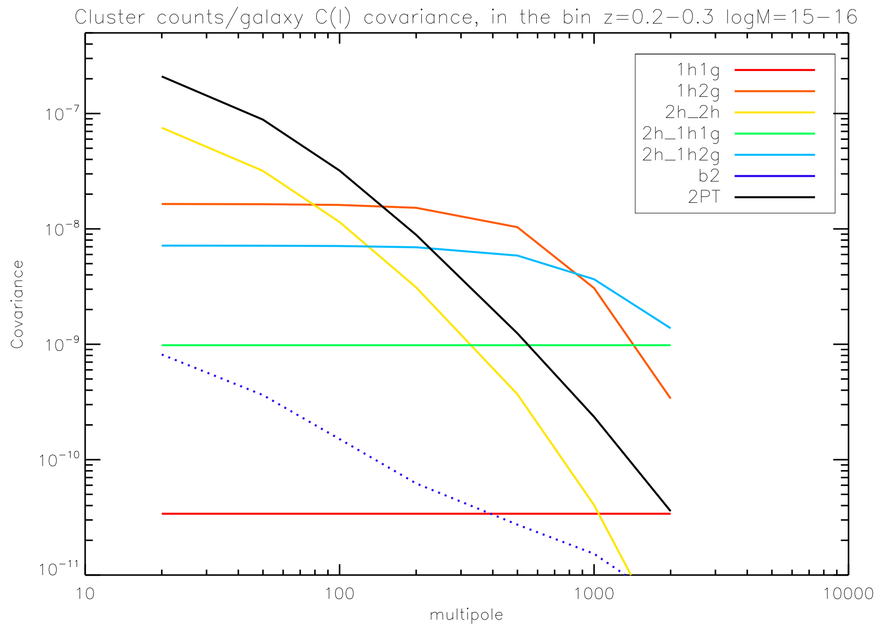
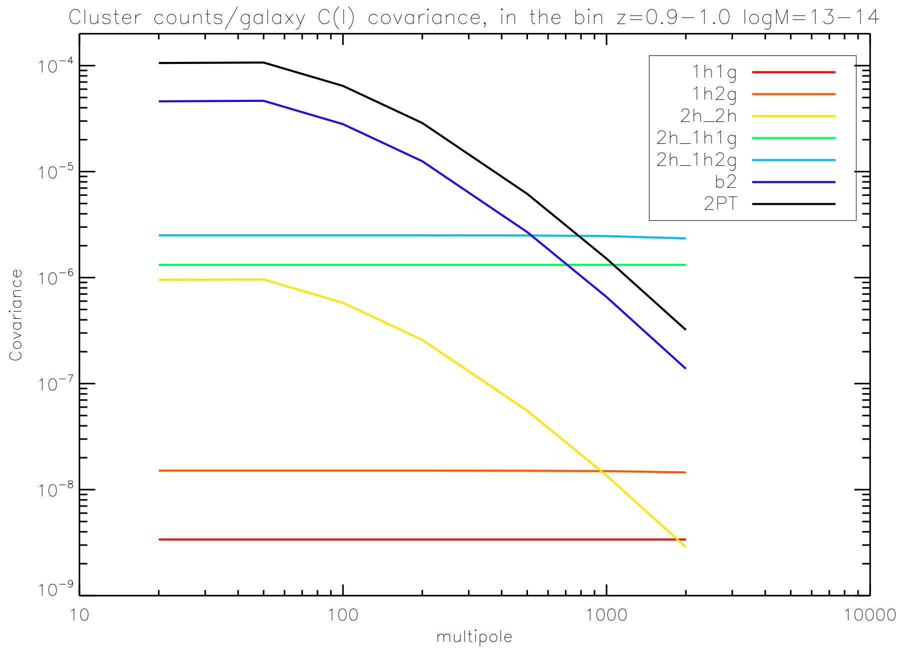




