What do numerical simulations tell us about stellar magnetic fields?

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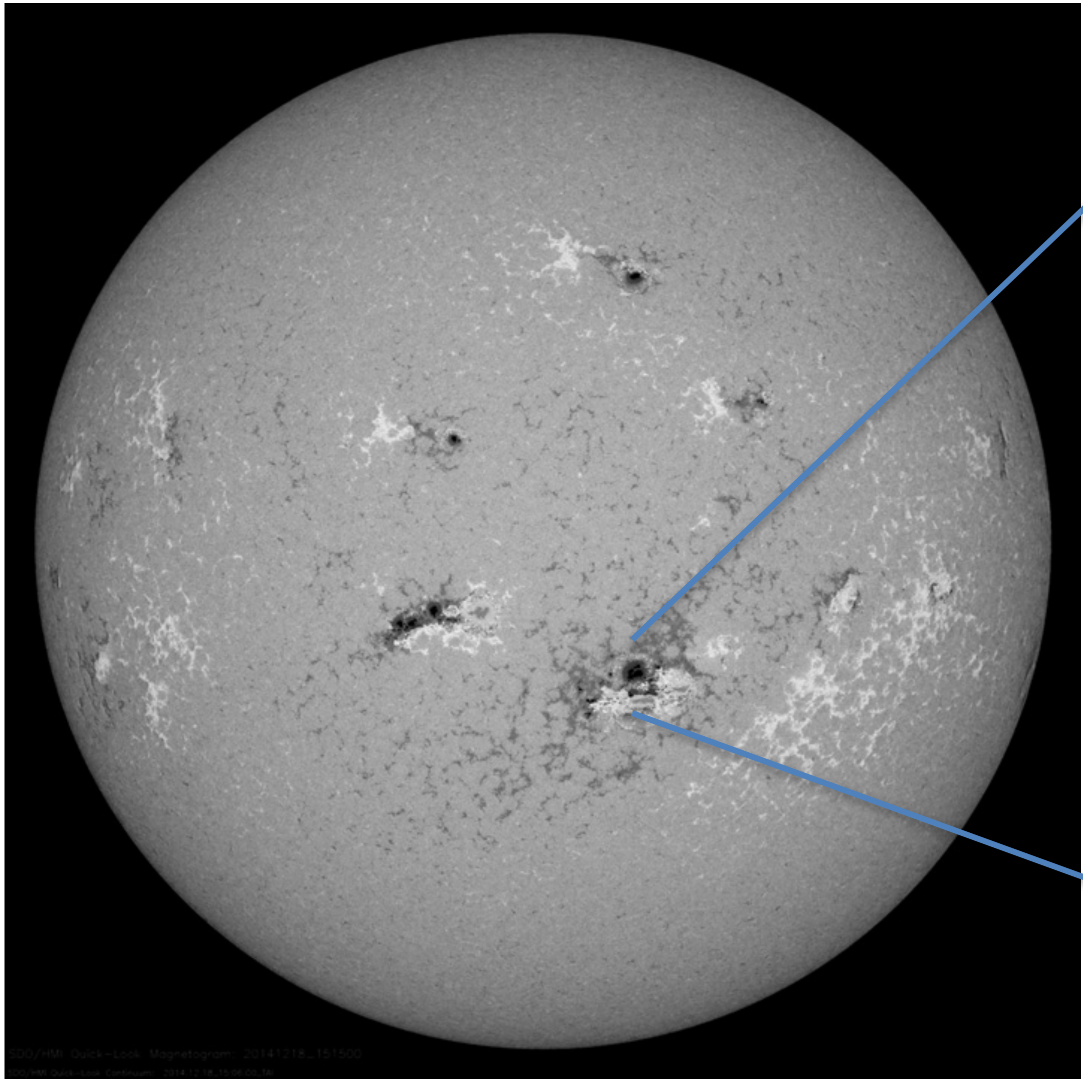
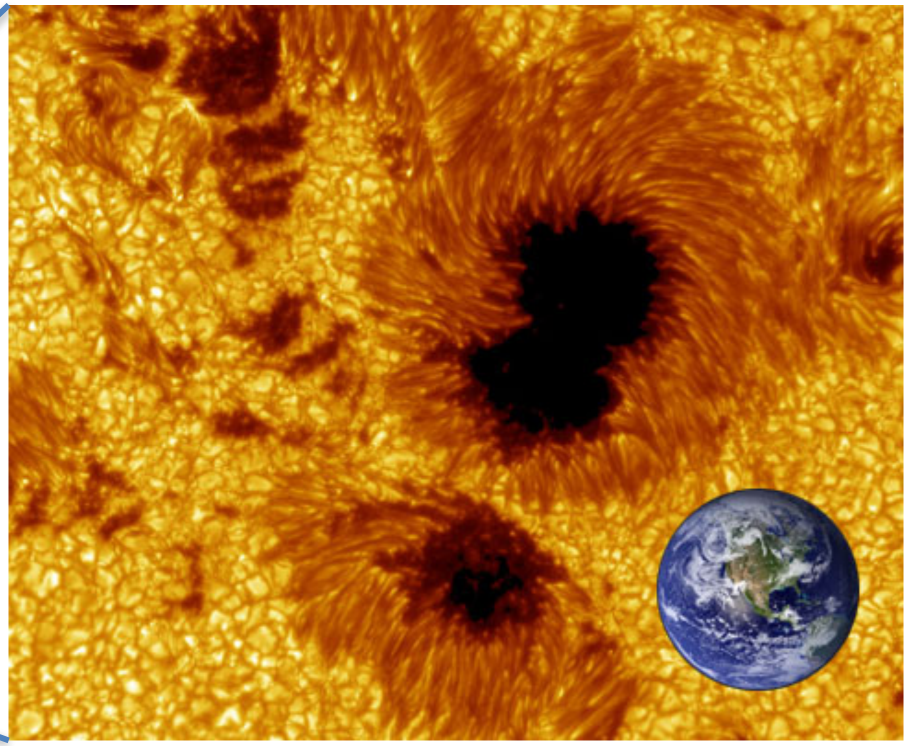
IAP, November 2020

The Sun: a magnetic star

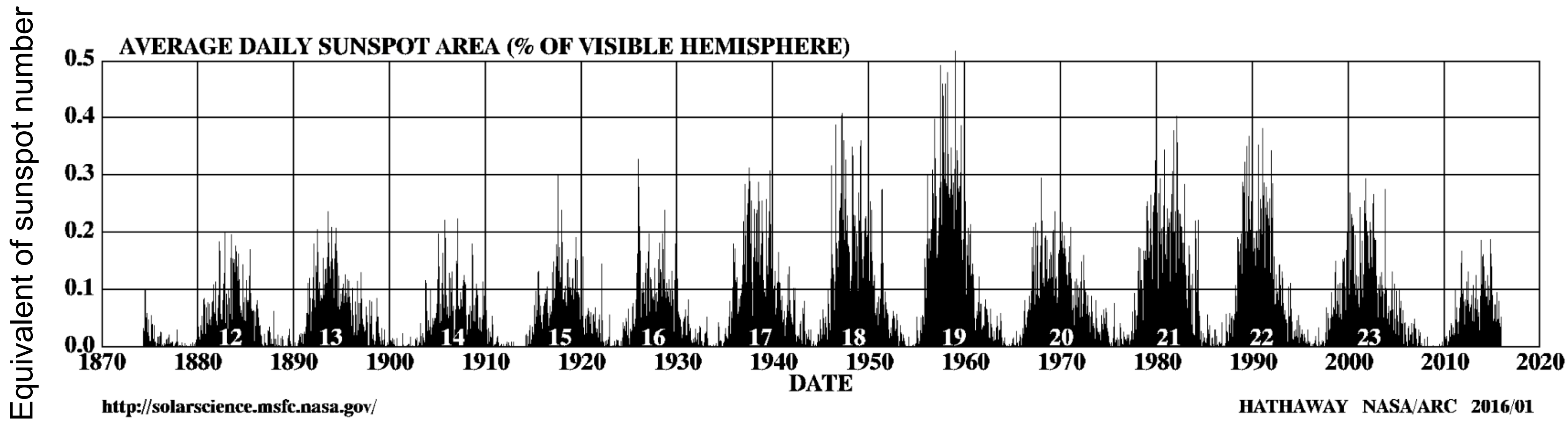


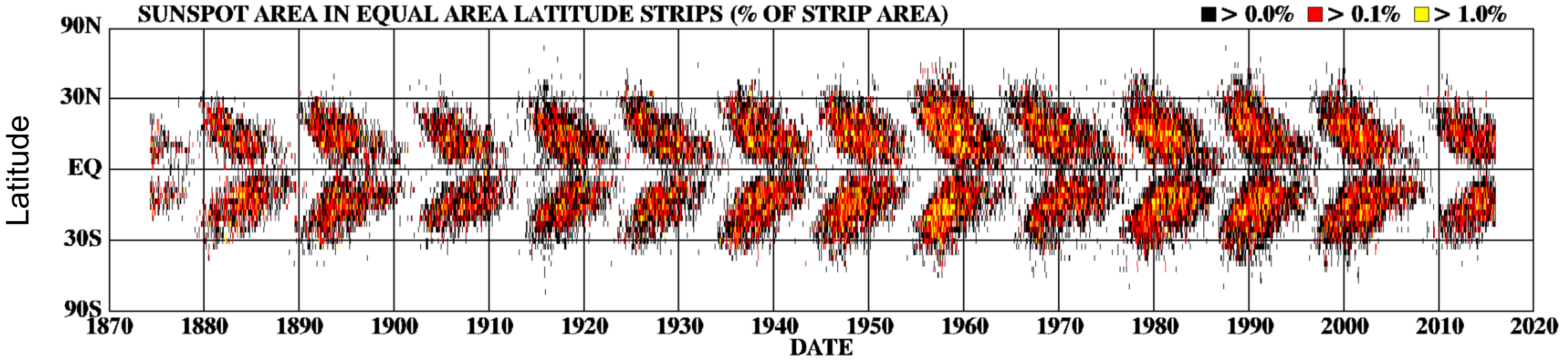


Zoom on a sunspot group

The Sun in 2014

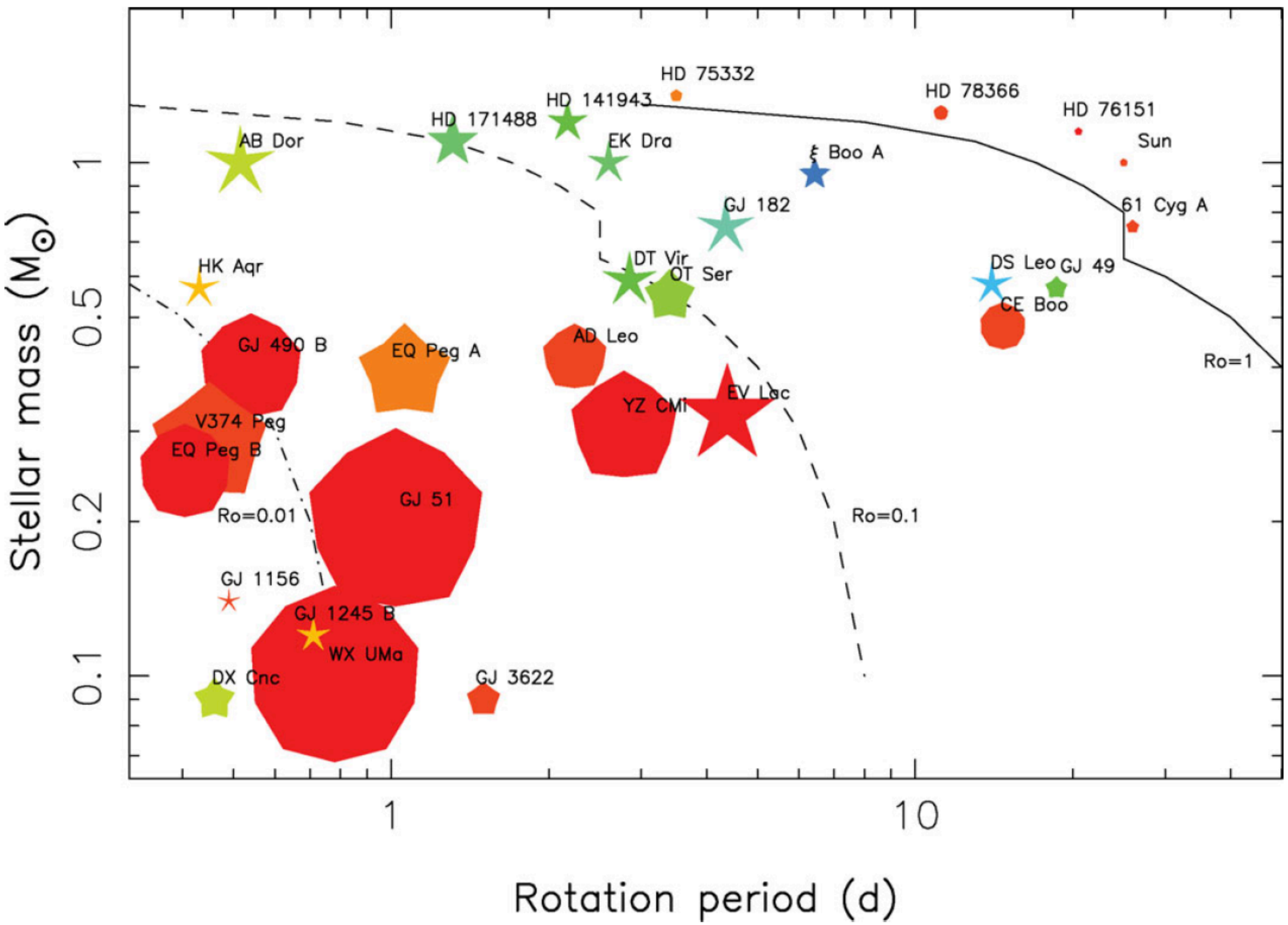
Sunspots: temporal evolution





Magnetic fields in cool stars

- ❑ Mostly multipolar for $M_\odot > 0.35$
- ❑ Mostly dipolar for $M_\odot < 0.35$
- ❑ Bistability for $M_\odot < 0.2$
- ❑ Field strength increases with rotation
- ❑ More and more toroidal with rotation

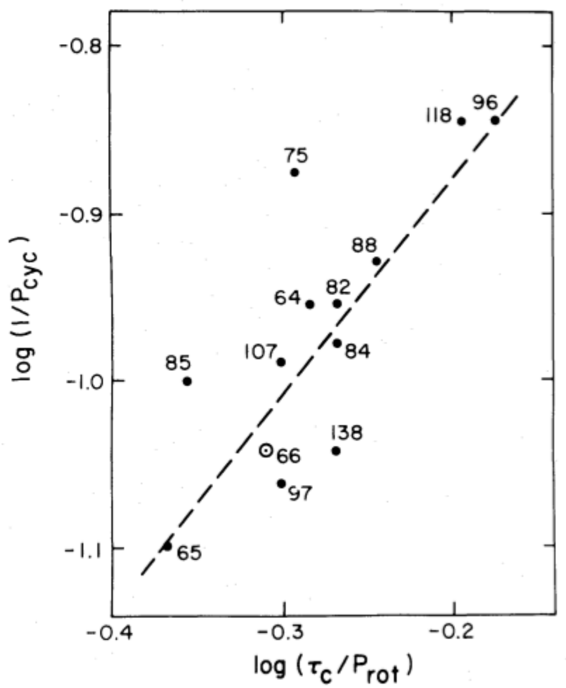


Morin, Donati et al. (2008-2010), Folsom et al. 2016
Petit et al. 2008, B cool survey (Marsden et al. 2014)

Observations of magnetic cycles on other stars

❑ **Indirect measurements:** chromospheric activity

Noyes et al. 1984



Chromospheric activity (Mount Wilson data, Ca II HK lines): $P_{cyc}=Ro^{1.28+/-0.48}$

where the Rossby number $Ro=P_{rot}/\tau$

=> P_{cyc} increases with P_{rot}

❑ **Recent direct measurements:** magnetic field

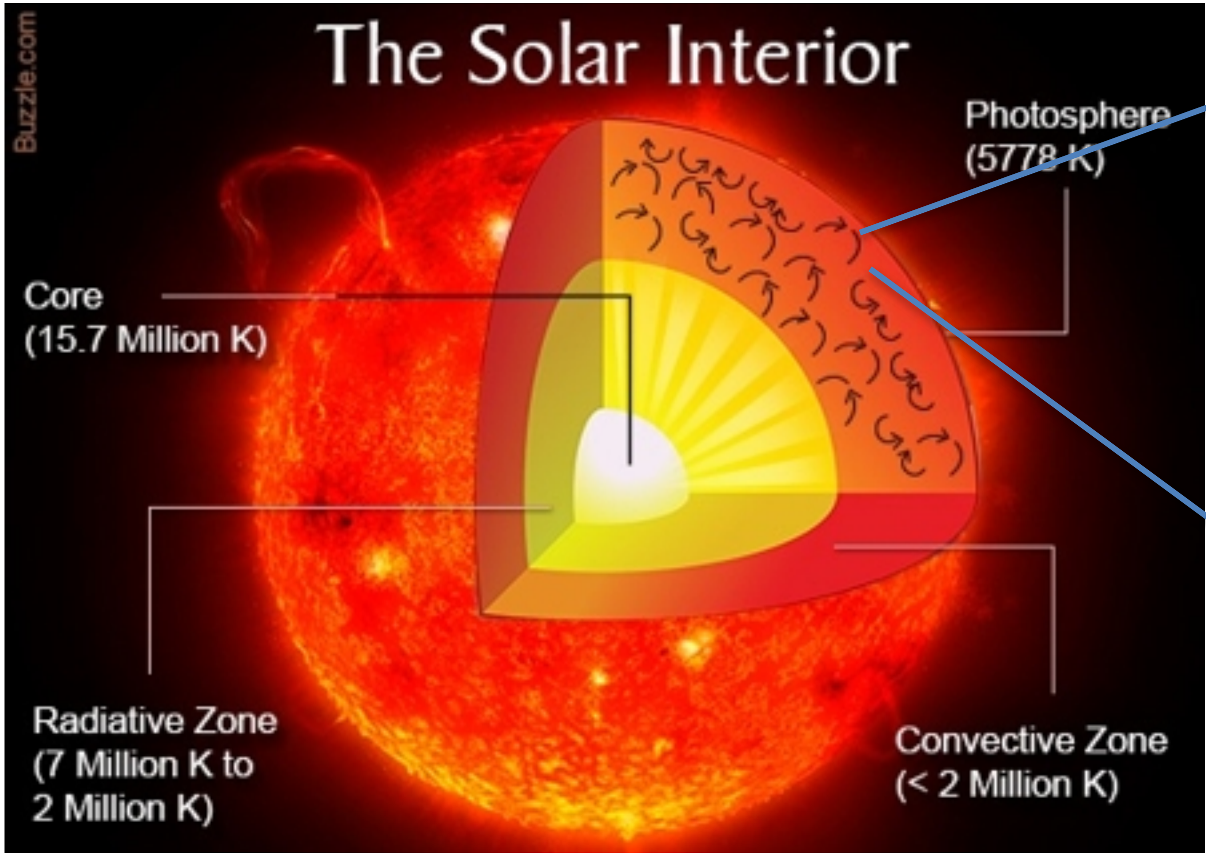
Donati et al 2008, Fares et al 2009, Mengel et al 2016: τ boo: 2 years

Petit et al 2009, Morgenthaler et al 2011: HD 190771 (complex variability)

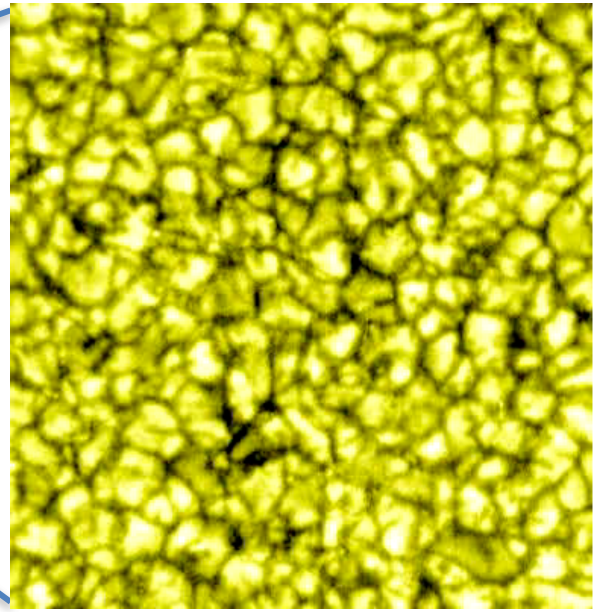
Garcia et al 2010, Salabert et al. 2016, Kiefer et al. 2017: asteroseismic signatures

Boro-Saika et al 2016: 61 Cyg A (solar twin): 14 years

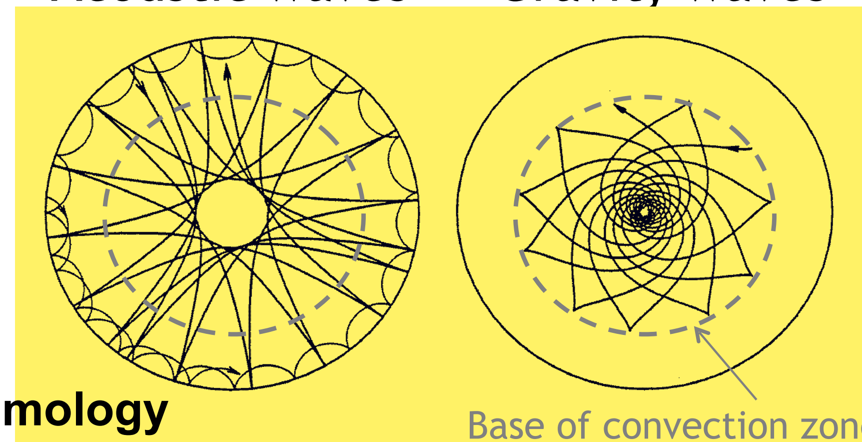
Solar interior and plasma flows



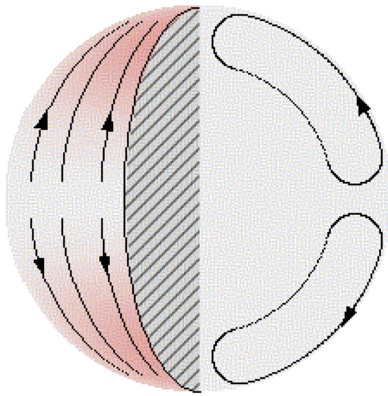
❑ Granulation (surface convection)



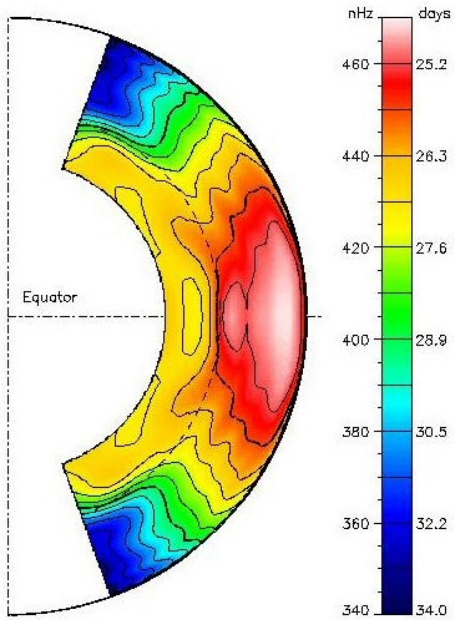
Acoustic waves Gravity waves



Helioseismology



❑ Meridional flow



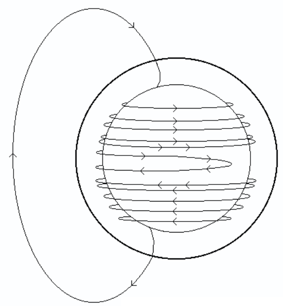
❑ Rotation

Theory: the induction equation (MHD)

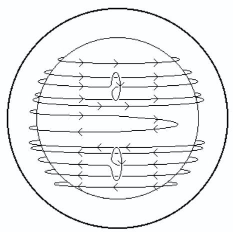
$$\frac{\partial \mathbf{B}}{\partial t} = \nabla \times (\mathbf{u} \times \mathbf{B}) - \nabla \times (\eta \nabla \times \mathbf{B})$$

$$\frac{\partial \mathbf{B}}{\partial t} = \underbrace{(\mathbf{B} \cdot \nabla)\mathbf{u}}_{Shearing\,of\,B} - \underbrace{(\mathbf{u} \cdot \nabla)\mathbf{B}}_{Advection\,of\,B} - \underbrace{\mathbf{B}(\nabla \cdot \mathbf{u})}_{Compressibility} - \underbrace{\nabla \times (\eta_m \nabla \times \mathbf{B})}_{Magnetic\,diffusion}$$

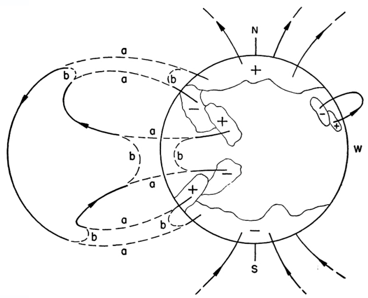
Source of magnetic field



√Ω-effect



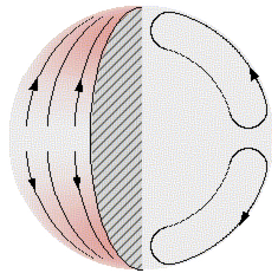
√α-effect



√ Babcock-Leighton source term

Transport of magnetic field

- √ Large-scale flows (meridional circulation)
- √ Downward pumping by penetrative convection



- √ Transport from the base of the convection zone to the surface

2D numerical simulations

Mean induction equation

- Simplified description of physical processes
- Fast and efficient tool
  Parametric studies

3D numerical simulations

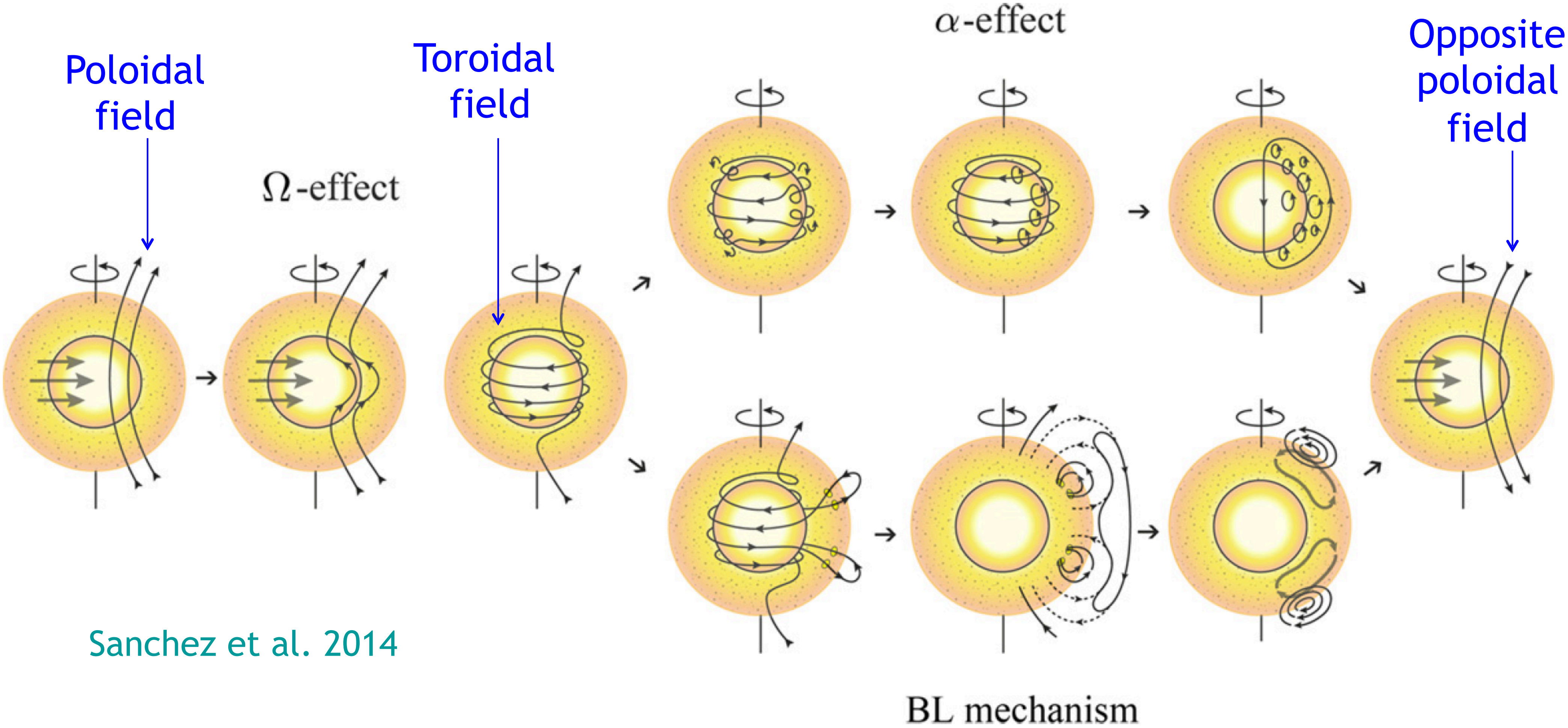
Full MHD equations

- Much more complex
- Self-consistent simulations

Kinematic dynamo ingredients

Basic solar dynamo ingredients (kinematic dynamo)