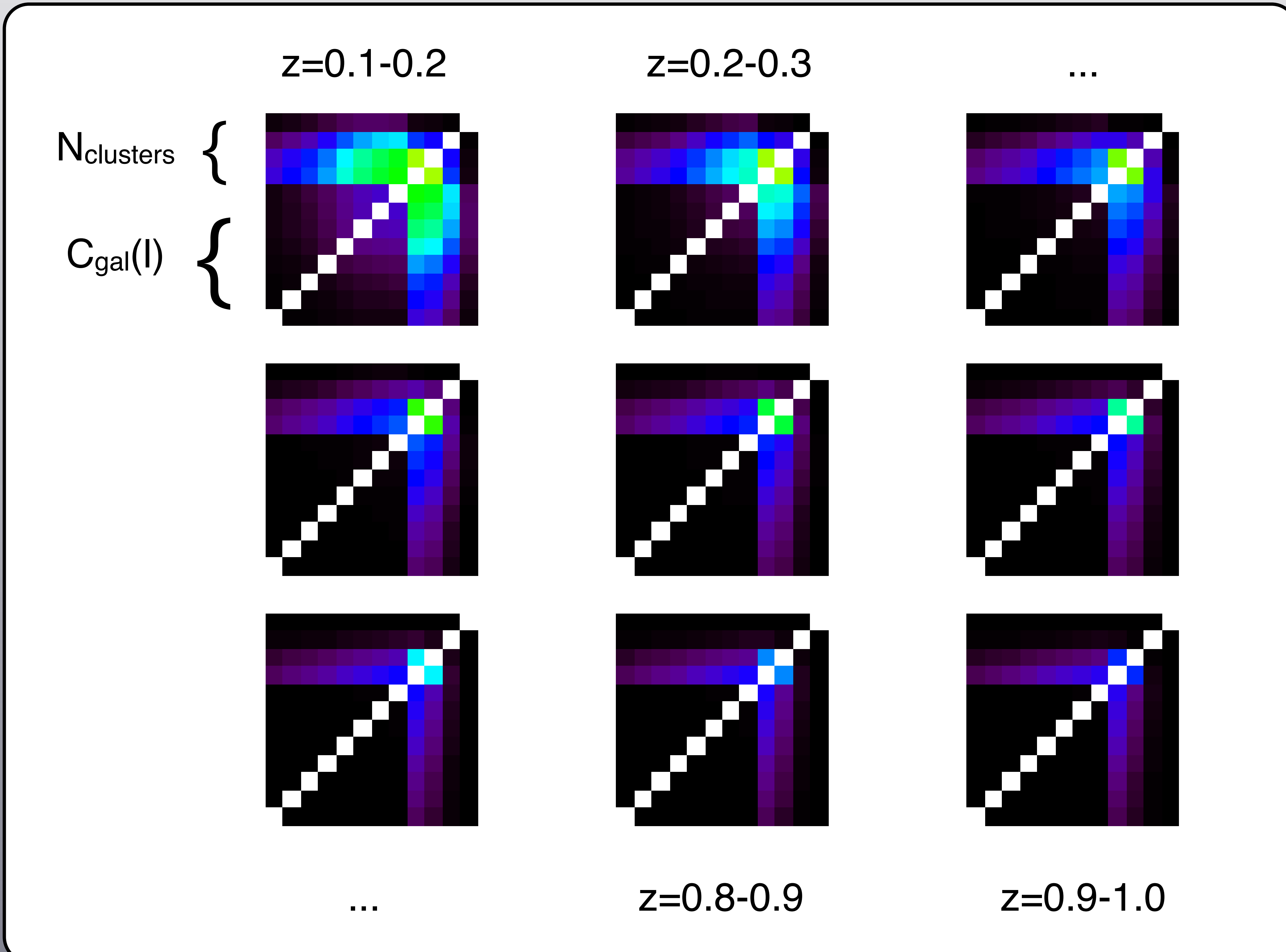
3

1 : z=0.2-0.3 and $\log(M/M_{sun})$ = 13-14

2 : z=0.2-0.3 and $\log(M/M_{sun})$ = 15-16

3 : z=0.9-1.0 and $\log(M/M_{sun})$ = 13-14

II. Ideal results II : joint covariance matrix



9 z-bins z=0.1-1.0

4 logM bins
logM = 14-16

8 multipoles
l=30-300

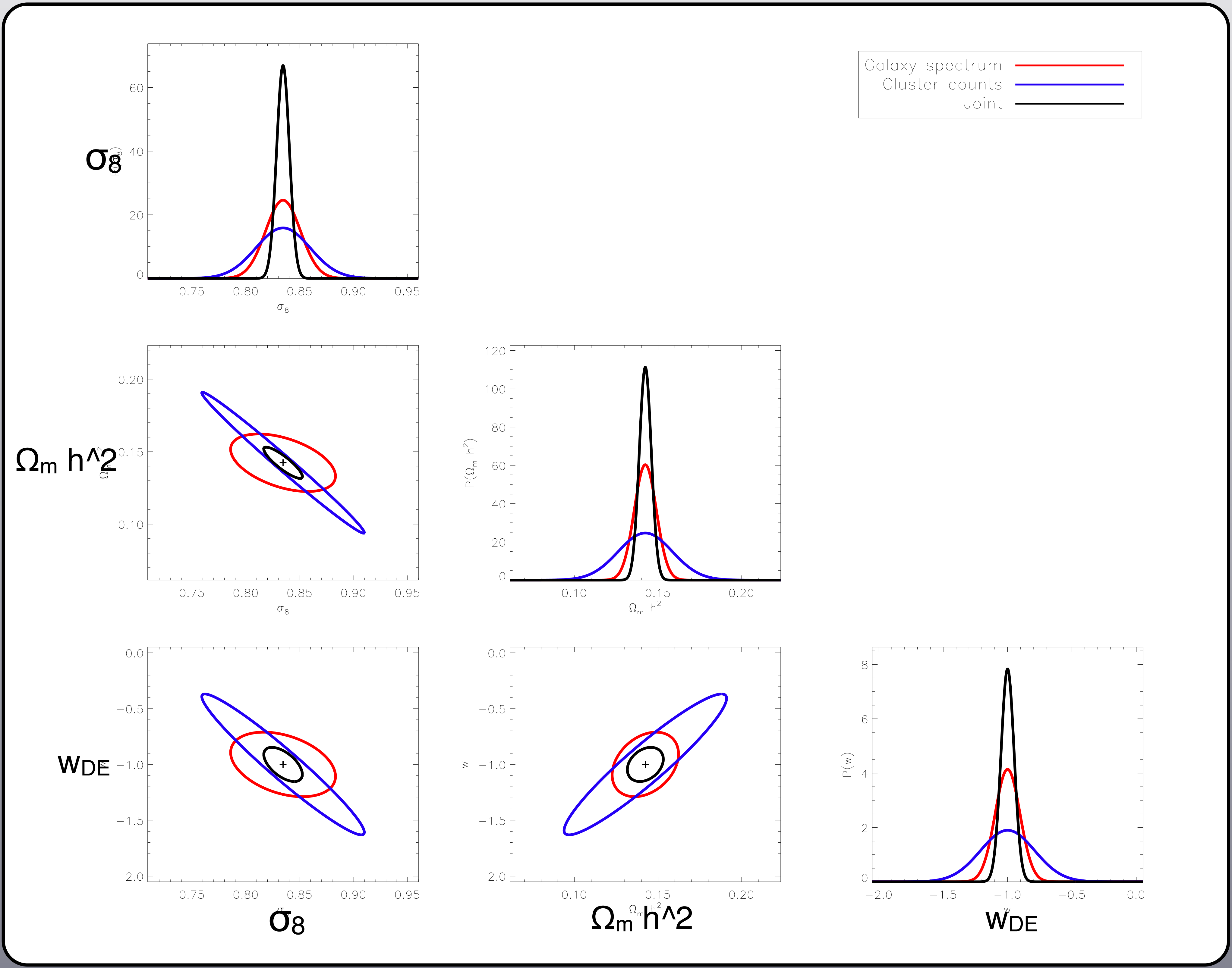
no photo-z errors, purity nor completeness

full cluster autocov (shot-noise, sample variance)

full crosscov

galaxy autocov : Gaussian, SSC, 1h trispectrum

II. Fisher analysis



after marginalisation over HOD parameters

$S/N(C_l^{gal}) = S/N(N_{cl})$

Joint constraints better than if probes were independent

III. Joint likelihood

Cluster counts follow a Poisson distribution
Galaxy correlation is more Gaussian

→ How to mix their likelihood ?
(cannot assume that the joint likelihood is Gaussian)