The solar dynamo: process through which the motions of a conducting fluid permanently regenerates a magnetic field

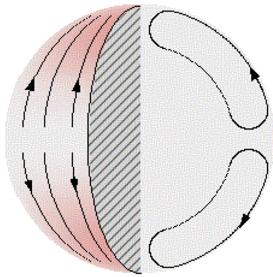


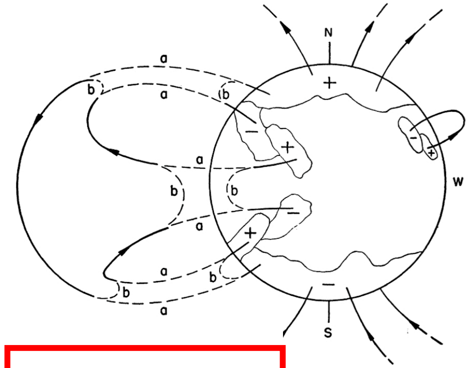
Sanchez et al. 2014

Babcock-Leighton

Magnetic cycles in 2D models

- Mean-field induction equation only
- Babcock-Leighton dynamo model
- 2 coupled PDEs

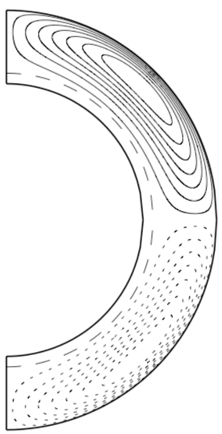




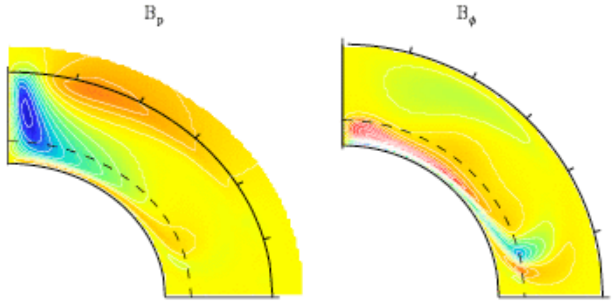
$$\frac{\partial A_\phi}{\partial t} = \frac{\eta}{\eta_t}\left(\nabla^2 - \frac{1}{\varpi^2}\right)A_\phi - R_e \frac{\mathbf{u}_p}{\varpi}\cdot\nabla(\varpi A_\phi) + C_\alpha \alpha B_\phi + C_s S(r,\theta,B_\phi)$$

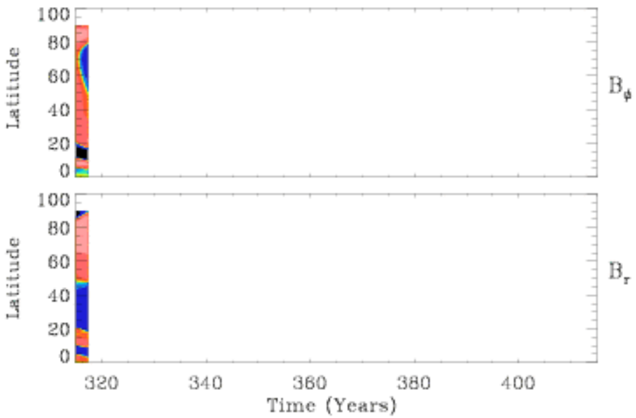
$$\frac{\partial B_\phi}{\partial t} = \frac{\eta}{\eta_t}\left(\nabla^2 - \frac{1}{\varpi^2}\right)B_\phi + \frac{1}{\varpi}\frac{\partial(\varpi B_\phi)}{\partial r}\frac{\partial(\eta/\eta_t)}{\partial r} - R_e \varpi \mathbf{u}_p \cdot \nabla\left(\frac{B_\phi}{\varpi}\right) - R_e B_\phi \nabla\cdot\mathbf{u}_p + C_\Omega \varpi(\nabla\times(\varpi A_\phi \hat{\mathbf{e}}_\phi))\cdot\nabla\Omega$$

Standard model:
single-celled
meridional
circulation



Dikpati & Charbonneau 1999
Jouve & Brun 2007





- Cyclic field
- Butterfly diagram

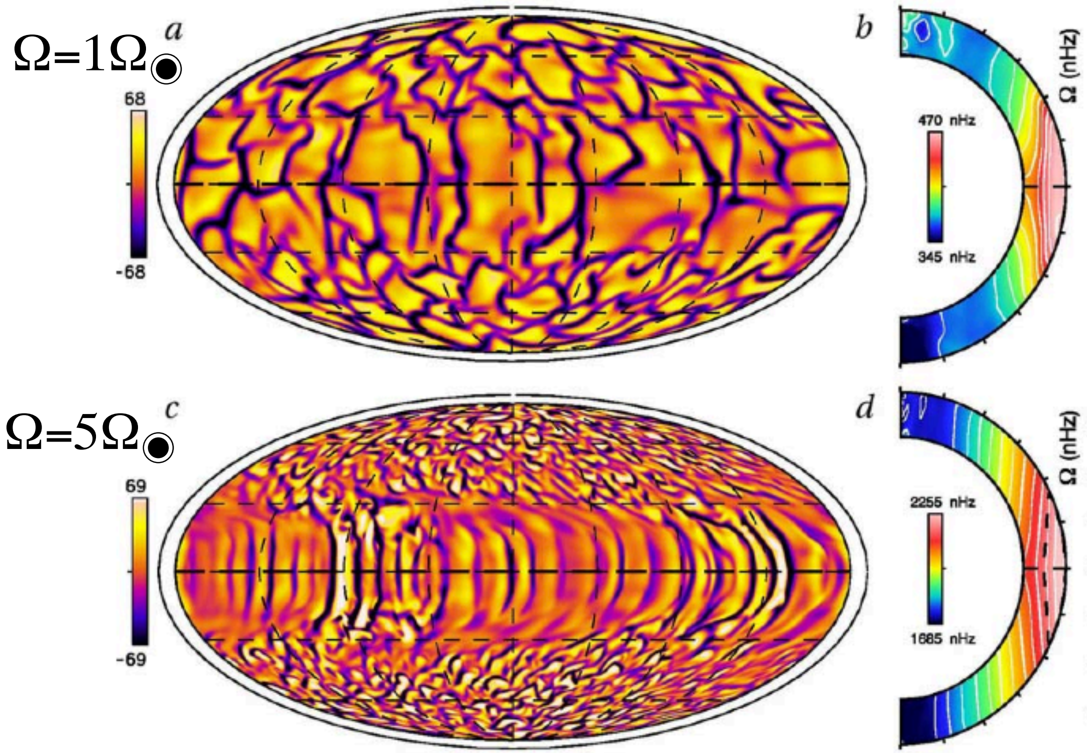
ok with observations

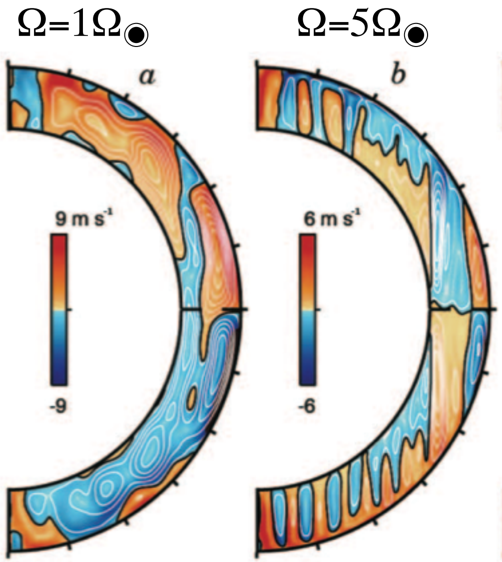
- Very strong dependence of cycle period on MC

ar $P_{cyc} = v_0^{-0.91} s_0^{-0.013} \eta^{-0.075} \Omega_0^{-0.014}$

Is this solar model applicable for rapidly-rotating solar-like stars?

Applying solar models to other stars: 2D models + prescriptions from 3D

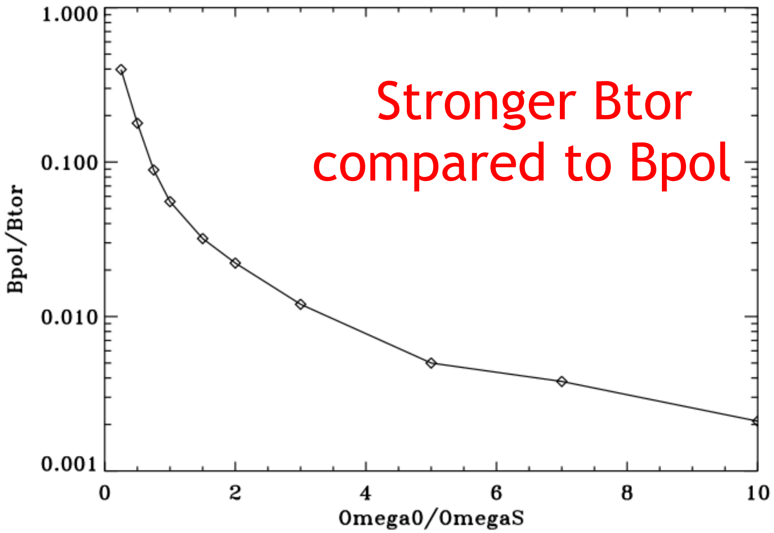


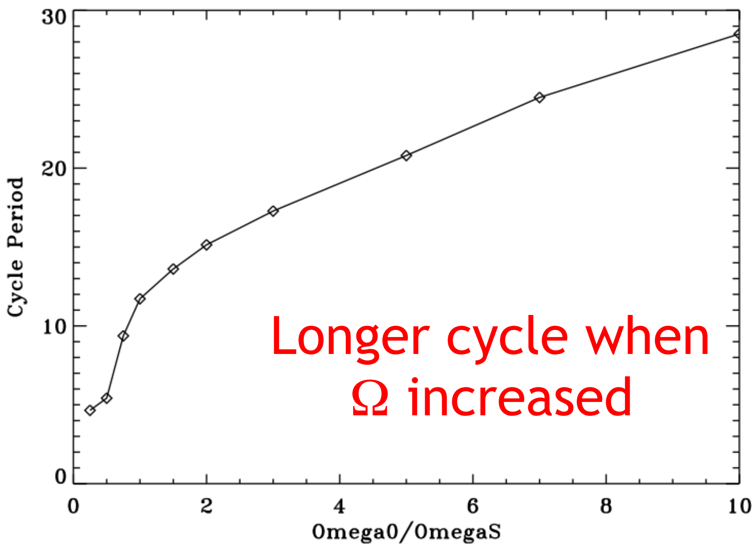


Prescriptions from Brown et al. 2008:

– Vp α $\Omega^{-0.9}$

– ΔΩ increases with Ω



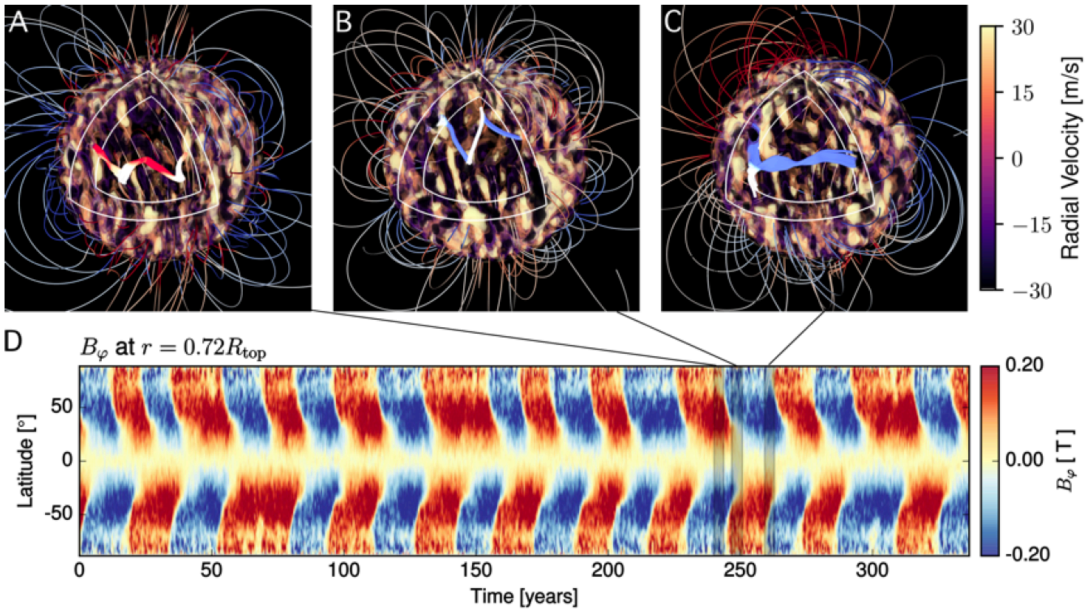


Jouve et al. 2010

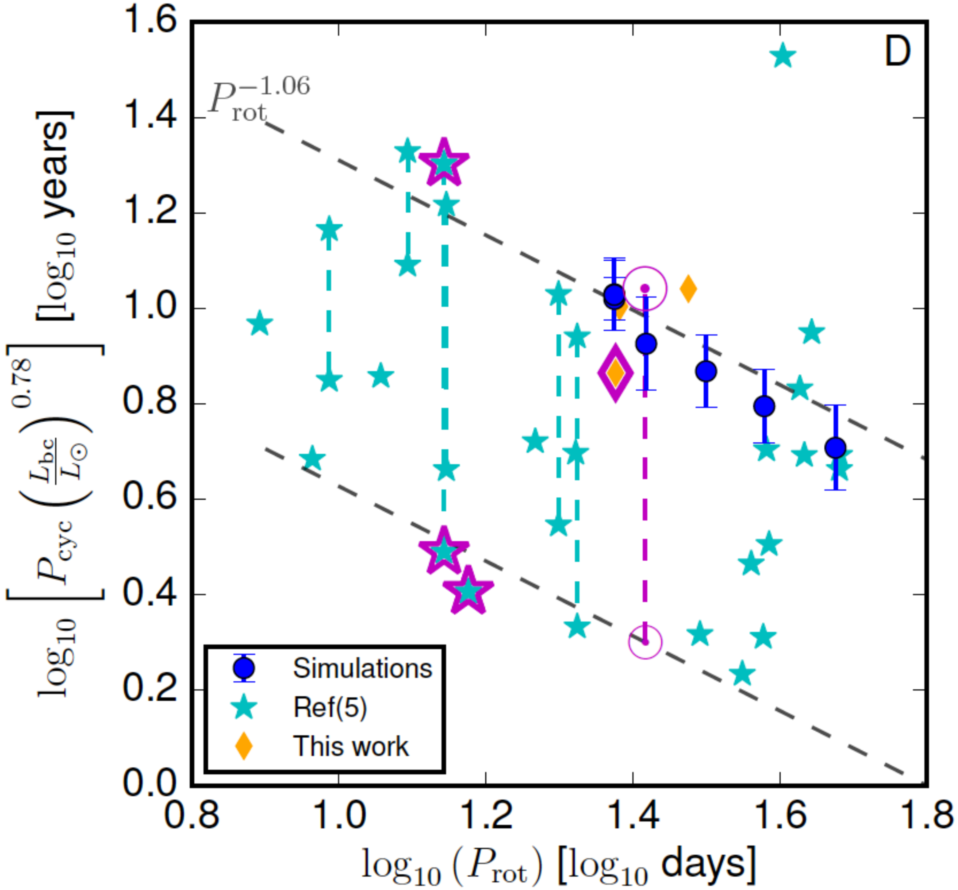
The MC profile needs to be **strongly** modified to reconcile models and observations