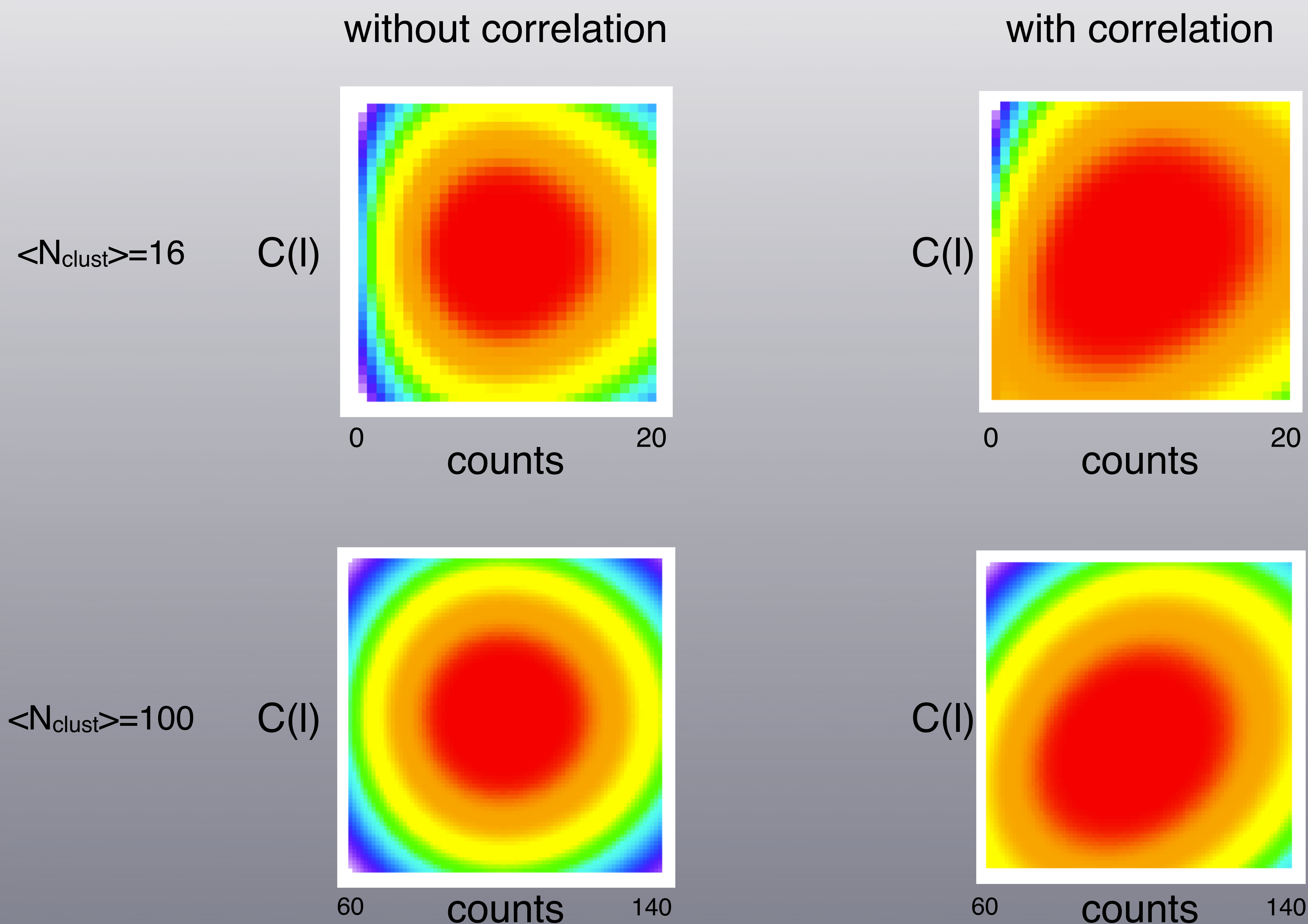
Edgeworth / Gram-Charlier expansion

$$P(x) = \exp\left[\sum_{n=1}^{+\infty} \left(\kappa_n(P) - \kappa_n(P_{\text{fidu}})\right) \frac{(-1)^n}{n!} \frac{d^n}{dx^n}\right] P_{\text{fidu}}(x)$$