Applying solar models to other stars: 3D more realistic models

$\Omega=\Omega_\odot$

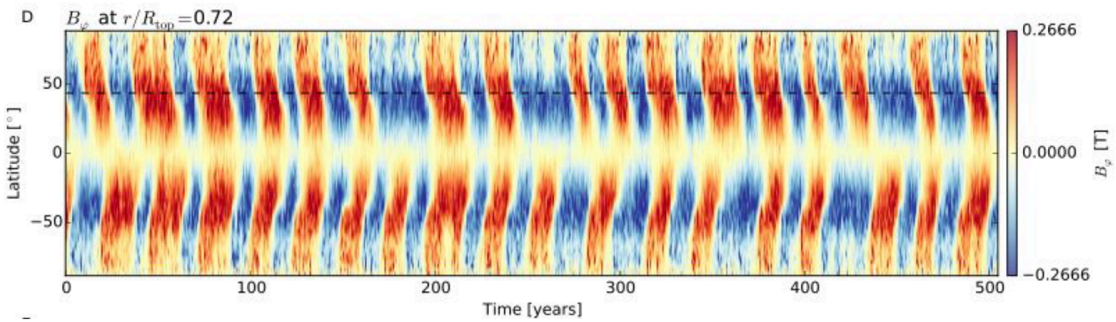


Strugarek et al. 2017



At fixed luminosity, slower rotation produces shorter magnetic cycles!

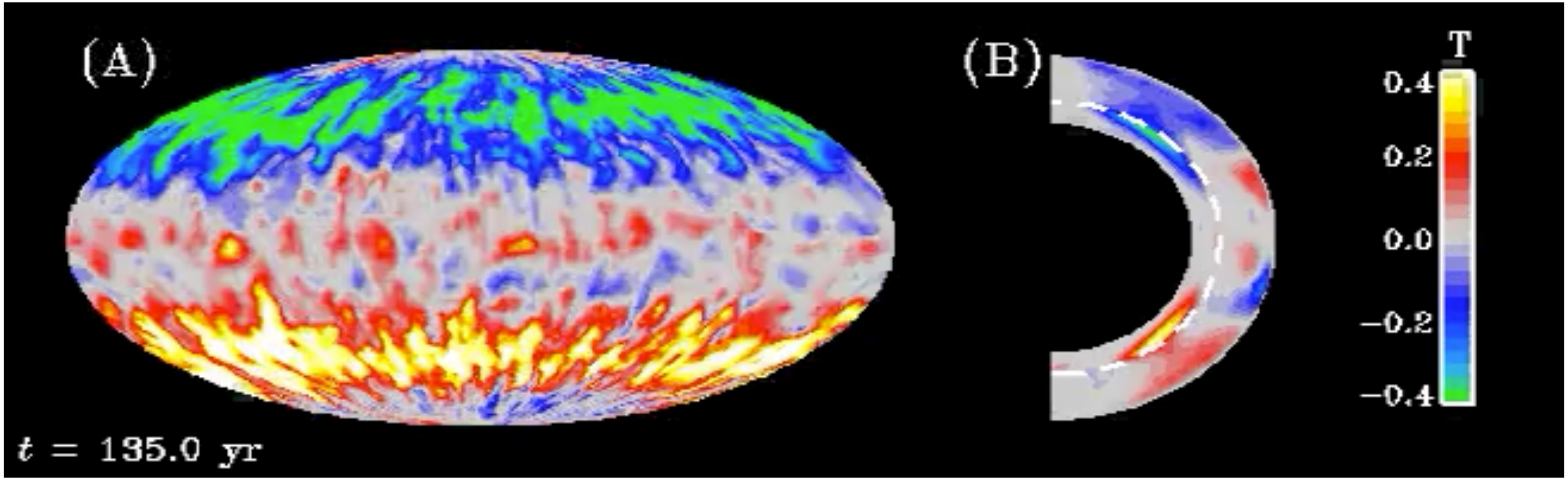
$\Omega=0.6\Omega_\odot$



- Corrected P_{cyc} scales with P_{rot}^{-1}
- **Not an $\alpha\Omega$ nor a BL dynamo**

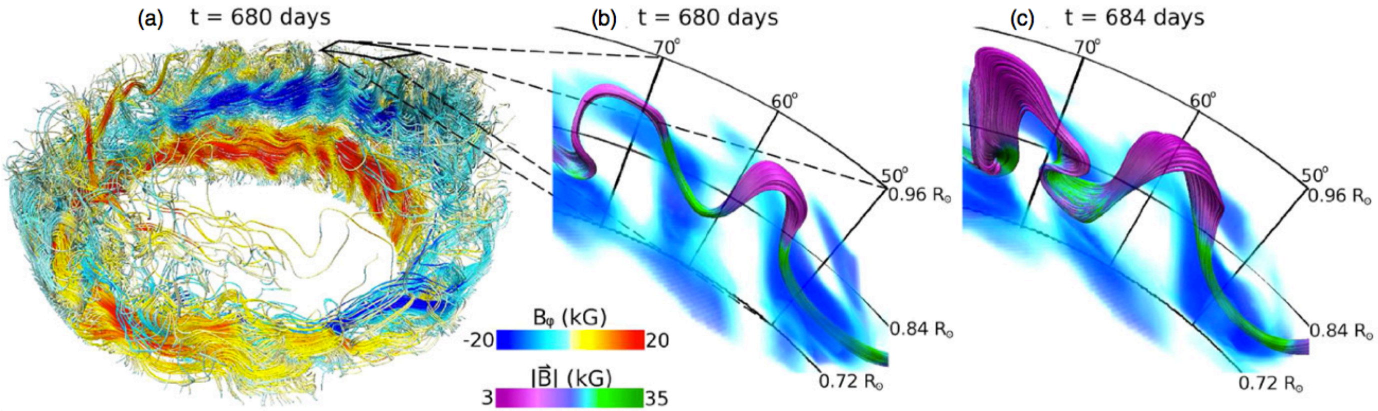
Spots in 3D models?

- 3D models produce magnetic cycles **without producing spots** and meridional circulation does not seem to set up the cycle period (Brown et al. 2011, Ghizaru et al. 2010, Nelson et al. 2013, Käpylä et al. 2013, Augustson et al. 2015, Hotta et al. 2016)



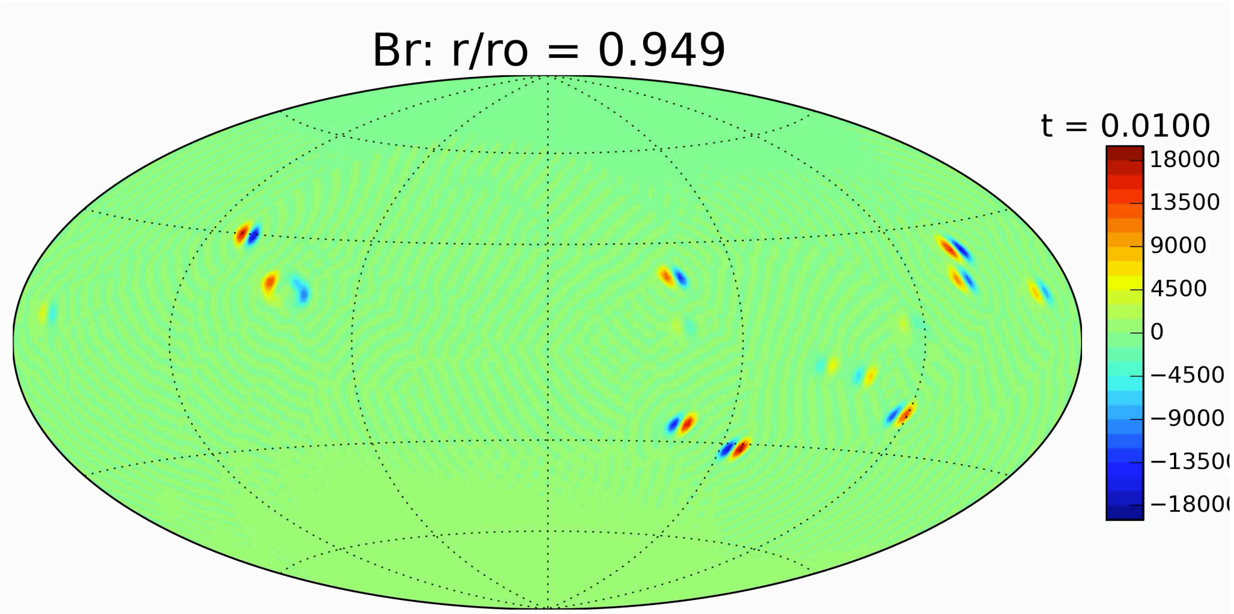
- Strong concentrations of toroidal field can still be built but buoyant structures do not make it to the top to produce spots!

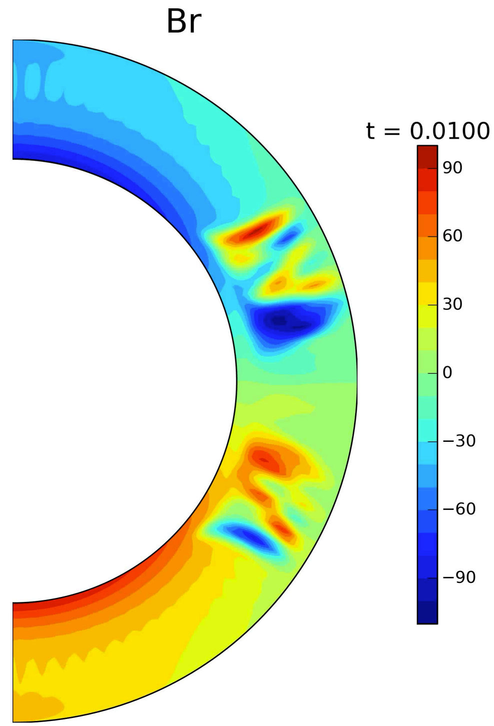
Nelson et al. (2011, 2014)

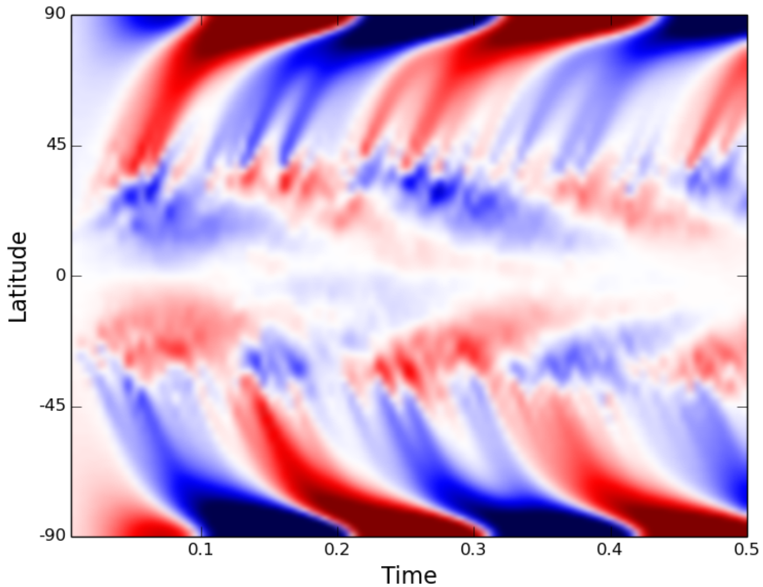


3D kinematic models: combining approaches

- Mean-field dynamo models + 3D flux emergence and spot formation (Yeates & Munoz Jaramillo 2013, Miesch & Dikpati 2014, Miesch & Teweldebirhan 2016, Kumar, Jouve, Pinto & Rouillard 2018)