Expand around independent case. Result :

$$\mathcal{L}(\text{counts}, C_\ell) = \exp\left[-\sum_{ij} \langle c_i\, C_{\ell_j} \rangle_c \left(\log \bar{c}_i - \Psi(c_i + 1)\right) \left({}^T C_\ell\, C^{-1} e_j\right)\right] \mathcal{L}(\text{counts}) \mathcal{L}(C_\ell)$$

III. Joint likelihood : functional form



B. Joint likelihood : conclusions / perspectives

- valid to combine any Gaussian and Poisson observables
(e.g. weak-lensing/counts)

- large counts and small crosscov limit :
Gaussian with correct covariance matrix

- extended to include cluster sample covariance

- inclusion of Bayesian hyperparameters
→ robustness to tension and error estimates
(in progress)

Conclusions

- Cluster counts - galaxy spectrum cross-covariance with a diagrammatic formalism
- Full non-linear model is needed : HM+HOD
- Cross-covariance is not negligible and creates a synergy between the probes
- Joint non-Gaussian likelihood
- Future : MC pipeline for realistic forecast and application to DES data

Thanks for your attention

Super-sample covariance

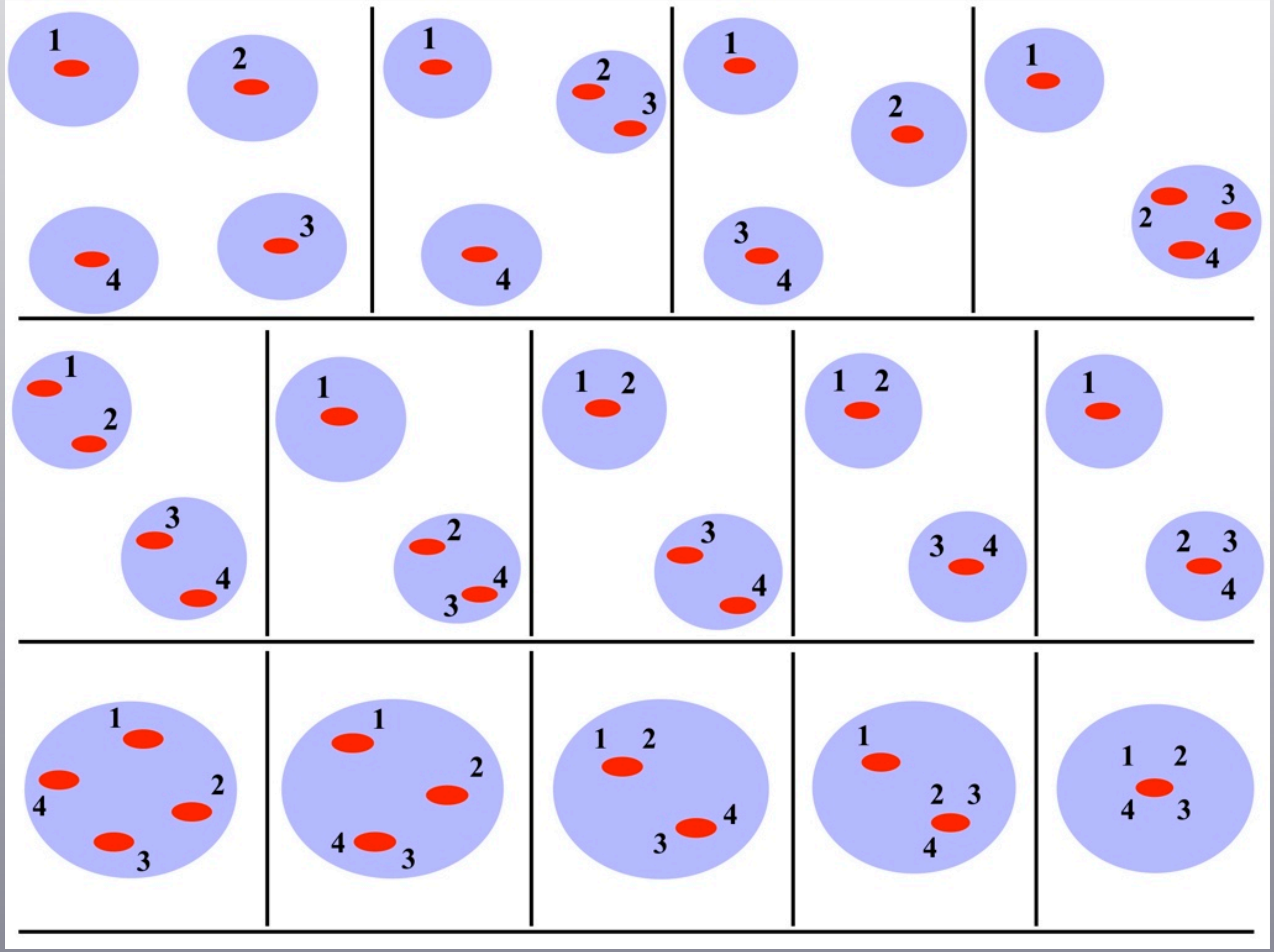
Reaction of observable to long wavelength modes (=background change)

$$\mathrm{Cov}_{\mathrm{SSC}}(C_\ell(i_z), C_{\ell'}(j_z)) = \int \mathrm{d}V_1\, \mathrm{d}V_2 \frac{\overline{n}_{\mathrm{gal}}(z_1)^2\, \overline{n}_{\mathrm{gal}}(z_2)^2}{\Delta N_{\mathrm{gal}}(i_z)^2\, \Delta N_{\mathrm{gal}}(j_z)^2} \frac{\partial P_{\mathrm{gal}}(k_\ell)}{\partial \delta_b} \frac{\partial P_{\mathrm{gal}}(k_{\ell'})}{\partial \delta_b}\, \sigma^2_{\mathrm{proj}}(z_1, z_2)$$

$$\mathrm{Cov}_{\mathrm{SSC}}\left(\hat{N}_{\mathrm{cl}}(i_M, i_z), \hat{C}_\ell(j_z)\right) = \int \frac{\mathrm{d}V_{12}\, \overline{n}_{\mathrm{gal}}(z_2)^2}{\Delta N_{\mathrm{gal}}(j_z)^2}\, \frac{\partial n_h}{\partial \delta_b}(i_M, z_1)\, \frac{\partial P_{\mathrm{gal}}(k_\ell|z_2)}{\partial \delta_b}\, \sigma^2_{\mathrm{proj}}(z_1, z_2)$$

Covariance of the density monopole (between two redshifts)

Diagrams for the galaxy trispectrum



Halo-galaxy-galaxy bispectrum : from 3D to 2D

$$b_{0\ell\ell}^{\rm hgg}(M_1, z_{123}) = \frac{\delta(z_2 - z_3)}{r_2^2 \frac{dr}{dz_2}} \frac{2}{\pi} \int k_1^2 dk_1 \, B_{\rm hgg}(k_1, k_2^*, k_2^* | M_1, z_1, z_2, z_2) \, j_0(k_1 r_1) \, j_0(k_1 r_2) \quad \text{with} \quad k_2^* = \frac{\ell + 1/2}{r(z_2)}$$

angular bispectrum

3D bispectrum

Bessel functions

Limber's approximation on k_2 and k_3

(bispectrum varies slowly compared to bessel's oscillations)

Ingredients

$$B_{\rm hgg}^{2h-1h2g}(k_{123}|M_1,z_{123}) = \frac{\delta_{z_2,z_3}}{\bar{n}^2_{\rm gal}(z_2)}\int dM\,\frac{d^2n_h}{dM\,dV}\langle N_{\rm gal}(N_{\rm gal}-1)(M)\rangle\,u(k_2|M)\,u(k_3|M)\,P_{\rm halo}(k_1|M_1,M,z_1,z_2)$$

halo mass function
(Sheth&Tormen or Tinker)