Self-consistent butterfly diagrams

Kumar, Jouve, Pinto & Rouillard , 2018

Kumar, Jouve & Nandy, 2019

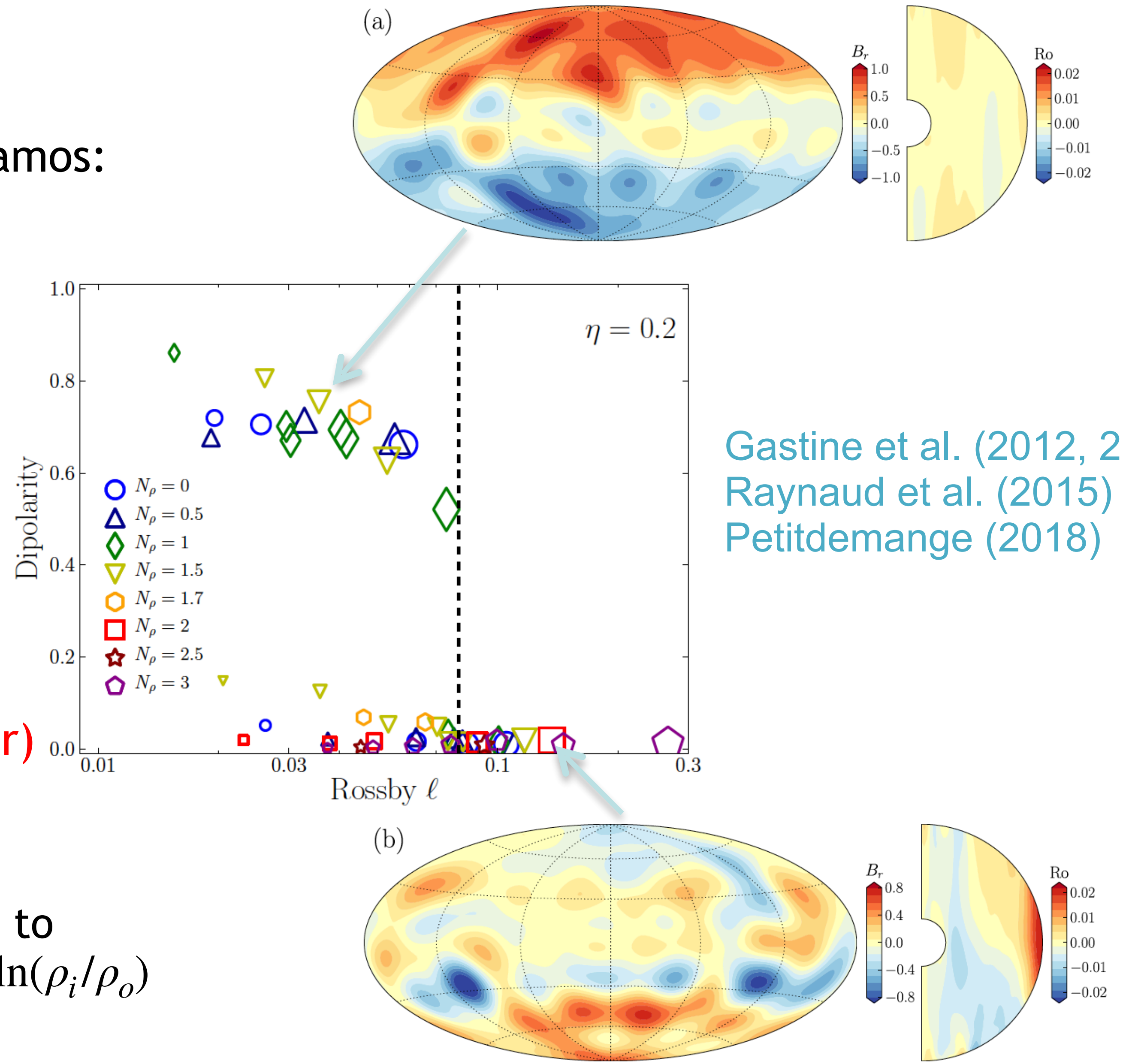
Magnetic topology in cool stars: influence of the Rossby number

- Change in Rossby
  Ro=inertia/Coriolis

(also seen in planetary dynamos: Christensen & Aubert 2006)

- Small Ro:
  Ordering role of Coriolis=dipolar (no role of shear)

- Large Ro:
  Inertia becomes dominant=multipolar (important role of shear)



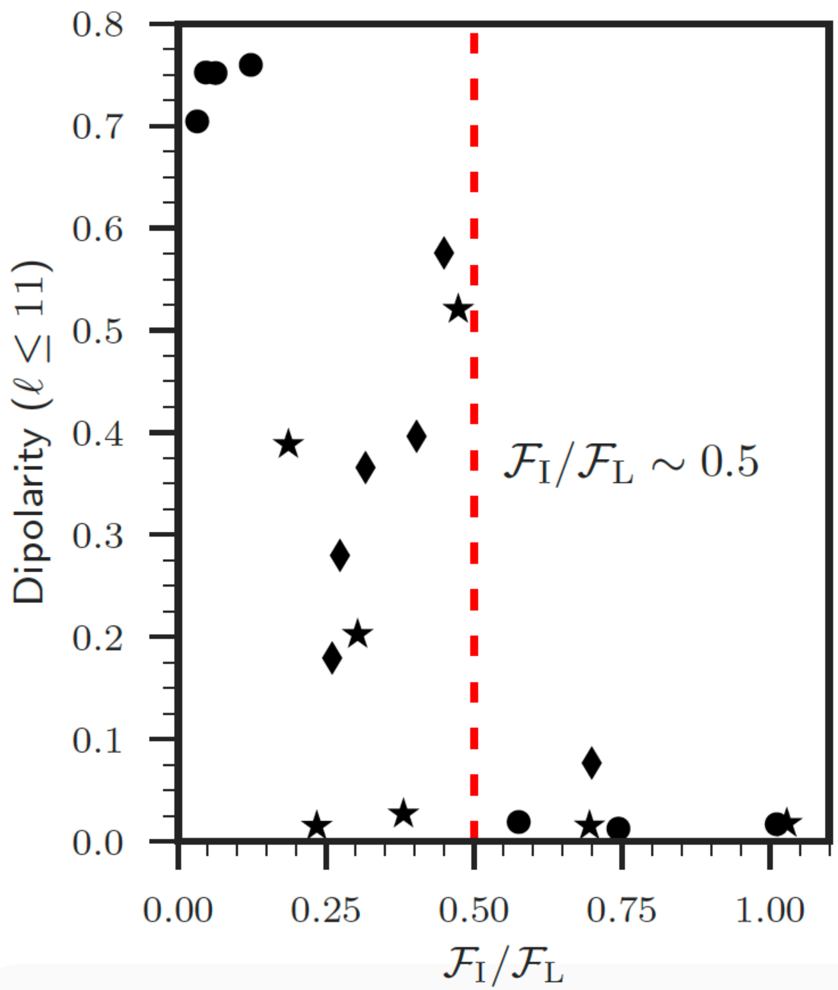
Gastine et al. (2012, 2013)
Raynaud et al. (2015)
Petitdemange (2018)

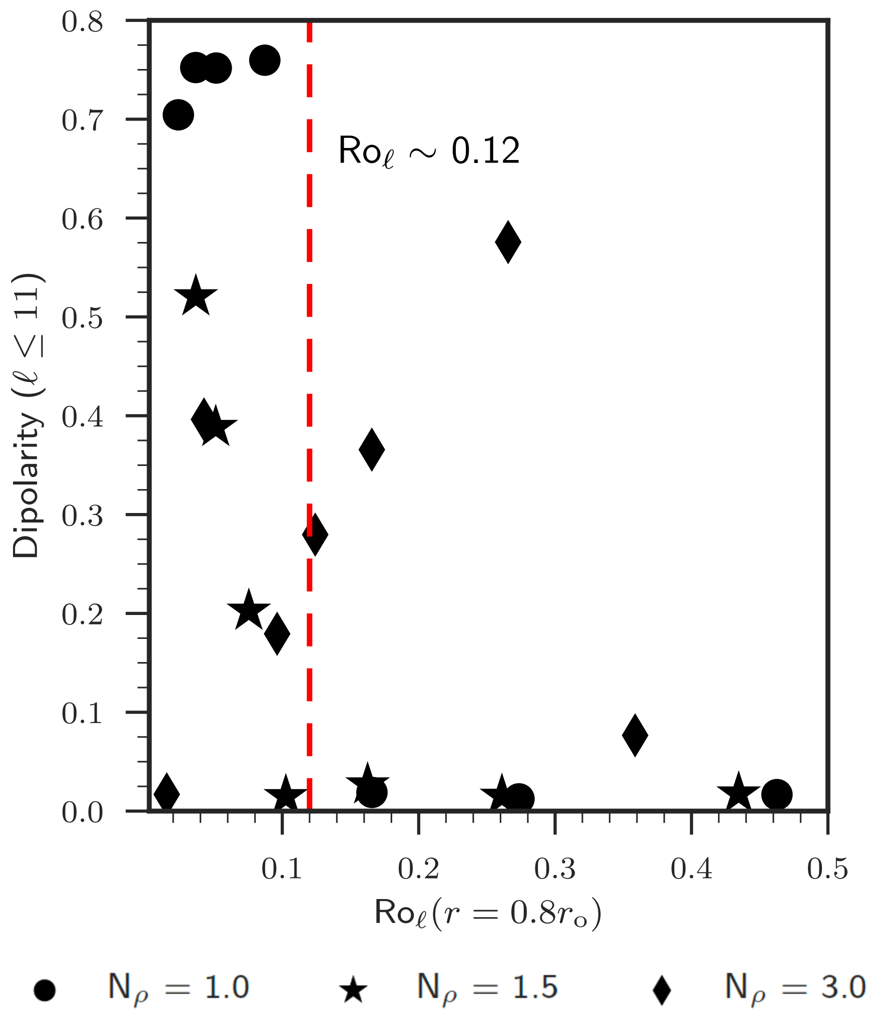
- Strong stratification leads to multipolar fields ? $N_\rho = \ln(\rho_i/\rho_o)$

Magnetic topology: influence of the Rossby number?

Zaire, Jouve & Gastine, in prep.

- ❑ With a forcing of convection slightly different (constant entropy gradient), dipole seems to survive at high Ro and high Nrho.

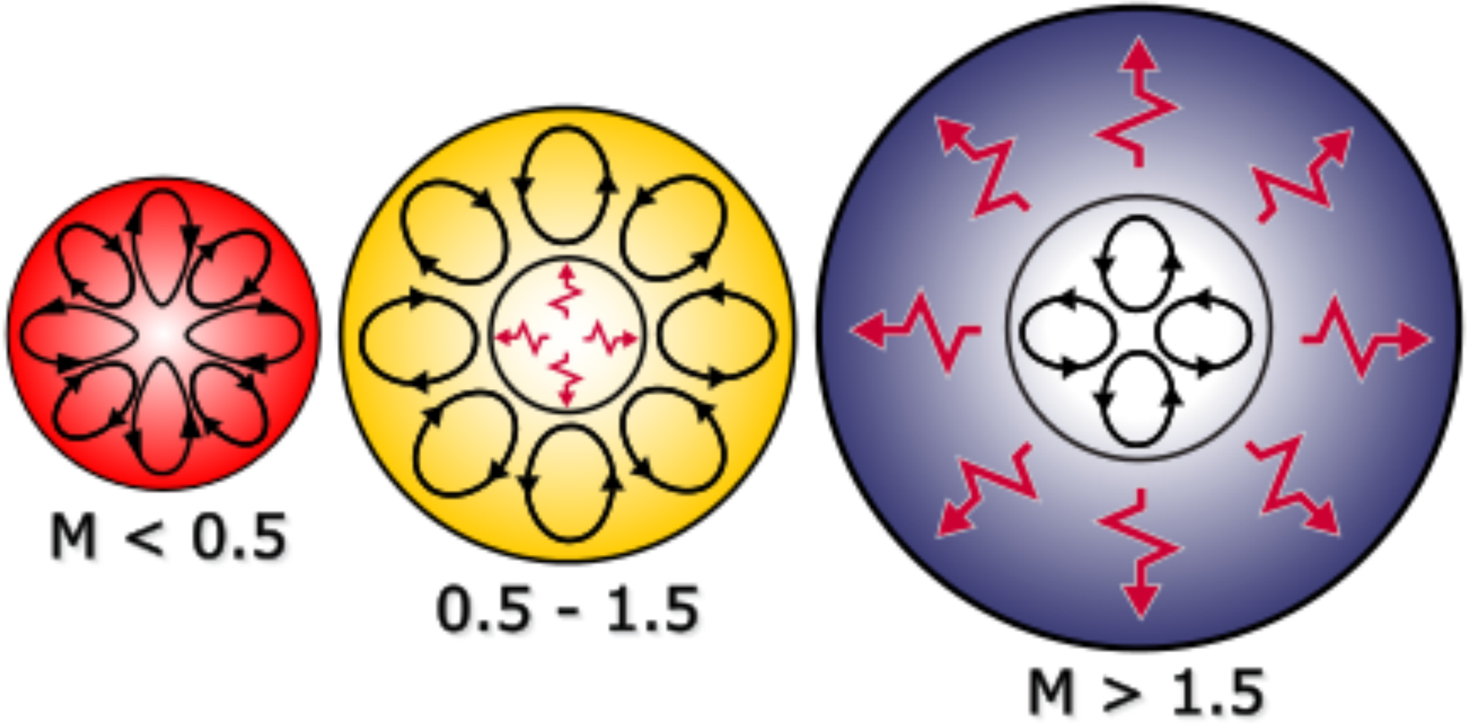




- ❑ The ratio of inertia to Lorentz forces (instead of inertia to Coriolis) seems to be a better indicator of dipolarity

Also seen in Boussinesq calculations of Menu et al. 2020

Magnetism of more massive stars



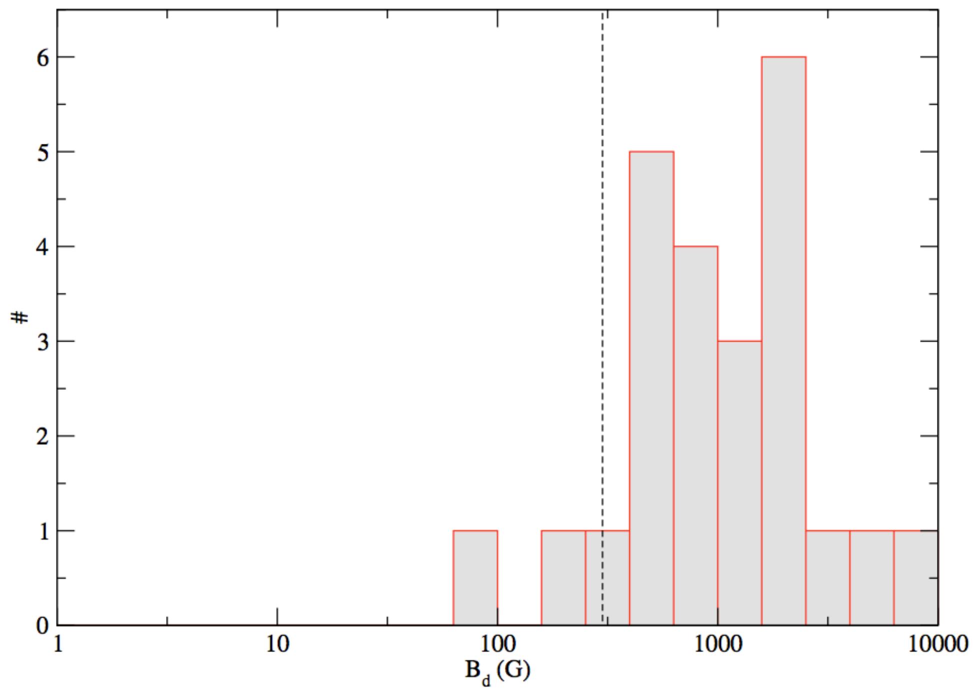
❑ **In more massive stars (with radiative envelopes)**

- Only 5 to 10% are found to possess a strong magnetic field, they are Ap/Bp stars
- Magnetic field starts to be detected on non-Ap stars: much weaker and complex

Ap/Bp stars magnetism

- Field configuration: inclined dipole (Lüftinger et al 2010)
- Field intensity: either strong fields (B > 300 G) or no field (Aurière et al. 2007)
- No detection on large sample of Am or HgMn stars (Aurière et al. 2010)