Halo Occupation Distribution
(Tinker&Wetzel 2010)

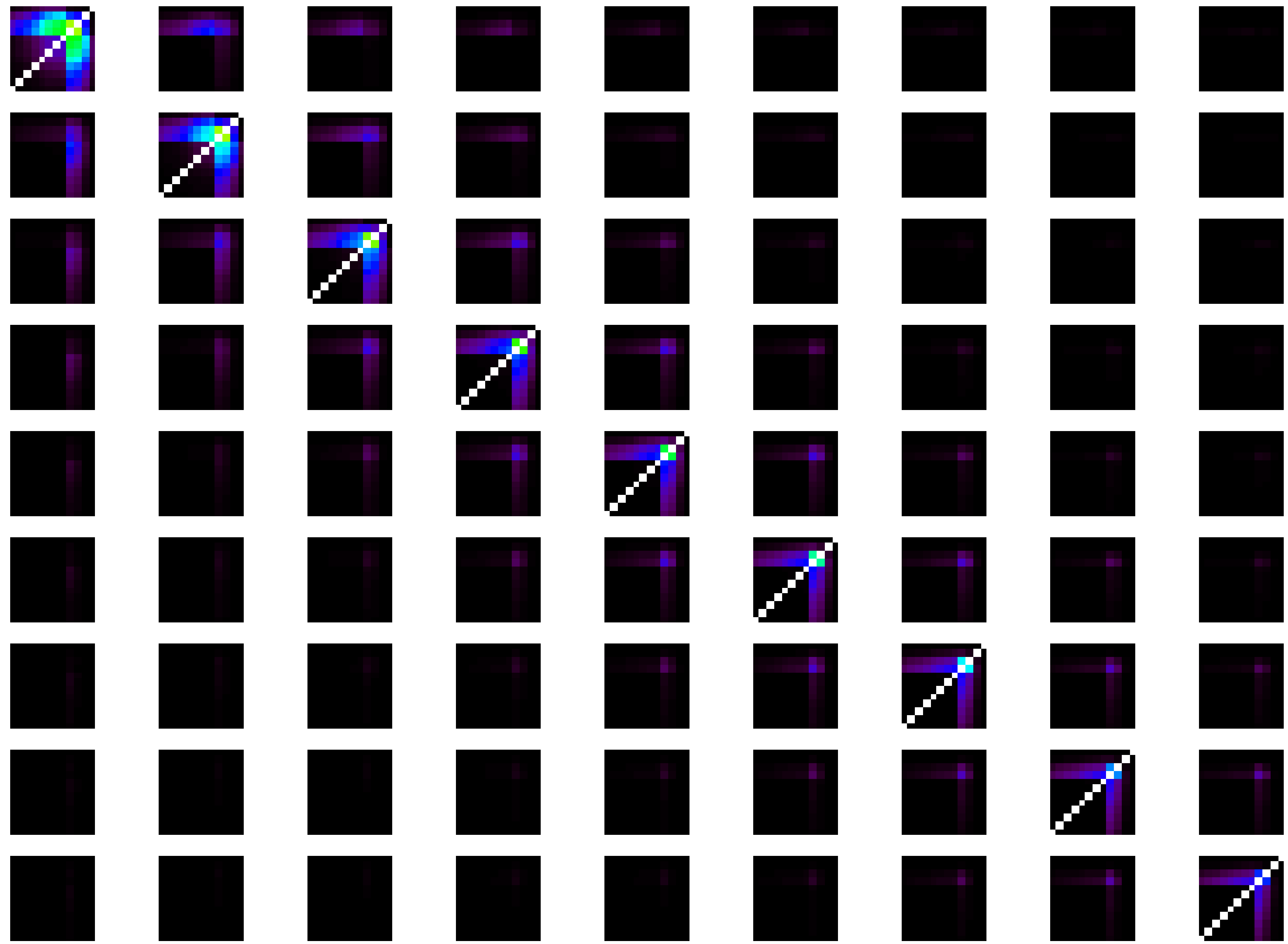
halo profile
(NFW)

halo bias
(Tinker : 1, Sheth&Tormen : 1&2)

matter power spectrum
(Eisenstein&Hu 98)

+ cosmology
(growth function, comoving volume...)