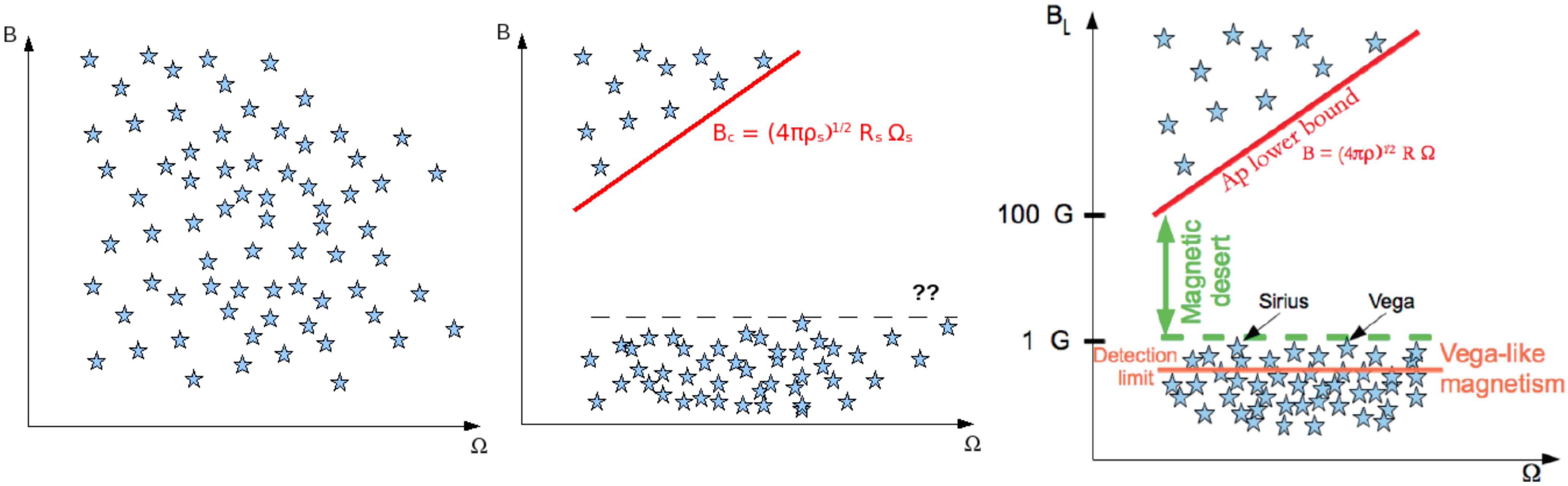
Musicos + NARVAL



- **Why such a threshold?** (Aurière et al. 2007)
  - Strong poloidal field → Differential rotation suppressed → Strong measured Bl
  - Weak poloidal field → Strong Bphi → Instabilities → Small horizontal scales Weak measured Bl
  - Structure dominated by toroidal field when $Max\left(\frac{B_\phi}{B_p}\right) \approx r\sin\theta\frac{\sqrt{4\pi\rho}\,\Omega}{B_p} \geq \alpha$
  - Possible instabilities for $Bp < Bc = r\sin\theta\sqrt{4\pi\rho}\Omega$

Theoretical argument

Courtesy: François Lignières

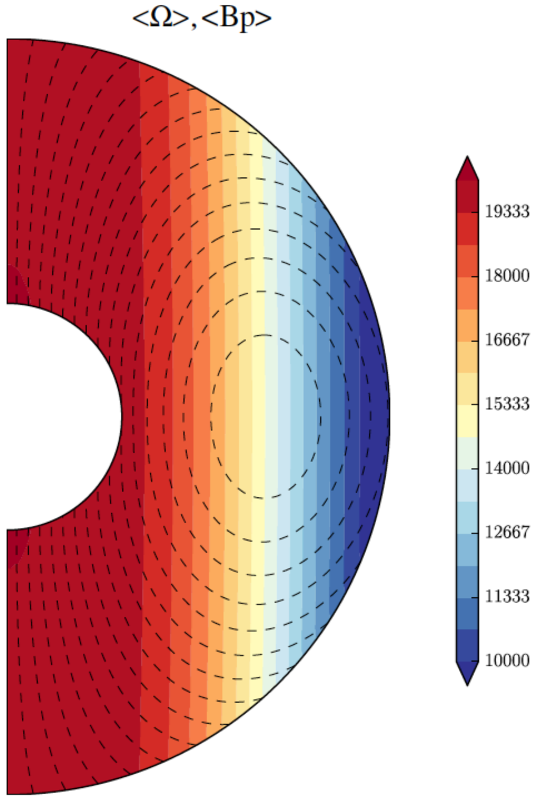


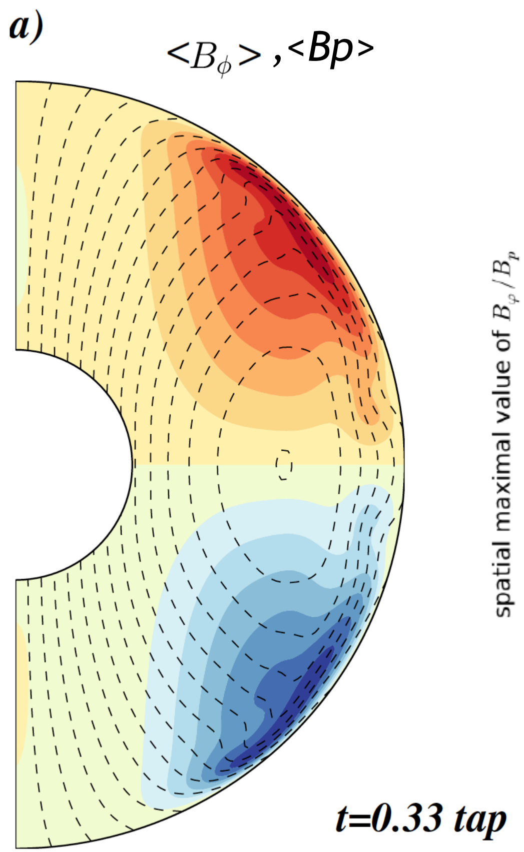
- Stellar formation: Fossil fields of variable intensities Bp, various rotation rates (and diff.rot.)
- For Bp < Bc → instabilities → Small longitudinal field (below detection limit).
- For Bp > Bc → Stable dipolar configurations (detected in Ap stars).

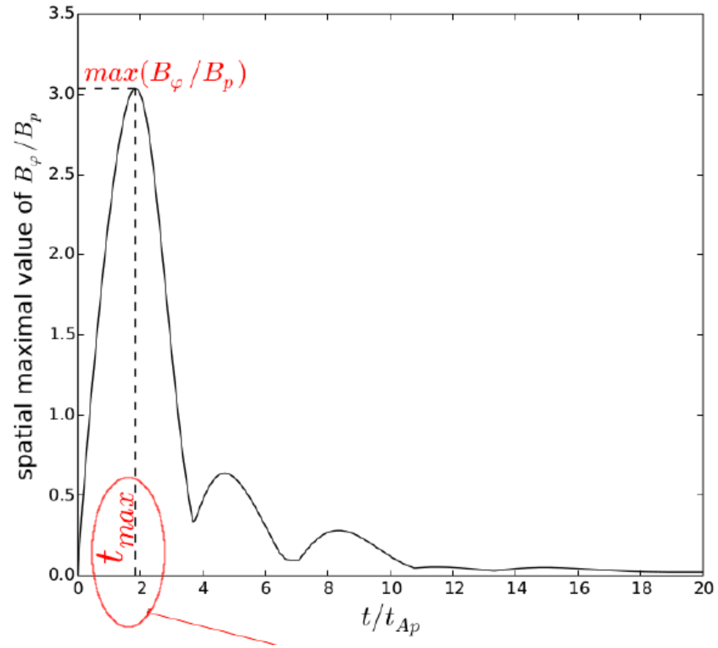
3D simulations to test theoretical scenario

MagIC Code
(pseudo- spectral 3D MHD)
Wicht 2002, Gastine & Wicht 2012

Gaurat et al. 2015







- Initial conditions:
Poloidal field (Lu)
Diff. rot.(Re)

$$L_u = \frac{B_0 d}{\sqrt{4\pi\rho}\eta}$$

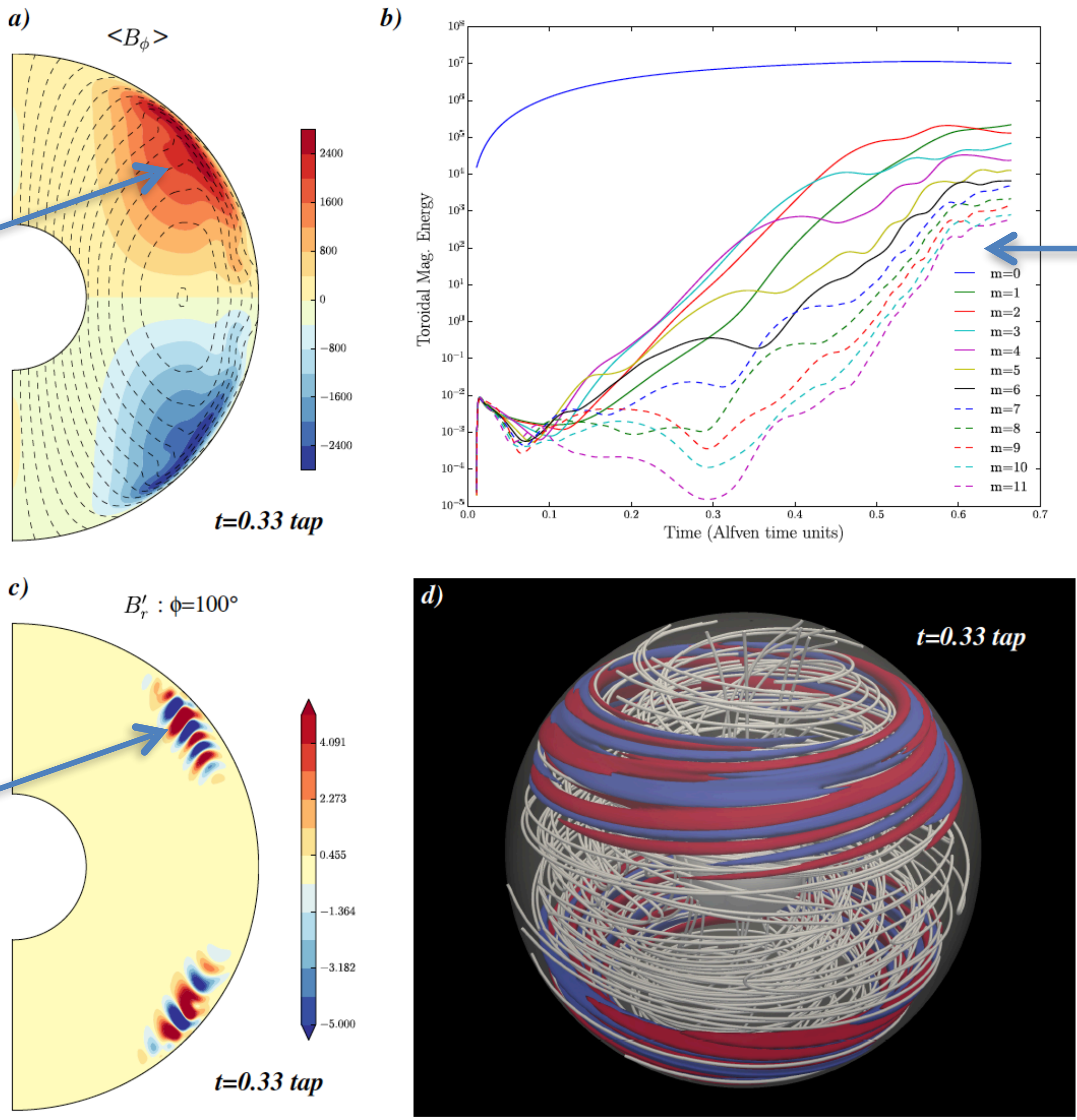
$$R_e = \frac{\Omega_0 d^2}{\nu}$$

$$Ro = \Delta\Omega/\Omega \approx 1$$

- A toroidal field is built which will then back-react on the differential rotation:
  - Is this configuration unstable?
  - Under which conditions is it triggered?
  - What are the consequences of this instability?

Evidence for a magnetorotational instability

❑ Typical case: Lu=60, Re=2 x 10^4: instability sets in around t=0.1 tap



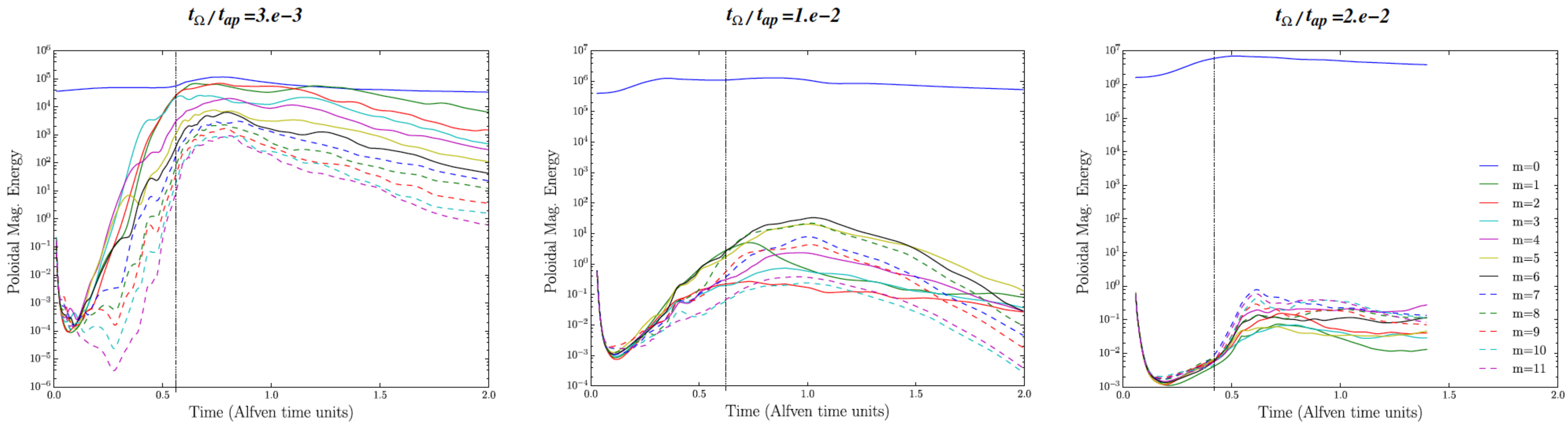
- Strong toroidal field, antisymmetric, close to the surface
- Instability around the regions of strong toroidal field
- Favored modes: m=4, 5 and 6

Jouve, Gastine Lignières 2015

What distinguishes stable from unstable cases?

- Background field evolves on poloidal Alfvén time scale t_{ap}
- Growth time of the MRI of the order of t_Ω ($\sigma = q\ \Omega/2$ with q around 1 here)

➡ Stable and unstable cases distinguished by the ratio t_Ω / t_{ap}



Effects of stable stratification

- Additional parameters:
  - degree of stratification measured by N/Ω
  - Ratio of viscosity to thermal diffusivity measured by Pr
  - In A stars, N/Ω is large (10^1-10^2) and Pr is small (10^{-6}-10^{-5})

- We expect strong effects of stable stratification:
  (less radial motions => more difficult for instabilities to develop)

- But a large thermal diffusion (small Pr) can help to reduce the effects of stratification:

  Thermal diffusion time $t_\kappa = L^2/\kappa$

  Buoyancy time $t_N = 1/N$

  => When $L^2 < Lc^2 = \kappa/N$ effects of stratification are reduced

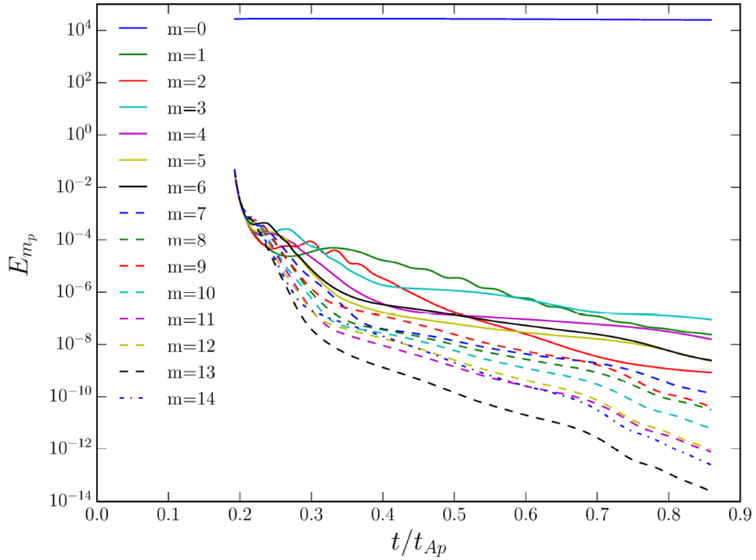
- In fact, our axisymmetric solutions depend only on Pr x $(N/\Omega)^2$ if

$$t_{Ap} >> t_\kappa >> t_\Omega >> t_N$$

Effects of stable stratification

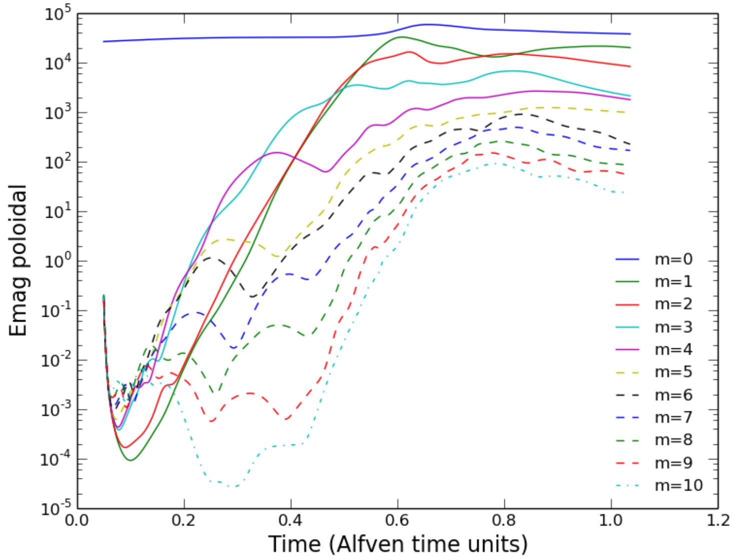
❑ N/Ω=5, Pr=1: instability is lost

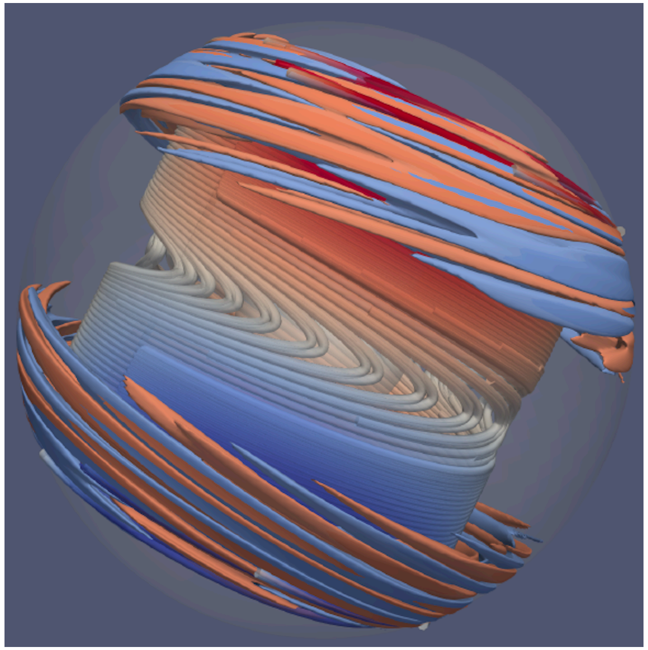
In this case, Lc is about 0.4% of the computational domain



❑ N/Ω=5, Pr=10^{-2}: instability is back

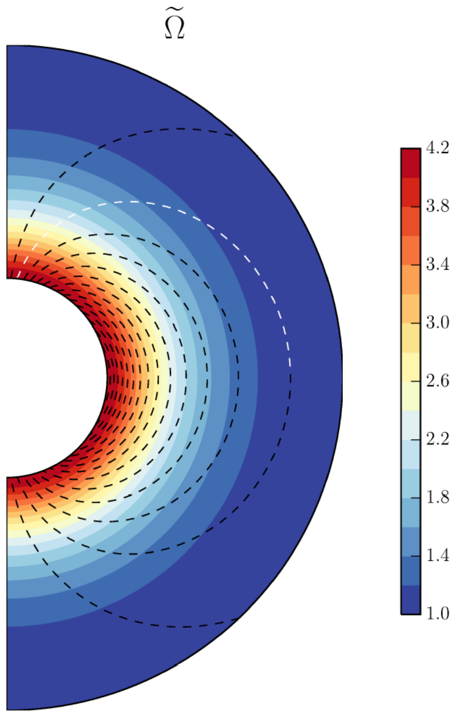
In this case, Lc is about 4% of the computational domain

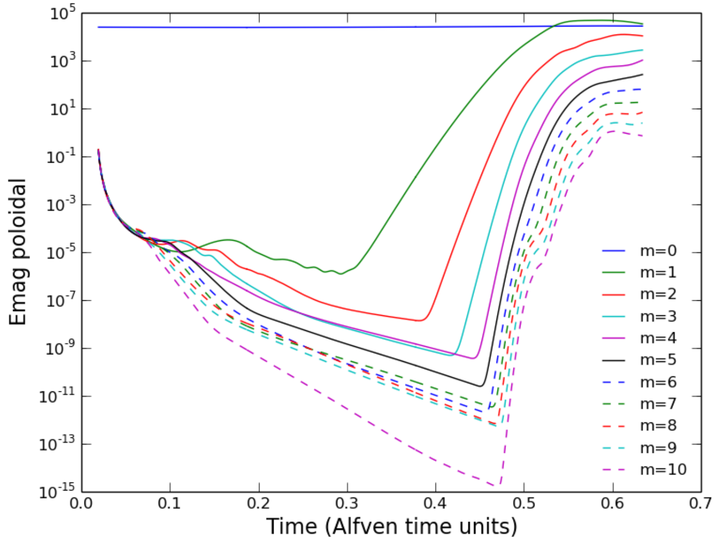




Effects of stable stratification

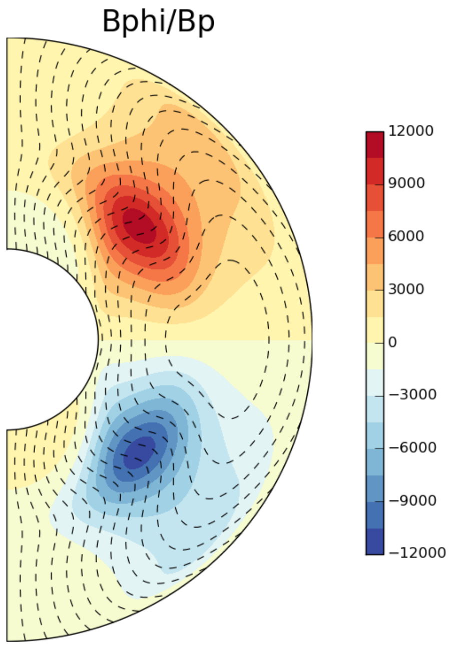
❑ Different initial differential rotation profile: radial instead of cylindrical

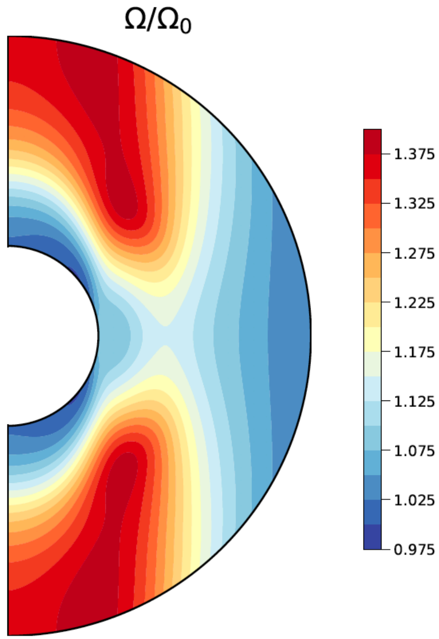


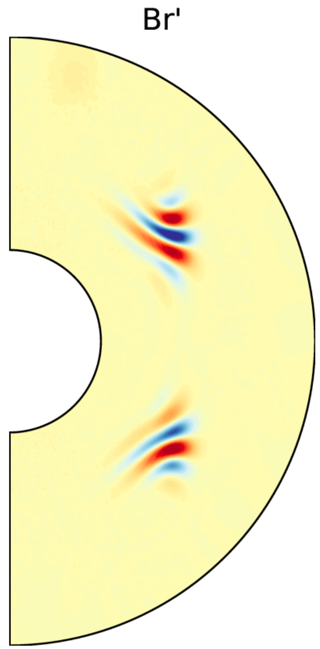


- m=1 is favored, appears after t_{Ap}
- Still a MRI but develops on latitudinal gradient of Ω

$N/\Omega=5$, $Pr=10^{-2}$





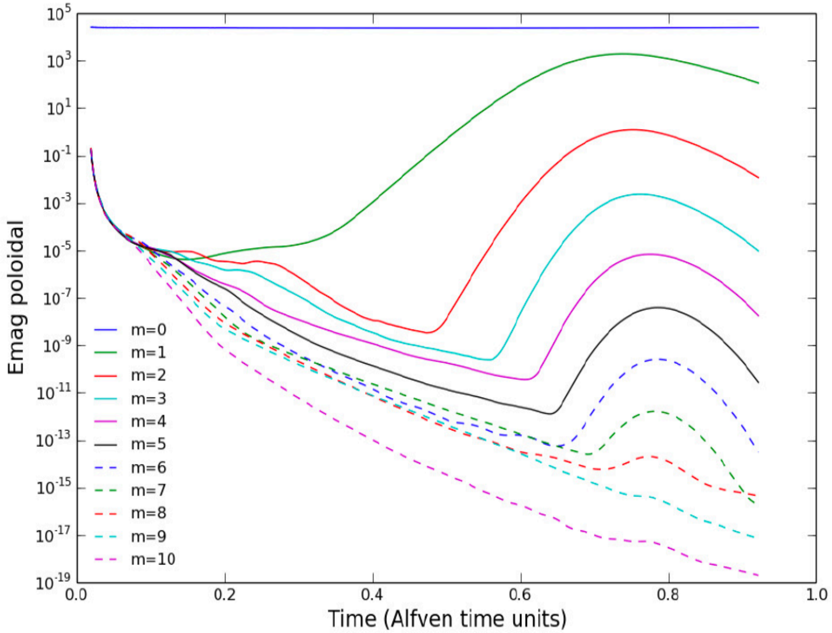


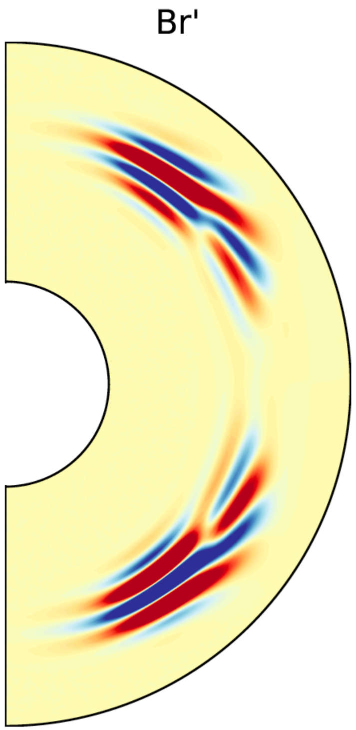
Effects of stable stratification

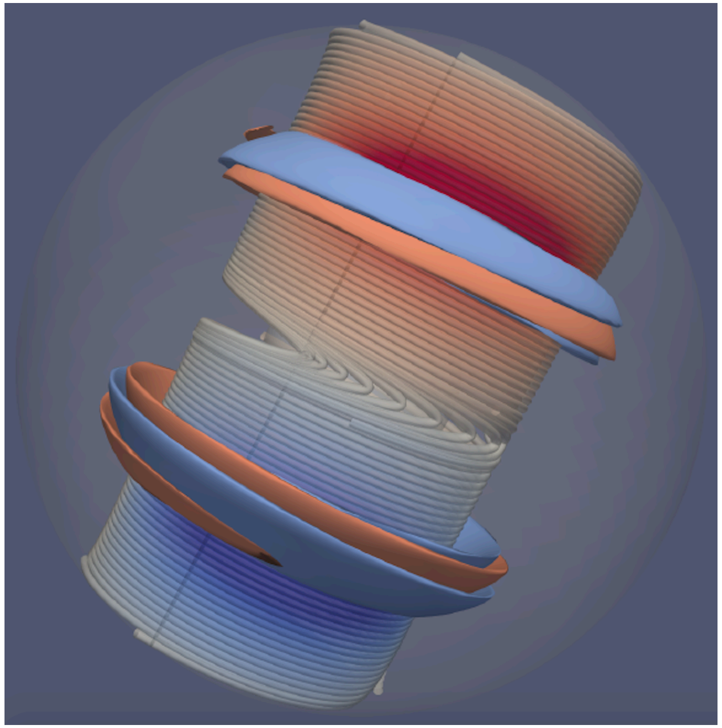
❑ Instability still present at Pr=1 !