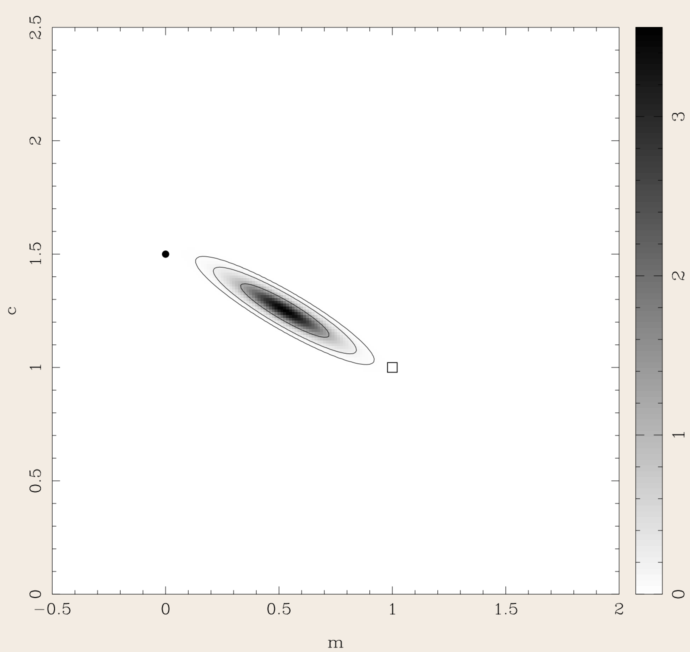
Counts - gal spectrum covariance, with cross-redshifts

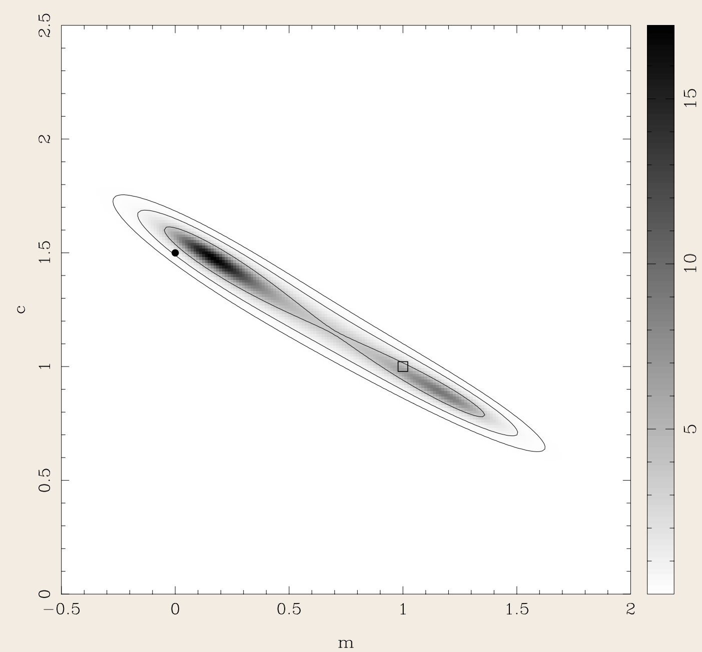


Bayesian hyperparameters

• Statistical method allowing to detect underestimation of error bars or inconsistencies between data sets

without HP





with HP

Hobson et al. 2002

• Idea : rescale error bars
one rescaling parameter per data set. These parameters are included in the MCMC exploration. Then marginalise over them.
• Only done for Gaussian distribution at the moment

Hyperparameters for a Poisson distribution

• not possible to satisfy all the properties of the Gaussian case
(i.e. keep the mean but rescale the variance)

• two possible approximate prescriptions
with a good asymptotic behaviour