- Stratification does not kill the instability
- Growth rate slightly reduced and unstable modes more horizontal

N/Ω=5, Pr=1



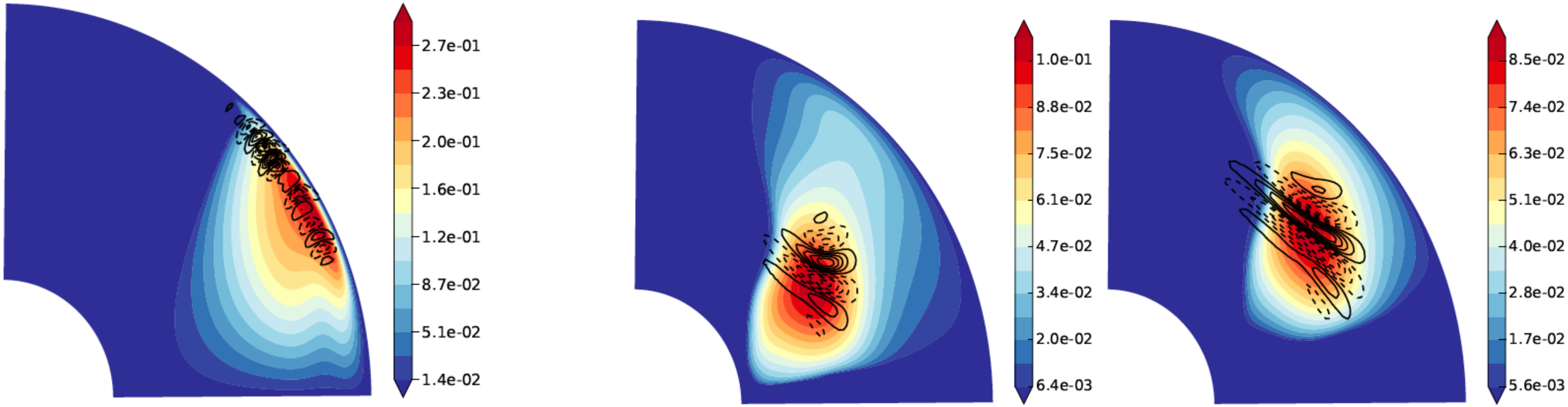




Differences radial/cylindrical diff. rot.

- In both cases, a local linear analysis gives good predictions of the location of the instability (and good estimate for growth rates): Acheson 78

Cylindrical, Pr=1e-2 — Radial, Pr=1e-2 — Radial, Pr=1



- Difference between 2 cases: origin of the instability

  - Radial gradient for cylindrical case and latitudinal gradient in the other
  - For a radial shear, vertical motions are needed to extract energy from shear
  - For a latitudinal shear, most unstable radial lengthscale can be independent of stratification (as in hydro situations, e.g. centrifugal instability with horizontal shear)

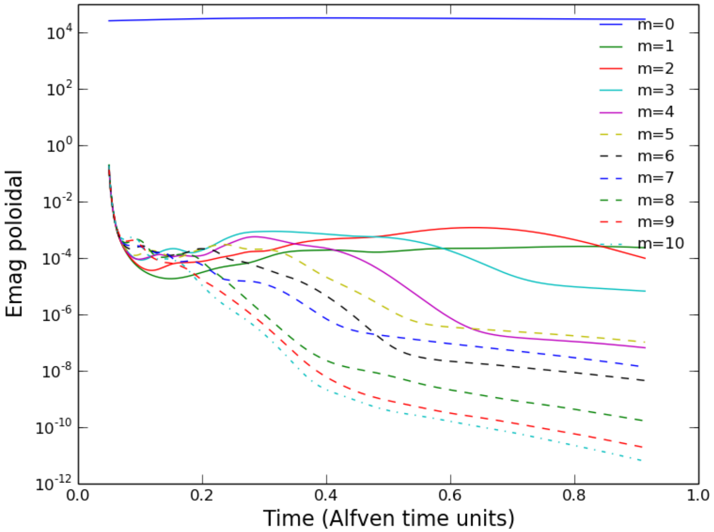
Effects of stable stratification

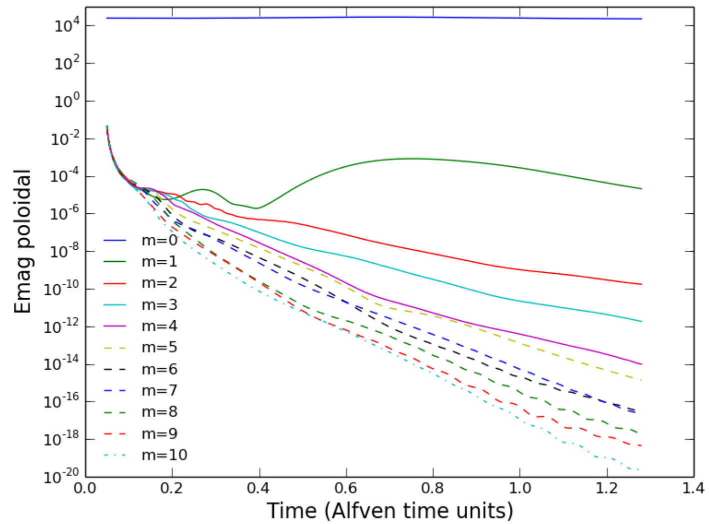
- Stable and unstable cases again distinguished by the ratio t_Ω/t_{ap}

$Pr\, N^2/\Omega^2 = 0.25$

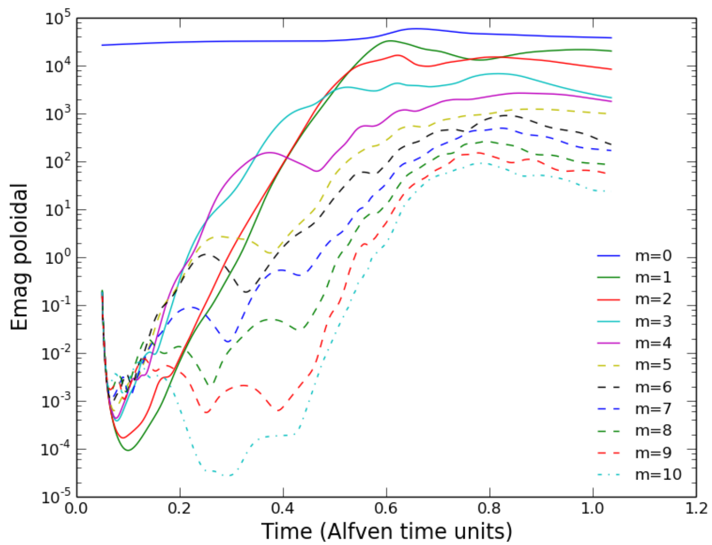
Cylindrical | **Radial**

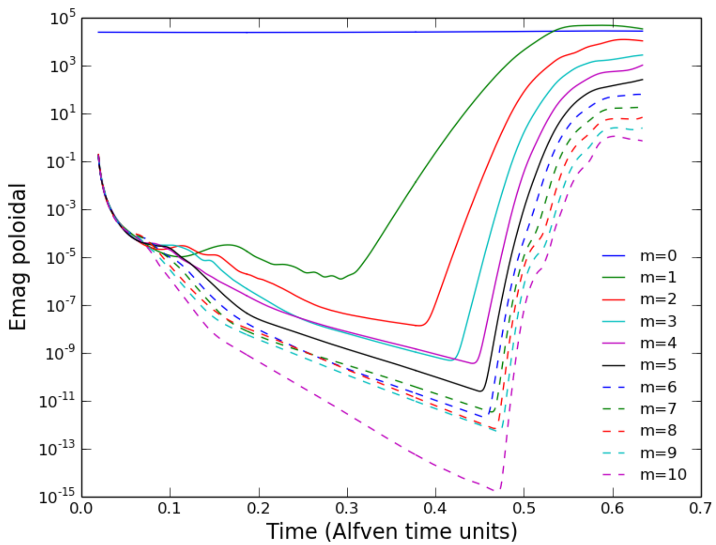
Large t_Ω/t_{ap}





Small t_Ω/t_{ap}



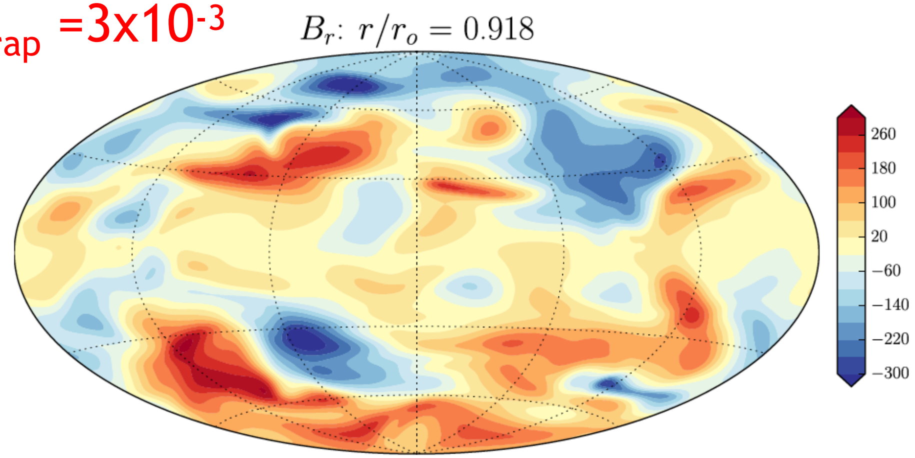


Application to A-type stars

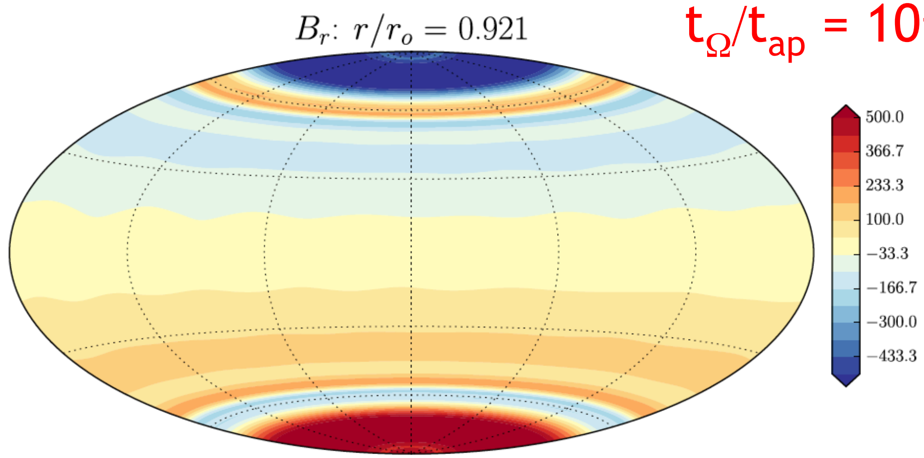
❑ Surface radial field: non-axisymmetric VS axisymmetric

- Unstratified cases

$t_\Omega/t_{ap} = 3\times10^{-3}$

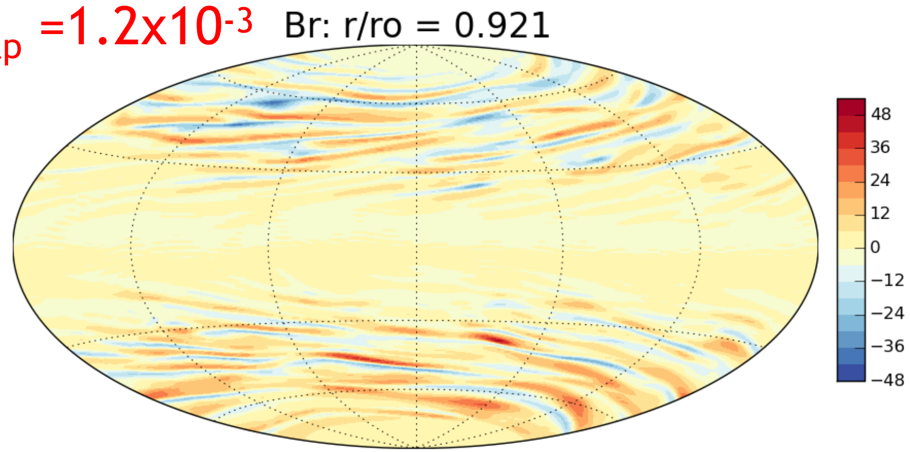


$t_\Omega/t_{ap} = 10^{-2}$

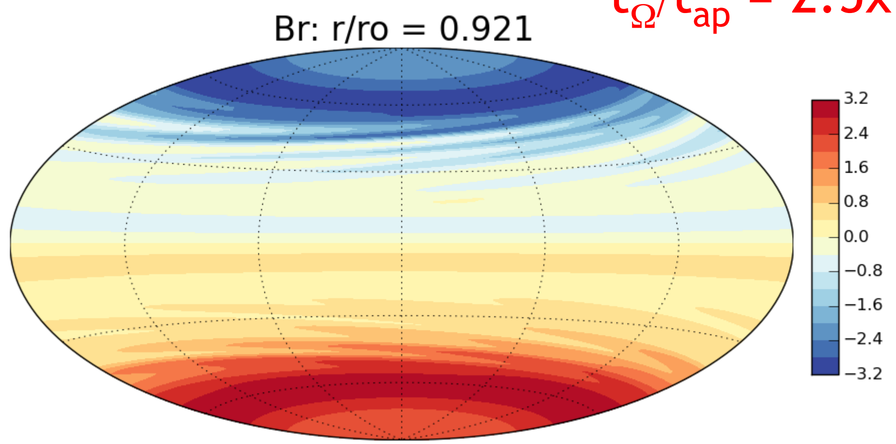


- Stratified cases

$t_\Omega/t_{ap} = 1.2\times10^{-3}$



$t_\Omega/t_{ap} = 2.5\times10^{-3}$



❑ Estimate of threshold field: $B_{0crit} = (10^{-2} - 10^{-3})\ \Omega_0 d \sqrt{\rho_0 \mu_0}$

❑ Proportionality with rotation rate also seen in observations (Lignières et al. 2014)

Conclusions

- Dynamo models of solar-like stars:

  - Magnetic cycle period VS rotation period: still unclear
  - What is missing in 3D models to actually produce spots?
  - Models commonly applied to the Sun challenged by other stars?

- Dynamo models of fully convective stars:

  - Change of geometry with Rossby number (or with internal structure?)
  - Bistable regime for late M
  - Dipoles could resist strong stratifications?

- Stellar radiative zones:

  - MRI unstable fields if t_{Ω}/t_{ap} weak enough
  - Strong modification of surface field in unstable cases
    => Dichotomy among A-type stars ?
  - Radiative zone dynamo?
  - Angular momentum transport by magnetic fields (red giants): ANR BEAMING

- More to come with SPIROU, Solar Orbiter, Parker Solar Probe, PLATO